The Private and Social Cost of Equity-Maximizing Debt Policy*

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Abstract

We analyze dynamic models of capital structure and asset prices in general equilibrium when managers follow equity maximizing debt policies. The model permits us to quantify both the private cost to firms of being unable to achieve firm-value maximization and the aggregate welfare cost due to increased default risk. Our setting encompasses time-varying economic conditions, as well as valuation under generalized preferences. The model provides an explanation for the procyclical use of unprotected debt: the cost to firms of the contracting friction increases in bad times. Likewise, expropriation incentives rise when firm valuations are low. Hence, without constraints on debt polices, leverage can be countercyclical, which amplifies the effect of excess debt on aggregate risk. In our baseline calibration, the social cost of unprotected debt is equivalent to 25% of the representative agent's income. This cost can exceed the private cost by a factor of two or more, and excess cyclicality accounts for half of the social cost.

Keywords: capital structure, covenant valuation, general equilibrium, social cost of contracting frictions

JEL CLASSIFICATIONS: E21, E32, G12, G32

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1. Introduction

This paper examines the valuation implications of noncontractible debt-equity conflicts in partial and general equilibrium. In particular, we compute and contrast the private and social costs that follow from managers being free to continually adjust debt and default policy to maximize equity value, rather than firm value. We contribute to an emerging literature that seeks to quantify the implications of contracting frictions both at the firm and aggregate level.

Our study examines one particular friction in a tractable and transparent framework. It has long been well understood that equity holders have incentives to increase firm debt to the detriment of creditors, possibly even to the point of lowering overall firm value. The prevalence of unprotected debt tells us that the ability to restrain debt issuance by managers (e.g., through covenants) is imperfect or costly to implement. This continues to be a timely and relevant topic, as leveraged loans and "cov-lite" credit have helped to drive U.S. corporate debt to its highest fraction of GDP in the post-War era.[1] Should policy makers be concerned? Do equity-maximizing capital structure decisions, in fact, have quantitatively important implications for aggregate risk and welfare? The current work presents (to our knowledge) the first investigation of this subject.

To this end, we study the problem of a firm with perpetual debt that can be costlessly adjusted, and cash flows that follow a pure jump process whose jumps are systematic events with idiosyncratic (firm-specific) magnitudes. We assume firms follow instantaneous linear adjustment policies. For the case of firm-value maximization (or full commitment), the result is a natural generalization of the seminal Goldstein, Ju, and Leland (2001) capital structure model with debt scaling up or down as a constant fraction of cash flow. To model equity maximization, we stay within the same class of policies but impose a consistency (or fixed point) condition, namely, that managers should have no

[1] At the end of 2018, 79% of the \$ 1.17 trillion outstanding leveraged loans in the U.S. were classified by S&P as cov-lite. Cross (2019).

local incentive to deviate within that class. We show in the appendix that this condition can be an equilibrium outcome of a game between equity-maximizing managers and the market. The fixed point condition is intuitive and yields unique stationary homogeneous solutions for a wide range of relevant parameterizations.

The model also allows firm and macroeconomic parameters to change over time. In addition, our analysis allows for generalized preferences. This enables us to quantify, in a fully-specified asset-pricing framework, the valuation consequences of equity maximizing debt policy.

Most importantly, the homogeneity of our framework permits aggregation across firms, meaning that we are able to close the model, and thus capture the feedback from corporate decisions to marginal utility and discount rates, which, in turn, affect the optimal policies. While absent in most treatments of contracting problems, these effects can be important because (as we show) lower asset values can increase the expropriation incentives of firms. Solving the general equilibrium allows us to quantify the social cost and the degree of externality stemming from the increased credit risk that is a consequence of equity maximization. In addition to the unconditional effects of increased debt levels, our solutions also permit us to offer insight into the implications for debt dynamics. In the model, firms have private incentives to shift to debt financing in bad times. Countercylicality induces further welfare costs, which we quantify.

While the general equilibrium analysis is the principal contribution of our work, the firm-level building blocks also contribute new insights to the line of research studying the determinants and value consequences of covenants in debt contracts. (See Roberts and Sufi (2009) for a survey of this research.) Our model offers a theory of how incentives to achieve debt protection vary over time (within a firm) and across firms with different characteristics, and thus when and why covenant use and strictness may vary.

Our main results are as follows.

In common with previous studies, our model produces greater leverage due to equity holders' expropriation incentives. Comparison of the equity-maximization decision with firm value-maximization reveals that the distortion incentives scale inversely with firm valuation multiples (e.g., Tobin's Q). The primary dimensions of the trade-off problem (default costs and tax benefits) scale with firm value, but the marginal expropriation benefit does not. To investigate magnitudes, we compute the loss of firm value in numerical examples. The surplus that accrues to firms that can credibly pledge to maximize firm value (e.g., through covenants) can be economically large, and agrees reasonably well with estimates in Matvos (2013) and Green (2019).

Next, we document that this surplus is countercyclical. This parallels our first finding. Intuitively, in good times when firm values are higher, the expropriation benefit from marginal debt issuance is relatively small compared with the marginal impacts from additional debt on tax benefit and default costs. This observation provides an explanation for the empirically observed procyclical use of unprotected debt.

Because expropriation incentives rise when firm valuations are low, under equity maximization, firms sell relatively more debt in bad times. The first-order trade-off incentive of higher expected default costs almost always implies that value-maximizing firms sharply reduce leverage in bad times. The same applies with equity maximization, typically also producing optimal decreases in leverage. But these decreases are significantly smaller, and optimal policies may actually feature increasing debt issuance.[2]

Since the countercyclical expropriation incentives are a novel prediction and are also a key mechanism in general equilibrium, we provide supportive empirical evidence. In an appendix, we present panel regressions, using a firm's debt covenant strictness as (an inverse) proxy for its managers' freedom to pursue equity-maximizing capital structure. The results show that firms with more unprotected debt increase their borrowing when economic growth is low or when valuation multiples are small. Back-of-the-envelope

[2] A similar result on countercyclical expropriation incentives in the context of (unprotected) sovereign borrowing is shown in Arellano and Ramanarayanan (2012).

calculations suggest that if all debt were protected, the effect on aggregate leverage variability would be non-negligible.

Solving the model in general equilibrium, the overall higher leverage under equity maximization imposes real costs due to higher credit risk: default rates and output volatility are higher. In our baseline calibration, the social cost of unprotected debt is equivalent to 25% of the representative agent's income. This cost can exceed the private cost by a factor of two or more, emphasizing the scale of the externality. In addition to the level of excess debt, the increased countercyclicality of debt under equity maximization provides a further level of amplification. This dynamic effect from time-varying economic conditions accounts for half of the social cost in the calibrated model.

The model economy omits important channels – including investment and labor market decisions – through which financial policies can have real effects.[3] While recognizing the limitations of drawing precise numerical conclusions in such a setting, our results are qualitatively robust across a range of parameterizations. In particular, the findings with respect to the degree of externality do not depend strongly on the preference specification.

1.1 Relation to Literature

Our paper extends the tractable stochastic environment introduced in Johnson (2020). That work is also concerned with the social cost of corporate debt decisions, and analyzes a general equilibrium trade-off model versus an alternative formulation embedding moral hazard. The form of the debt contract in that paper precludes expropriation, however. So it does not address the potential real effects studied here.

A related strand of literature, in a partial equilibrium setting, examines firm behavior when equity holders cannot commit to firm-value maximization.[4] This problem in debt

[3]We also acknowledge that, since tax shields drive debt incentives, a complete welfare analysis should take into account effects of commitment versus noncommitment that flow through government fiscal policy.

[4]This noncontractibility leads to equity maximization. We use the terms commitment and no-

policy decisions was first formally modeled in Bizer and DeMarzo (1992) in the context of sequential borrowing. Brunnermeier and Oehmke (2013) show that lack of commitment in maturity structure leads to shorter average maturity of debt because a firm has an incentive to shorten one creditor's debt contract to expropriate other creditors. Recently, Besbes, Iancu, and Trichakis (2018) and Iancu, Trichakis, and Tsoukalas (2017) have shown that debt expropriation incentives can lead equity-maximizing managers to distort inventory and product pricing decisions.

Focusing specifically on debt quantities, Admati, DeMarzo, Hellwig, and Pfleiderer (2018) have shown, in a two-period setting as well as numerically illustrating in a dynamic setting, that the inability to commit to firm value maximization has profound impact on firms' leverage dynamics. Without commitment, shareholders always want to increase debt following increases in cash flow. Equity-maximizing managers are never willing to reduce debt no matter how large the potential increase in overall firm value would be – the so-called "leverage ratchet effect.". The mechanism is that existing creditors capture all the firm value enhancement that debt reduction would achieve. By contrast, in Dangl and Zechner (2020) equity maximizing managers may find it optimal to reduce leverage following bad news. This incentive only arises with shorter maturity debt outstanding. Hence maximizing this effect (and minimizing roll-over costs) leads to an interior optimal maturity structure of debt. DeMarzo and He (2020) derive leverage dynamics when managers cannot commit to firm value maximization, with a general cash flow process in continuous time. They analyze a "smooth" equilibrium where leverage changes are gradual (of order dt). They affirm that equity-maximizing managers never actively reduce debt. They also speak to the effect of no-commitment on debt maturity structure choice as well as the mutual reinforcement between a firm's investment and financing strategies if it faces commitment problems on both dimensions. We discuss in more detail the

commitment to refer to firm and equity value maximization, respectively, throughout the paper.

relation between our (partial equilibrium) results and these models in Section 2.[5]

More broadly, the present work belongs to the literature examining the aggregate implications of contracting problems. Important contributions include Cooley, Marimon, and Quadrini (2004), who examine the inability of lenders to enforce repayment from entrepreneurs. Their model implies that limited contractual enforceability amplifies aggregate shocks through the cyclicality of entrepreneurs' hold-up power. Lorenzoni (2008) models two-sided limited commitment by lenders (households) and borrowers (entrepreneurs), in a three-period economy. As in our model, inefficient liquidation leads to welfare losses. In contrast, Gale and Gottardi (2015) model a social benefit to debt as default allows efficient firms to benefit by buying bankrupted firms' assets at fire-sale prices. Their equilibrium features too little, rather than too much debt, resulting in underinvestment and imposing a welfare cost. In a general equilibrium model with endogenous innovation, Geelen, Hajda, and Morellec (2019) include both a positive effect of debt through the effect of tax subsidies on entry, and a negative effect from inefficient default and underinvestment.

To summarize, this study contributes both new tools and new insights towards the effort to examine corporate finance problems and macroeconomic outcomes within the same setting. We quantitatively evaluate the implications of debt-equity contracting frictions in a model that includes time-varying business conditions and generalized preferences. We compute endogenous prices and quantities of debt and equity under firm- and equity-value maximizing policies. Our general equilibrium analysis highlights the joint determination of these privately optimal policies and aggregate risk and discount rates. We find that both the firm level and economy-wide costs of equity maximization are large, and that the latter significantly exceed the former.

[5]It is perhaps worthwhile to distinguish the contracting problem studied in these papers (and ours) from a separate literature that investigates the implications for financial policy of the principle-agent problem when managers maximize neither firm nor equity value, but rather their own private value. This problem is modeled in Morellec (2004). There, the manager's private value is maximized with inefficiently *low* leverage.

2. Model

This section and the next analyze a continuous-time economy in which the contracting technology may not allow firms to implement value-maximizing capital structure policies. This section presents the basic model in partial equilibrium, defines the condition that characterizes firm value maximization, and discusses properties of solutions.

2.1 Firms, Debt, and Discount Rates

Each firm in the economy consists of a single project that produces a non-negative stream of goods. Let $Y^{(i)}$ denote the instantaneous output flow of project i. We assume $Y^{(i)}$ follows the pure-jump stochastic process

$$\frac{dY_t^{(i)}}{Y_t^{(i)}} = \mu\ dt + d\left[\sum_{j=1}^{\mathcal{J}_t}\left(e^{\varphi_j^{(i)}} - 1\right)\right]. \tag{1}$$

Here $\mathcal{J}_t$ is a regular Poisson process with intensity λ, and the percentage jump size is $\varphi_j^{(i)}$. If a jump occurs at time t, the sign of the jump is a Bernoulli random variable (with both outcomes having equal probability). The jump sizes $\varphi^{(i)}$ are drawn from gamma distributions defined over the positive or negative real line, depending on the sign of the jump. A discontinuous cash flow process with random jump sizes is convenient for modeling credit risk because jumping below any given bankruptcy threshold is always a possibility.[6] We denote the mean of the log jumps $\varphi_j^{(i)}$ as σ since, in general equilibrium, this will determine the volatility of the aggregate cash flow process Y. A second parameter, L, determines the standard deviation of the firm-specific jump size.[7] For numerical analysis, we will assume that the (log) upward jumps are drawn from an exponential distribution satisfying $\mathrm{E}[Y'/Y|\mathrm{up}] = 1/\mathrm{E}[Y'/Y|\mathrm{down}]$. In this sense, the jump distribution

[6] The analysis below goes through completely in the case of mixed jump-diffusive dynamics.

[7] The pdf of the gamma distribution is usually written as $f(x; a, b) = x^{a-1}e^{-x/b}\ b^{-a}/\Gamma(a)$ with mean ab and standard deviation $\sqrt{a}b$. So in the notation used here $\sigma = ab, L = 1/\sqrt{a}$.

is symmetrical.[8]

Following Admati, DeMarzo, Hellwig, and Pfleiderer (2018) and DeMarzo and He (2020), we assume that firms are allowed to issue or repurchase debt at any time at no cost. Equity finance is also assumed costless. Increases in debt are paid to equity holders; decreases are funded by equity holders. There is no physical capital in the model, and, to keep the focus on financial policy, the firm's stock of (intangible) capital is fixed.

The form of the debt contract is restricted to being a perpetual note. Extending the model to allow finite maturity debt is straightforward. However, the firm faces no choice on maturity, or on any dimension of contract design, and can issue only one class of debt.

Following the usual assumptions in the capital structure literature, we assume the firm receives a tax deduction for coupon interest paid, and that this deduction is realized continuously as long as the firm is alive. When owners of the firm choose to abandon, we assume the project's income stream is permanently reduced by a factor, α, and that creditors inherit the rights to this stream. As usual in this class of models, one non-contractibility built into the set-up is that the firm management cannot commit in advance *not* to act in the interests of equity holders by defaulting when optimal (for equity) to do so.

To value the firm's securities, the economy is endowed with a pricing kernel, denoted Λ. Its dynamics (which will be derived in equilibrium below) are as follows

$$\frac{d\Lambda}{\Lambda} = \eta\ dt + d\left[\sum_{j=1}^{\mathcal{J}_t} \left((u^{-\gamma} - 1)1_{\{j,+\}} + (d^{-\gamma} - 1)1_{\{j,-\}}\right)\right].$$

Here γ is the coefficient of relative risk aversion of the representative agent, and η, u, and d are constants to be determined in equilibrium. Intuitively, $u > 1$ and $d < 1$ correspond to the aggregate fractional change in output on a jump event. These numbers determine

[8] The distribution of the upward jumps in output plays no role in determining capital structure or solving the model.

the economy's riskless interest rate as

$$r = -\eta - \tfrac{1}{2}\lambda[(u^{-\gamma} - 1) + (d^{-\gamma} - 1)].$$

2.2 Linear Debt Policies

The firm's financial policy at time-t consists of an amount of debt (the coupon payment per unit time), C_t, and an abandonment decision, i.e., whether not to default at t. Because adjustment is costless, the firm will optimize its policy continuously, given the current state. Since the economy's innovations are independent and identically distributed (*i.i.d.*), the firm's current state is summarized by the level of output[9], Y_t, and the inherited level of debt, C_{t-}.

Given the *i.i.d.* stochastic structure, the homogeneity of preferences, and the absence of transaction costs, it seems natural to conjecture that the firm will adopt a linear debt policy: $C = cY$. Indeed, for a value maximizing firm, the inherited debt should not affect the currently optimal debt since restructuring results only in payment between shareholders and creditors. Homogeneity, then, implies a constant ratio C/Y.

Below we will also restrict equity maximizing managers to strategies in this class. First, however, we show that following such a debt policy results in very tractable and convenient asset prices and optimal default condition. We begin with the latter.

Within our pure-jump economy, firms will only default at the instant of a sufficiently negative downward jump. Let $\underline{Y} = \underline{Y}(Y_t, C_{t-})$ be the largest level of post-jump cash flow for which the firm defaults, and define the critical log jump level $\underline{\varphi} < 0$ via $\underline{Y}/Y = e^{\underline{\varphi}}$.

Lemma 1. *The optimal default policy is for owners to abandon the firm on a jump of V_t below F_{t-}. For a firm following linear debt policy $c = C/Y$, assume that, prior to default, V is linear in output, Y, and let $v = V/Y$. Also assume F is linear in C with*

[9] In this section, we drop the firm-specific superscript (i). (Above we used Y to denote aggregate output.)

$p = F/C$. Then the critical default threshold is

$$e^{\underline{\varphi}} = F_t/V_t = cp/v. \tag{2}$$

Note: all proofs appear in Appendix A.

Besides characterizing the optimal default policy, the lemma is notable in that it provides an alternative interpretation of the parameter $\underline{\varphi}$ as simply the log of the market leverage ratio under linear policies. We will freely use this interpretation below, and occasionally just refer to $\underline{\varphi}$ as "market leverage."

Next, we use the pricing kernel to compute the value of the firm and the bond, conditional on $\underline{\varphi}$. These values, in turn, fix the optimal coupon using the result in the lemma. Thus, under linear policies, the firm's capital structure problem collapses to a one-dimensional optimization.

Proposition 1. *Given a fixed default boundary, $\underline{\varphi}$, bond value is linear in the coupon amount, $F = pC$, and firm value and debt quantity are linear in output, $V = vY$, and $C = cY$. Assuming the regularity conditions given in Appendix A hold, the unit debt value $p = p(\underline{\varphi})$ and the value-output ratio $v = v(\underline{\varphi}, p)$ are given by the closed-form expressions (24) and (26) in the appendix. Since the conditions of the lemma are satisfied, $c = c(\underline{\varphi}) = (v/p)\ e^{\underline{\varphi}}$.*

The expressions for p and v are straightforward to evaluate but are omitted for brevity. Notice that the proposition establishes the equivalence of the assumption of a linear debt policy and constant default threshold, $\underline{Y}/Y$.

Our focus on the linear strategy space for debt policy has roots in the classic dynamic capital structure model of Goldstein, Ju, and Leland (2001). Assuming firms can restructure (perpetual) debt upwards and pay proportional issuance costs, they show that, under geometric Brownian motion cash flow, the optimal coupon and bankruptcy threshold are linear in cash flow. Moreover, by a backwards-induction argument, they show that managers have no incentive to deviate from the linear policies. More recently, in a model of agency conflict, Lambrecht and Myers (2017) show that, under power utility, managers optimally set debt to be linear in the firm's net worth. In a setting similar to ours, without adjustment costs and with downward jumps in asset values, Lambrecht and Tse (2019) analyze the effect of alternate default resolution regimes on a bank whose equity maximizing managers follow a debt policy that is linear in asset value.

2.3 Equity Maximization

We now define a particular condition on capital structure policies that has a natural interpretation as the outcome of equity maximization. The rest of the paper will study properties of firms implementing policies that satisfy this condition.

First, to clarify terminology, when there are no contracting frictions, the firm's initial owners will select the debt policy (a coupon function and its associated default threshold) that maximizes firm value and contract with managers to implement that policy in all future states. Hence, firm-value maximization is equivalent to the ability to attain managerial "commitment". By contrast, without the ability to credibly commit to implement the value maximizing policy, managers will seek to maximize equity value even when doing so will reduce firm value. Hence, "no-commitment" and equity-value maximization are equivalent. Equity maximization, however, is constrained by the market's anticipation of managers' lack of commitment to *any* policy. That constraint takes the form of limiting the set of policies for which prices will be provided.

Suppose that a manager who is free from any commitment deviates from $C_t = \bar{C}(Y_t, C_{t-})$ to an alternate policy function $\tilde{C}(Y_t, C_{t-})$, leading to the firm and unit debt price $\tilde{V}_t = V(Y_t, \tilde{C}_t)$ and $\tilde{p}_t = p(Y_t, \tilde{C}_t)$, respectively. The pre-issuance equity value attained is:

$$\tilde{V}_t - \tilde{p}_t\tilde{C}_t \ + \ \tilde{p}_t[\tilde{C}_t - C_{t-}] = \tilde{V}_t - \tilde{p}_t C_{t-}.$$

There is a gain to deviating if the above quantity exceeds the previous value $V_t - p_t C_{t-}$, with V and p computed under the original policy. However, clearly the market cannot value the firm's claims under the assumption of the manager following a future policy that he has an immediate incentive to deviate from. This gives rise to the following definition.

Definition 2.1. *A* **fixed-point equity maximizing policy** *is a debt policy, C, and its associated optimal default policy, if and only if, for all points in the state space (Y, C_-), C is a fixed point of the mapping*

$$C^*(Y, C_-) = \arg\max_{C'} \left(V(Y, C') - p(Y, C')\, C_-\right)$$

satisfying $V - pC > 0$, where V and p are the pricing functions implied by the policy.

Belonging to the set of fixed points is equivalent to the condition on a policy that there are not immediate incentives to deviate to an alternative policy. In that sense, the pricing functions are justified and the policy is self-enforcing. Fixed-point policies are a reasonable representation of equity maximization in that the objective function is equal to the firm's equity value.

In Appendix B we provide a formal derivation of fixed point policies as an equilibrium of a game between the manager and the market wherein the market provides continual price schedules (conditional on quantities) for the firm's claims. Managers maximize equity value given these schedules, and the market breaks even in the sense that, at the firm's chosen quantities, prices are equal to the risk neutral discounted expected value of each claim's future cash flows under correct beliefs about the firm's future actions. This provides a foundation for using this definition as our equilibrium concept under no commitment.

While the definition given above is straightforward, it does not lend itself to constructing solutions since, in general, C may be any measurable function of the state variables.

Hence, the maximization is not feasible. Our approach is to restrict the strategy space to linear policies, as described in the previous subsection. With this restriction, the definition is simply a fixed point of the mapping

$$c^*(c_-) = \arg\max_{c'}\{v(c') - p(c')\,c_-\}$$

where now the pricing functions are those derived in Proposition 1, and the maximization is just a one-dimensional optimization over the positive real line.[10] As our numerical work will illustrate, we can find unique linear fixed-point policies for a range of reasonably parameterized and interesting examples, and in Appendix A we present sufficient conditions for this to occur.

The restriction to linear strategies for equity maximizing managers is not without loss of generality. However the benefits of doing so include straightforward pricing formulas and an implementable formulation of equity maximization. Also, since a key objective of the analysis is to compare and contrast the equity-value maximizing and firm-value maximizing solutions, it is useful to have both policies in the same class.

2.4 Solution Properties

What effect does equity maximization have on firm policies? Intuitively, if a firm issues debt without imposing restrictions on managers, the debt is unprotected against actions that can transfer value from creditors to equity holders. In a two-period trade-off model it is easy to show that this situation *always* arises: for any non-zero amount of initial debt at $t = 0$, managers will optimally sell more debt at $t = 1$, even if nothing else changes. This is the "ratchet effect" described by Admati, DeMarzo, Hellwig, and Pfleiderer (2018). Equity holders optimally take on more leverage than firm-value maximization would

[10] If there are multiple fixed points, then managers will select the one that maximizes initial equity value.

imply. We can readily prove an analogous result (with the aid of some simplifying, but not necessary, conditions).

Corollary 2.1. *Assume that* $\alpha = 0$, $r > 0$, *and that the probability density function of negative jumps,* $g^{-}(x)$, *satisfies* $\frac{1}{g^{-}} \frac{dg^{-}}{dx} \leq -\frac{\tau}{1-\tau}$ *and* $g^{-}(0) > \frac{\tau}{1-\tau} \frac{r+\tilde{\ell}_d}{\tilde{\ell}_d}$. *Then, with commitment, a unique optimal capital structure exists, characterized by* $\underline{\varphi}^C$. *If a unique fixed-point policy exists, then it is characterized by higher market leverage:* $\underline{\varphi}^{NC} > \underline{\varphi}^C$ *and lower firm value.*

The following result provides sufficient conditions (given explicitly in the appendix) for the existence and uniqueness of such a policy in the further simplified case where the distribution of the down jump fraction, Y'/Y, is uniform.

Corollary 2.2. *Assume the conditions of Corollary 2.1 hold, and, in addition assume the conditions enumerated in Appendix A. Then a unique linear fixed-point policy exists.*

For our equilibrium to exist, it must be the case that, for some high level of leverage, it is in the interest of equity-maximizing managers to reduce debt. It is worth observing that the conditions described in the appendix can indeed fail. Our numerical work illustrates, however, that a fixed-point solution does exist for many plausible parameter values.

An important topic for our subsequent analysis is to understand what factors drive the incentive for extra leverage when managers are free to maximize equity value. Some helpful intuition emerges from examining the first-order condition for firm-value maximization and comparing it to the analogous condition under equity maximization. The effects are most transparent in the special case of uniformly distributed down jumps[11] with zero recovery. In this case, a manager inheriting a coupon amount $\bar{c}$ will solve the equation

$$\frac{1-2\tau}{1-\tau}\, e^{\underline{\varphi}} - \frac{\tau}{1-\tau} \frac{r}{\tilde{\ell}_d} = \left(\frac{p(\underline{\varphi})}{v(\underline{\varphi})} \right)^2 \bar{c}. \tag{3}$$

[11] This corresponds to exponentially distributed percentage jumps with $L = \sigma = 1$.

Here the left side are the terms that are set equal to zero under firm value maximization. They represent the standard trade-off effects, i.e., the marginal cost of default losses and the marginal benefit of tax shields. With full commitment, we have a simple closed-form solution for optimal market leverage, $e^{\underline{\varphi}}$, which yields p, v and the optimal coupon c. The right side is the wedge introduced by the contracting friction. This is the marginal benefit to equity holders of *decreasing* the value of debt. For any value of $\bar{c}$, the equity maximizing policy will be at a strictly greater value than the firm value maximizing policy, since the right side is positive. Our fixed-point requirement is that the optimal coupon and the current coupon coincide. This means we solve (3) with $\bar{c} = c = e^{\underline{\varphi}}(v/p)$. Hence the right side becomes $p\ e^{\underline{\varphi}}/v$, which represents the equilibrium expropriation incentive. While this remains a nontrivial function of $\underline{\varphi}$ even in our simplified example, we can still observe that anything that increases its value pushes the equity maximizing policy further away from firm value maximization.

Here the key observation is the inverse dependence on v. This tells us that, other things equal, the wedge is small when firm value is large, and large when firm value is small. So the intuition to bear in mind is that the threat of expropriation is likely to be most significant under adverse conditions for the firm.

Like this paper, DeMarzo and He (2020) (hereafter DH) study the commitment problem in a continuous-time setting with Leland-type debt and no adjustment costs. They focus on debt policies characterized by slow adjustment, $dC = G\ dt$. A natural question is how their policies relate to ours. In particular, in side-by-side comparisons, do they lead to economically different conclusions about the impact of commitment on leverage?

A seemingly stark contrast between the DH policies and ours is that in their solution, as in Admati et al (2017), debt issuance is always positive, whereas our policies feature both issuance and repurchase.[12] However, both models imply that leverage ratios have

[12] Optimal debt reduction by equity-maximizing managers is also a feature of the equilibrium in Dangl and Zechner (2020). In the sovereign debt model of DeMarzo, He, and Tourre (2021), a risk-neutral government may optimally repurchase its debt from risk averse investors when the risk premium is high.

stable dynamics. In the DH model, issuance goes to zero as C/Y increases, leading to mean reversion when the rate G/C falls below the cash flow growth rate, μ. Hence, in their model, as in ours, equity-maximizing managers optimally reduce leverage in some region of the state space.

To perform an exact comparison, in the appendix we explicitly solve the DH model with stochastic jumps and examine solutions of the two models using the same cash flow parameters.[13] The left-hand panel of Figure 1 shows the issuance rate and the credit spread for the DH model as the debt level C varies, holding current cash flow fixed at $Y = 1$. In this specification[14], the growth rate is $\mu = 0.03$, and the issuance rate is seen to hit that value at about $C = 1.57$, which defines the steady state. Using the same parameters, our model implies a constant ratio $C/Y = 1.22$ meaning that the expected issuance rate (i.e., aside from the jump adjustments) is equal to μ. The corresponding credit spread for off-equilibrium values of debt is shown in the left-hand panel of Figure 2.

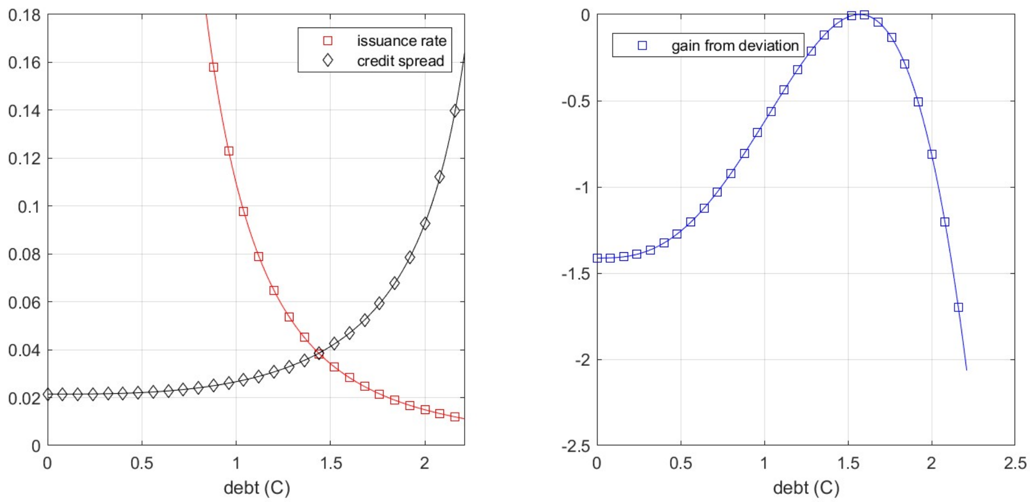
Our model is seen to imply somewhat lower debt and and substantially lower credit spreads than the DH model at the steady state(50 basis points versus 450). The higher credit spreads are not entirely due to increased default risk: if we evaluate the DH solution at $C/Y = 1.22$, the default threshold is actually lower than our model's (0.54 vs 0.62) and the credit spread is still over 300 basis points. In the DH model, bondholders are subject to continuous expropriation in the sense that $dC > 0$ in all states and $dp/dC < 0$. Bondholders require compensation for this, whereas in our model the steady state is obtained at time-zero and leverage never varies.

It is also notable that the positive issuance property of the DH solution remains in the presence of cash flow jumps. In particular, their equilibrium does not collapse to ours

[13]For technical reasons, the generalization in the appendix retains Brownian shocks. In addition, numerical solutions in the appendix converge as the Brownian diffusion coefficient goes to zero. Also we set the debt retirement rate to zero. Results reported here are for this limit.

[14]All of the parameters are given in the figure caption.

Figure 1: Demarzo-He Solution with Cashflow Jumps



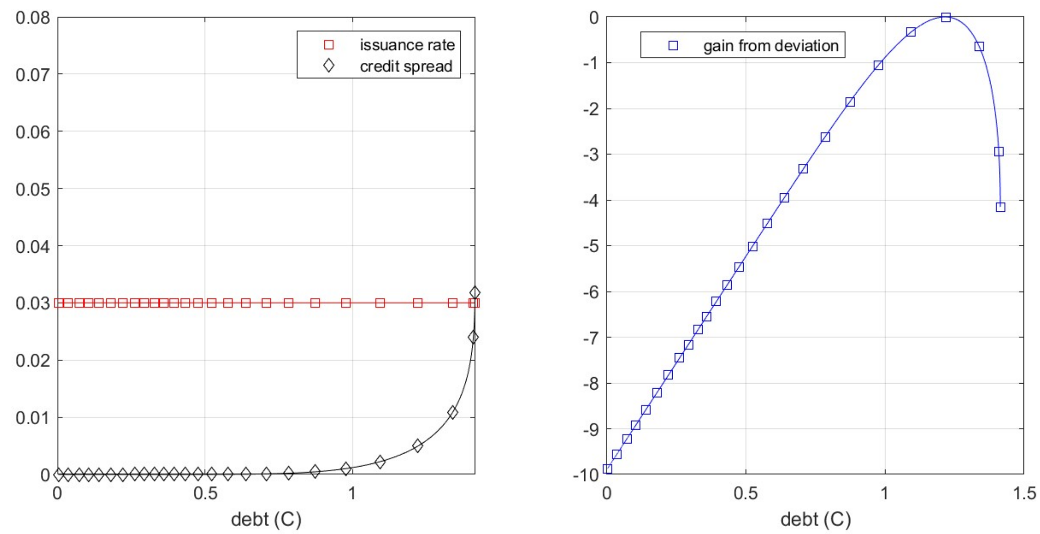
The figure shows properties of the solution of the model of Demarzo and He (2020) as generalized in the appendix to the case of jump dynamics for the cash flow process and perpetual debt. The parameters are $\sigma = 0.1, \mu = 0.3, L = 1, \alpha = 0, \lambda = 1, \tau = 0.3, \gamma = 4, r = 0.05, d = 0.97, u = 1/d$. Functions are evaluated for $Y = 1$. The horizontal axis is the debt quantity C. The left hand plots the equity-maximizing issuance rate G/C and the credit spread $1/p - r$ for the debt. The right panel plots the gains to equity (as a fraction of cash flow) from a one-time deviation from the steady-state policy $C = 1.57$.

with instantaneous adjustment to the steady state. In effect, the manager in the DH model is committed to maintaining a gradual adjustment policy whereas our manager is committed to maintaining constant leverage.

Interestingly, the first-order condition for equity-maximizing issuance implies that the debt price p must equal $-\frac{\partial V^e}{\partial C}$, where V^e is the value of equity (which is $V - pC$ in our notation). This is mathematically equivalent to the first order condition for our model's fixed point, discussed above. In the DH model, too, the condition assures that the issuance policy will always represent a fixed-point in the sense that, given any inherited debt level $\bar{C}$, the equity payoff from adjusting to C is $V^e(C) + p(C)(C - \bar{C})$ is maximized at $C = \bar{C}$. The equivalence is illustrated in the right panels of Figures 1 and 2 where we compute the gains from deviation from both model's steady states.[15] Thus the two

[15]Note that both plots are computing the gains from a one-time deviation to another policy within

Figure 2: Linear Fixed-Point Solution



The figure shows properties of the solution of the model of Section 2.2. The parameters are $\sigma = 0.1, \mu = 0.3, L = 1, \alpha = 0, \lambda = 1, \tau = 0.3, \gamma = 4, r = 0.05, d = 0.97, u = 1/d$. Functions are evaluated for $Y = 1$. The horizontal axis is the debt quantity C. The left hand plots the equity-maximizing issuance rate and the credit spread for the debt. The right panel plots the gains to equity (as a fraction of cash flow) from a one-time deviation from the steady-state policy $C = 1.22$.

equilibrium concepts have similar economic foundations.

As a practical matter, the path-dependent nature of the DH model solution would render it too complex for the main purposes of the present paper.[16] A key advantage of our linear policies is that their simplicity permits us to achieve the goal of extending the analysis of the commitment problem to general equilibrium and time-varying economic conditions.

the same class (a gradual adjustment policy in the DH case and a linear policy for us) which is then subsequently followed. Neither computation admits deviation to alternate classes of policy.

[16]It is also worth remarking that the DH model does not nest firm-value maximization (or full commitment) as a subcase.

3. Extensions

We next generalize the basic model to the two key extensions that will permit us to address our primary research questions. First, we treat the case of of time-varying economic conditions. Second, we show how to aggregate the economy and solve for the full general equilibrium.

3.1 Time-Varying Parameters

The model developed above will allow us to analyze the comparative static implications of equity maximizing capital structure. A natural question is whether the model can be extended to encompass time-variation in the firm's environment. We now show that it can, allowing us to analyze the dynamic implications of equity maximizing capital structure as well.

We consider a two-regime version of the model, in which the regimes are indexed by $s \in 1, 2$. In principle, all the parameters of the model $\sigma, \mu, \lambda, \alpha, L, u, d, r$ are allowed to vary with s. This encompasses variation in either (or both) of the firm-specific, and macroeconomic variables. The state index itself follows a Markov switching process with intensity denoted $\omega(s)$. This process is assumed to be independent of the instantaneous output process Y, implying that Y does not change when a state switch occurs.

This setting leads us to conjecture that, under linear policies, the claims prices remain linear in output, but with coefficients $c(s)$, $v(s)$ and $p(s)$ that depend on the state. It is then straightforward to show that the solution for the optimal default boundary is again given by (2) so that $e^{\underline{\varphi}(s)} = c(s)p(s)/v(s)$.

That threshold determines the firm's response to an adverse output jump. One subtlety that then arises is whether it is *also* ever optimal for owners to abandon upon the event of a switch from one state to the other. Nothing in the set-up precludes this possibility, and modifying the results below to deal with it is straightforward. For simplicity, we will present results for the case in which this is not optimal. This imposes a regularity condition on the model parameters summarized in the following definition and lemma.

Definition 3.1. *For a given capital structure policy, and implied claims values, denote as* **Condition S** *the following pair of inequalities:*

$$v(2)/v(1) > \exp(\underline{\varphi}(1)), \qquad v(1)/v(2) > \exp(\underline{\varphi}(2)).$$

Lemma 2. *If* **Condition S** *holds, then it is not optimal for owners to abandon the firm upon a switch between states.*

Even though output doesn't change on a state switch, it will in general be true that discount rates change. Specifically, letting $\mathcal{I}_t$ denote the Poisson process counting state switches (whose intensity is $\omega(s)$), the kernel now can be written

$$\frac{d\Lambda}{\Lambda} = \eta(s)\ dt + d\left(\sum_{j=1}^{\mathcal{J}_t}\left((u(s)^{-\gamma}-1)1_{\{j,+\}} + (d(s)^{-\gamma}-1)1_{\{j,-\}}\right)\right)$$
$$+ d\left(\sum_{i=1}^{\mathcal{I}_t}\left((\xi(1)-1)1_{\{i,1\}} + (\xi(2)-1)1_{\{i,2\}}\right)\right). \tag{4}$$

where $1_{\{i,2\}}$ indicates a switch from state 2 to state 1 , and $\xi(2) > 0$ is the ratio of marginal utility in state 1 to that of 2 (and vice versa for $1_{\{i,1\}}$ and $\xi(1)$). If $\xi(1) = \xi(2) = 1$, the regime switches can be viewed as idiosyncratic, or unpriced, events.

We then have the following generalization of Proposition 1.

Proposition 2. *Given a pair of values $\underline{\varphi}(s)$, in each regime, bond value is linear in the coupon amount, $F = p(s)C$, and firm value and debt quantity are linear in output, $V = v(s)Y$, and $C = c(s)Y$. Assuming the matrices $K_p(\underline{\varphi})$ and $K_v(\underline{\varphi})$ defined in Appendix A* *are are both positive definite, then the bond value p and firm value v are the solutions to*

$$K_p p = 1_2 \tag{5}$$

$$K_v v = (1-\tau)1_2 \tag{6}$$

where $1_2 = \{1,1\}'$. The optimal coupon in each state is then $c = (v(s)/p(s))\ e^{\underline{\varphi}(c)}$.

As in Section 2, the proposition simplifies the analysis of alternative policies. We can view the set of all optimal linear policies as being defined over the set of pairs $(\underline{\varphi}(1), \underline{\varphi}(2))$. With commitment to debt policy, a firm that starts life in state s then chooses the pair $\underline{\varphi}$ to maximize $v(s)$.[17] Without commitment, we again require an equity maximizing policy to satisfy a consistency condition.

Specifically, under linear policies the condition is that a policy vector $\underline{\varphi}$ together with associated price vectors v and p determined by Proposition 2, and coupon vector c determined by the default condition in Lemma 1, is, in each state, a fixed point of the mapping

$$c^*(c_-; s) = \arg\max_{c'} v(c'; s) - p(c'; s)\, c_-,$$

conditional upon the policy in the other state, and, in both states, and $v(c;s) - p(c;s)c(s) > 0$. Intuitively, unless a linear policy has the fixed point property, then managers have an incentive to deviate instantaneously.

We will use the two-regime model in Section 4 to contrast the cyclical properties of debt policy with and without commitment.

[17] Note that there are, in general, two full-commitment solutions for otherwise identical firms, depending on their birth state, because the same pair of default thresholds will not maximize both $v(H)$ and $v(L)$.

3.2 General Equilibrium

A primary focus of the paper is to examine how equity maximizing capital structure may affect the economy as a whole. This section explains how the model can be aggregated to compute the quantities of interest in general equilibrium.

Aggregation is feasible when it is not necessary to keep track of the distribution of firm characteristics. Hence we assume that the economy is endowed with a continuum of stochastically identical project-firms, whose measure is denoted M. Specifically, the output jump-counting process, $\mathcal{J}_t$ and the sign of the jump are assumed to be common across firms. Thus a jump is a systematic event. Conditional on the sign, the individual jump incidences are $i.i.d.$ across firms.

Ignoring entry and exit for the moment, we can then integrate over firms to obtain the dynamics of aggregate output Y in state s as

$$dY_t = \mu(s)\ Y_t\ dt + d\ \int_i^M\ Y_t^{(i)} \left(\sum_{j=1}^{\mathcal{J}_t} \left(e^{\varphi_j^{(i)}} - 1 \right) \right)\ di$$

$$= \mu(s)\ Y_t\ dt + Y_t\ d \left(\sum_{j=1}^{\mathcal{J}_t} \left(\mathrm{E_t} \left[e^{\varphi_j^{(i)}} | \varphi_j > 0; s \right] 1_{\{j,+\}} + \mathrm{E_t} \left[e^{\varphi_j^{(i)}} | \varphi_j < 0; s \right] 1_{\{j,-\}} - 1 \right) \right). \tag{7}$$

where $1_{\{j,+\}}$ and $1_{\{j,-\}}$ are indicators for the sign of the jth jump. Applying a law of large numbers, the stochastic term is

$$Y_t\ d \left(\sum_{j=1}^{\mathcal{J}_t} \left(\Phi^+(t) 1_{\{+\}} + \Phi^-(t) 1_{\{-\}} - 1 \right) \right)$$

where $\Phi^{\pm}(s)$ are the exponential integrals over the positive and negative jump size distributions. Thus aggregate output follows a binomial process. Conditional on the state and the sign of the jump, the size of aggregate shocks is not random. For up-jumps, we conclude that the aggregate parameter $u(s)$ is equal to $\Phi^+(s)$.

On a down-jump event, we have already shown that it will be optimal for owners of firms to default if they experience a drop of $\log(Y_t^{(i)}/Y_{t-}^{(i)})$ below $\underline{\varphi}(s)$. We have assumed that for all such firms output is thereafter reduced by the factor $\alpha(s)$, which may be zero. The effect of exit on output is simply to alter the downward aggregate jump size. In equation (7) above, we replace the down-jump expectation $\mathrm{E_t}\left[e^{\varphi_j^{(i)}}|\varphi_j^{(i)} < 0; s\right]$ with

$$\mathrm{E_t}\left[e^{\varphi_j^{(i)}}|\underline{\varphi} < \varphi_j^{(i)} < 0; s\right] + \alpha \mathrm{E_t}\left[e^{\varphi_j^{(i)}}|\varphi_j^{(i)} \leq \underline{\varphi}; s\right].$$

This is the expected fractional decline in firm-specific output on a down jump. Aggregation implies that it is the (nonstochastic) economy-wide decline in output, denoted d. General equilibrium thus links firms' optimal policies to aggregate risk.

We saw above that equity maximization results in amplifying the firm-value maximizing leverage. Higher leverage feeds through to higher aggregate default, lowering d. In general, lower d will then feedback to induce somewhat less debt. In the firm's problem, lowering d results in a higher marginal utility increase on down jumps, which makes debt more expensive to issuers. This can illustrated explicitly in the simplified special case with uniform jumps and no recovery. Under firm-value maximization, optimal leverage then turns out to be directly proportional to d^γ. A one percent lower d entails a γ-percent drop in leverage. Thus general equilibrium effects may mitigate the contracting friction.

However, this logic omits the general equilibrium effect on the distortion itself. Recall from Section 2.4 that the expropriation term in the fixed point expression was proportional to p/v. Increased aggregate risk will, in general, lower v. In the special case, it is straightforward to show that p actually rises with risk as the lower interest rate dominates the higher risk premium. Thus both terms strictly increase the expropriation incentives, providing a positive feedback channel. We investigate the quantitative implications of these effects in Section 5.

Turning to entry, we assume that households use their resources to create a flow of new projects, which increases the mass dM/M at an exogenous rate. The flow of new projects shows up as an additional term in the growth rate, μ, of aggregate output, dY/Y. When new projects are created, they are distributed uniformly across households. Each household sells its projects to all the others. Each firm then sells its initial quantity of debt, the value of which passes to the equity holders. These financial transactions between households and themselves result in no net flow of real goods. Households' aggregate income is assumed equal to Y_t.[18]

Next, we assume there is a representative household characterized by preferences of the stochastic differential utility class (Duffie and Epstein (1992), Duffie and Skiadas (1994)), the continuous-time analog of Epstein and Zin (1989) preferences. Specifically, agents maximize the lifetime value of the consumption stream C, defined as

$$J_t = \mathrm{E_t}\left[\int_t^{\infty} f(C_u, J_u)\, du\right].$$

where

$$f(C, J) = \frac{\beta C^{\rho}/\rho}{((1-\gamma)\; J)^{1/\theta - 1}} - \beta\theta J.$$

Here β is the rate of time preference, γ is the coefficient of relative risk aversion, $\rho = 1 - 1/\psi$, where ψ is the elasticity of intertemporal substitution, and $\theta \equiv \frac{1-\gamma}{\rho}$. (We assume $\gamma \neq 1, \rho \neq 0$.) There is no savings technology so $C = Y$. Since Y's dynamics have already been determined, it is straightforward to solve for the value function, which determines marginal utility, and hence the pricing kernel and the interest rate.[19]

Proposition 3. *The household's value function is $J = j(s)\; Y^{1-\gamma}/(1-\gamma)$, where $j(s)$ is the solutions to two coupled algebraic equations given in the appendix. The pricing kernel*

[18] A government sector is assumed to collect corporate taxes, net of tax shields, and rebate any surplus to households.

[19] The results below will continue to assume that it is not optimal for firms to exit upon a switch of state in either direction. Hence Condition S needs to be verified in any given solution. The assumption is sensible in general equilibrium since the alternative is the simultaneous default of all firms in the economy.

takes the form given in (4) with

$$\eta(s) = \beta\theta\ [(1 - \frac{1}{\theta})\ j(s)^{-\frac{1}{\theta}} - 1] - \gamma\mu$$

$$\xi(1) = (j(2)/j(1))^{1-\frac{1}{\theta}}, \qquad \xi(2) = (j(1)/j(2))^{1-\frac{1}{\theta}}.$$

The riskless interest rate is then given by

$$r(s) = -(\eta(s) + \tfrac{1}{2}\lambda(s)((d(s)^{-\gamma} - 1) + (u(s)^{-\gamma} - 1)) + \omega(s)(\xi(s) - 1)).$$

We will use Proposition 3 in Section 5 to compute the welfare loss to the economy when managers choose capital structure to maximize equity value.

4. The Private Value of Capital Structure Commitment

We now turn to the examination of the model's quantitative properties. To start, this section focuses on the cost to the firm itself of lack of commitment to maximizing firm value. We illustrate the characteristics that lead to greater or lesser cost, both in the cross-section and in the time-series. The analysis in this section is set in partial equilibrium; general equilibrium implications are examined in Section 5. Our goal is to identify the drivers behind valuation effects and to connect to empirical evidence. In particular, there is an extensive literature on the use and consequence of covenants. While we recognize that the mapping is imperfect and covenants are used for many purposes, narrowing the scope for expropriation is prominent among them. Hence, covenant use may be interpreted as, in part, at device to achieve capital structure commitment, while a lack of covenants renders managers relatively free of commitment and able to pursue equity value maximization.

4.1 Cross-Sectional Implications

To start, we numerically illustrate the difference between firm-value maximizing and equity-value maximizing capital structure policies for a range of firm parameters for the single-regime case when there is no time-variation in parameters.[20] The exercise holds the economy-wide parameters (u, d, r, γ, τ) fixed. Also, for simplicity, all numerical examples throughout the paper will assume that intensity of output jumps is $\lambda = 1$, i.e., one jump is expected per year.

Our analysis in Section 2 showed that the equity maximizing policy produces more leverage than firm value maximization. Table 1 shows the magnitude of the distortion in debt policy for a range of firm characteristics. Columns with and without commitment are labeled C and NC, respectively. The first two columns report the quantity of debt scaled by output(or "book leverage"). The middle columns report the market value of debt scaled by the market value of assets. The last two columns show the credit spread on the debt. The first row shows results for a baseline set of firm parameters. Each of these parameters is then varied to a higher and lower value in the three subsequent pairs of rows.

From the first row, we see that firms without commitment take on about 50 percent more debt than those with commitment. This excess debt increases the default risk and more than doubles the credit spread on the firm's debt. Because of the lower price (higher yield), the increase in debt looks less dramatic in market value terms, but still results in a leverage increase of 22 percent of the unlevered asset value.

From the two middle rows, we see that in terms of market leverage, the distortion is much higher when firm growth is low, but is not much changed with the level of output volatility or of the recovery rate. While the quantity of excess debt increases with σ and

[20] Recall that we refer to the two cases as "with commitment" and "no-commitment" for short-hand.

α, the lower price of the debt essentially off-sets the increase.

Table 1: Capital Structure : Committed *vs* Non-Committed

	book leverage (C/Y)		market leverage (F/V)		credit spread $(y-r)$	
	C	NC	C	NC	C	NC
$\mu = 0.03$, $\sigma = 0.10$, $\alpha = 0.25$	0.95	1.52	0.55	0.77	2.30	4.82
$\mu = 0.00$	0.53	1.18	0.55	0.93	2.30	9.37
$\mu = 0.06$	4.52	4.96	0.55	0.59	2.30	2.65
$\sigma = 0.05$	1.03	1.42	0.62	0.78	1.11	2.61
$\sigma = 0.15$	1.01	1.65	0.54	0.76	3.50	6.38
$\alpha = 0.0$	0.77	1.28	0.48	0.69	2.06	4.38
$\alpha = 0.5$	1.27	1.85	0.67	0.85	2.73	5.24

The table reports theoretical firm properties under capital structure commitment (columns labelled C) and no-commitment (NC) using the model of Section 2. The baseline firm parameters are shown in the top row left-hand column. Other values are $L = 3, \lambda = 1, \tau = 0.3, r = 0.07, d = 0.97, u = 1.03, \gamma = 4$.

Table 2 shows the valuation consequences of the excess leverage under no-commitment. The first two columns report the firm valuation multiplier; the next column gives the discount expressed as a multiple of income; the right-most column expresses the discount as a percentage of the value under commitment. The firm parameter values in each row are the same as in the preceding table.

Two main observations from this Table are (1) that the loss of investor surplus can be economically large; and (2) these theoretical values are plausible relative to empirical estimates. Matvos (2013) estimates the value of an array of bond covenants in a sample of syndicated loans. In terms of pricing, he finds that including two covenants (the median in his sample) reduces the credit spread on the loan by 50 percent, on average, which accords with our findings in Table 1. His methodology also produces estimates of the issuing firm's surplus from including protective covenants, relative to the alternative of issuing unprotected debt. On average, the surplus is 52 percent of the credit spread. This can be compared with our valuation units in Table 2 by capitalizing the perpetuity value of the credit spread. Using the parameters in the first row, protected debt carries an interest rate of 9.3 percent and the riskless rate is 7.0 percent. So a risky unit perpetuity has a price discount of 3.53 ($1/.07 - 1/.093$). The firm has a debt quantity of 0.95 times income. So the price discount is 3.36 times income. One-half this amount (Matvos's estimate) is 1.68 times income, which is almost precisely the surplus value shown in the table (1.67).

Recently, Green (2019) has estimated the valuation of the full covenant packages found in high-yield bonds. His point estimate is 2.4% of firm value, which, while economically large, is smaller than most of the values in the right-hand column of Table 2. His model (and structural estimation) allow covenants to have real costs, however, which in our setting they do not.[21] While he interprets the cost in terms of inefficient restrictions (e.g. on asset sales or investments) due to limitations in available covenants, they could also include direct costs of monitoring and enforcement. Green's estimates are thus net benefits, whereas the numbers in the table are gross.

In terms of cross-sectional variation, the table also highlights firm characteristics that make the ability to commit relatively more or less valuable. Most prominently, the model implies very little surplus from covenants for high-growth firms, and very large surplus for low-growth firms. The model does not embed a depiction of the contracting technology – and its cost function – that could lead some firms to optimally pay to achieve protected debt, while others choose not to do so. However, unless that contracting cost itself is a steeply declining function of firm profit growth, the model does offer the prediction that covenant usage should be rarer in high-growth firms and more frequent in low-growth ones.

This prediction also finds some empirical support. See Nash, Netter, and Poulsen

[21] Indeed, Green estimates that a full array of high-yield covenants would have a (small) net negative impact on the value of an investment-grade firm.

Table 2: Cost of Unprotected Debt

	firm value (V/Y)		value loss	
	C	NC	difference	percent
$\mu = 0.03$, $\sigma = 0.10$, $\alpha = 0.25$	18.42	16.75	1.67	10.00
$\mu = 0.00$	10.30	7.75	2.55	32.82
$\mu = 0.06$	87.58	86.64	0.94	1.09
$\sigma = 0.05$	20.60	19.12	1.48	7.76
$\sigma = 0.15$	17.76	16.21	1.55	9.57
$\alpha = 0.0$	17.79	16.39	1.40	8.52
$\alpha = 0.5$	19.62	17.75	1.87	10.51

The table reports theoretical firm properties under capital structure commitment (columns labelled C) and no-commitment (NC) using the model of Section 2. The baseline firm parameters are shown in the top row left-hand column. Other values are $L = 3, \lambda = 1, \tau = 0.3, r = 0.07, d = 0.97, u = 1.03, \gamma = 4$.

(2003), Demiroglu and James (2010), and Reisel (2014).[22] The rationale sometimes offered in the literature is that covenants may be unsuitable for high-growth firms because they impose limits on flexibility and may impede growth opportunities. Interestingly, our model's result has nothing to do with flexibility or investment. Instead, as described in Section 2, the expropriation incentive that leads to excess leverage scales inversely with firm value. Intuitively, managers of very valuable (high-growth) firms have much more incentive to get the first-order trade-off between tax-shields and default costs right, and less incentive to worry about transfers from creditors.

4.2 Dynamic Implications

We now turn to the dynamic model developed in Section 2.2 to understand the model's implications for time-series variation in the capital structure distortion due to uncommitted debt policy. We consider two types of applications. First, we consider (idiosyncratic)

[22]Billett, King, and Mauer (2007) and Bradley and Roberts (2015) report evidence that covenant use increases with growth opportunities, however.

variation in the firm's parameters, holding the economy-wide parameters fixed. We next consider simultaneous variation in firm and macroeconomic conditions (as in Hackbarth, Miao, and Morellec (2006) and Bhamra, Kuehn, and Strebulaev (2010)). All the cases will adopt the assumption that there is a "good" state that is unconditionally more likely, with a half-life of 5 years, and a short-lived "bad" state, with a half-life of 5 quarters.

A first finding is that the model implies more countercyclical leverage under equity-maximization. The basic logic of the trade-off setting implies that when risk increases or growth decreases, increasing potential default costs lead to less leverage. In Table 3, we quantify this via the ratio of debt (or leverage) in the bad state to that in the good state. The first column tells us that, for these parameter values, firms with commitment lower their debt quantities in bad times by multiples from 0.48 to 0.87. The third column indicates similar contractions in market leverage, except in one case (the first row) where lower debt leads to bond prices that are high enough that the market leverage increases. These policies contrast with the no-commitment cases in the second and fourth columns. There we see debt quantities and leverage that can sometimes *increase* in bad times. In the cases where there is still a decrease, that decrease is markedly smaller than in the corresponding case with commitment. Again, the intuition behind this result is that expropriation incentives decrease when the firm value is higher.[23]

Given that capital structure is relatively more distorted without commitment in bad times, it is not surprising that the surplus loss to firms is also higher in those times. This is shown in Table 4, where the percentage loss of firm value in the no-commitment cases is reported in each state. The losses are only mildly larger in the bad state in the first three rows where the states are idiosyncratic. However the next three rows, where macroeconomic parameters also vary, show a notably larger surplus loss in bad times.

[23]Countercyclical leverage also arises in the private information model of Hennessy, Livdan, and Miranda (2010). In a separating equilibrium, bad-type firms are always equity financed, and good-type firms need to increase their leverage to signal following negative outcomes to counteract the effect of observable information. In Levy and Hennessy (2007) leverage can be countercyclical because of the need to maintain high managerial ownership in bad times to solve agency problems.

Table 3: Cyclicality of Capital Structure Policy

	debt quantity bad-state/good-state			
	book leverage		market leverage	
	C	NC	C	NC
switching variables:				
$\mu^{(i)}$	0.8689	1.0060	1.0161	1.1559
$\sigma^{(i)}$	0.7470	1.0411	0.7483	0.9974
$\mu^{(i)}, \sigma^{(i)}$	0.7077	0.8788	0.8171	0.9891
r, d, u, ξ, and				
$\mu^{(i)}$	0.6402	0.9046	0.8590	1.1217
$\sigma^{(i)}$	0.6110	0.8983	0.6557	0.9096
$\mu^{(i)}, \sigma^{(i)}$	0.4827	0.8863	0.6531	1.0563

The table compares the optimal debt policies for the 2-regime model with and without capital structure commitment. The numbers reported are the ratios of debt amounts in the bad state to the amount in the good state. The first two columns measure debt as coupon expense as a fraction of output (C/Y in the model). The second two columns measure debt as market value of debt as a fraction of firm value (F/V). In the first three rows, the aggregate state does not switch, and is described by the parameters $r = 0.06, d = 0.95, \xi = 1$. The firm-specific growth rate and volatility switch, and take on the [good-state, bad-state] values values $\mu = [0.06, 0.00]$ and $\sigma = [0.05, 0.15]$. In the bottom three rows the aggregate state also switches, and is characterized by the pairs $r = [0.07, 0.01], d = [.95, .88], \xi = [2.0, 0.5]$. All cases use $u = 1/d, \alpha = 0.25, L = 3, \lambda = 1, \gamma = 4$, and $\omega = [0.14, 0.56]$.

This finding also finds support in the empirical literature. Again acknowledging the imperfect mapping between use of covenants and capital structure commitment, it is noteworthy that countercyclical covenant use is documented by Bradley and Roberts (2015) and Helwege, Huang, and Wang (2017). Likewise, procyclical issuance of "cov-lite" debt is noted by Becker and Ivashina (2016) among others. The fact that investors apparently exhibit an increasing appetite for unprotected debt in expansions is sometimes viewed as evidence of irrational exuberance or "reaching for yield." Our model offers a different perspective. Since the relative benefits of covenants are smaller in good times, it follows that we should expect fewer firms to use them.[24]

[24] We note again the qualifier that we have no explicit model of variation in contracting costs across firms or economic states. If costs of writing and monitoring covenants also go up in bad times, then the model's prediction about their adoption could be ambiguous.

Table 4: Cyclical Cost of Unprotected Debt

	percent reduction in firm value without commitment	
	good state	bad state
switching variables:		
$\mu^{(i)}$	4.44	5.26
$\sigma^{(i)}$	9.67	11.29
$\mu^{(i)}, \sigma^{(i)}$	2.46	3.24
r, d, u, ξ, and $\mu^{(i)}$	17.85	21.98
$\sigma^{(i)}$	15.74	18.82
$\mu^{(i)}, \sigma^{(i)}$	15.86	21.40

The table shows the percentage decrease in firm value without commitment to debt policy for the 2-regime model. In the first three rows, the aggregate state does not switch, and is described by the parameters $r = 0.06, d = 0.95, \xi = 1$. The firm-specific growth rate and volatility switch, and take on the [good-state, bad-state] values values $\mu = [0.06, 0.00]$ and $\sigma = [0.05, 0.15]$. In the bottom three rows the aggregate state also switches, and is characterized by the pairs $r = [0.07, 0.01], d = [.95, .88], \xi = [2.0, 0.5]$. All cases use $u = 1/d, \alpha = 0.25, L = 3, \lambda = 1, \gamma = 4$, and $\omega = [0.14, 0.56]$.

In Appendix E, we take the empirical evidence further and document the novel fact that, in panel regressions, tightness of covenants is associated with *more* procyclicality of leverage. In addition to contributing to the literature on the uses and effects of covenants, this finding support the assertion that the theoretical mechanisms in our model may have economically important effects both at the firm and aggregate levels.

5. The Social Cost of Noncontractibility

Our model has described how equity maximizing firms (or without commitment) adopt financial policies that feature more – and more countercyclical – leverage. An implication of this is that such policies lead to higher default rates and real losses in bad states of the world, compared to a world where managers are constrained to act in the interests of the entire firm. Quantifying the resulting welfare losses in general equilibrium provides an estimate of the social cost of the contracting friction. In this section, we analyze these costs in our model utilizing the solution for the value function of the representative household, as developed in Section 3.1. We have already computed (in partial equilibrium) the losses to firms due to noncontractibility. Here we extend the analysis to the entire economy, taking into account the externality that excessive debt imposes on aggregate risk.

The model economy is, of course, stylized and omits many channels through which financial policies could have real effects. So it is worthwhile to clarify that our objective here is to study differentials effects between versions of the model. In particular, our interest is in the incremental welfare cost of moving from an economy with commitment to one without commitment. Moreover, we wish to compare this differential to another one: the analogous private cost borne by owners of individual firms. We will also contrast social costs across versions of the model with and without fluctuations in aggregate moments. Since, in effect, we are quantifying "diff-in-diffs" of welfare, our conclusions may be robust to generalizations that could well have important implications for outright levels.

To undertake this exercise, we start by presenting four calibrations of the model that can achieve reasonable agreement with the data on certain important dimensions. Specifically, we will utilize both the single-regime version of the model (with constant moments) and the two-regime version (with time-varying macroeconomic parameters); and for each version, we present the general equilibrium solution with commitment (C) and without (NC). The models all utilize the same preference parameters, recovery and tax rates, and the same unconditional moments of the firm-specific and aggregate cash flow processes. In this sense, comparisons between them are clean: the only variation comes from changing either the contractability or the cyclicality.

Table 5 compares features of these models to analogous statistics from the data. There is substantial leeway in defining "the data" for purposes of such comparison, both because of differences across samples of firms and time periods, and because of differing possible interpretations of how to map model quantities to the real world. In the model, aggregate corporate cash flow, Y_t, is also output and consumption, as well as (up to a constant) dividends and earnings. The table uses the latter two quantities and presents first and second moments from 1926 through 2019. Key moments for modeling debt are averages of leverage, default rates, and credit spreads. Key for understanding firm risk are volatilities of levered firm equity returns and also of equity index returns. In addition, we include measures of firm valuation: the price-dividend and price-earnings ratios for levered equity.[25] In order to assess the calibration of the regime-switching models, in addition to unconditional values, the table reports the ratios of the moments across "good times" and "bad times". The data equivalent of these regimes is taken to be NBER expansions and recessions, with a couple of exceptions described in the notes in the appendix.

The parameter choices – given in the table caption – are guided both by the aim of matching features of the data and by the prior literature. The preference parameters are standard for macro-finance models, although we use a relatively low coefficient of risk aversion of four. (Sensitivity to these choices are discussed below.) Debt incentives are strong enough in the no-commitment case that $\alpha = 0$ is required to obtain realistic, albeit still high, levels of leverage. We follow Gomes and Schmid (2021) in using a marginal tax rate of $\tau = 0.20$. The parameters σ and L determine the degrees of systematic and idiosyncratic risk, respectively. We choose these to target the volatility of (levered) equity returns at the market and firm levels.

Reviewing the models' implications, from the unconditional moments in the single regime case, the model with commitment, C1, underfits on leverage, default risk, credit spreads, and equity volatilities, while the no-commitment version, NC1, tends to overfit them. In the corresponding models with business cycle risk, C2 produces too little

[25] Details on the samples and sources of all data quantities in Table 5 are given in Appendix D.

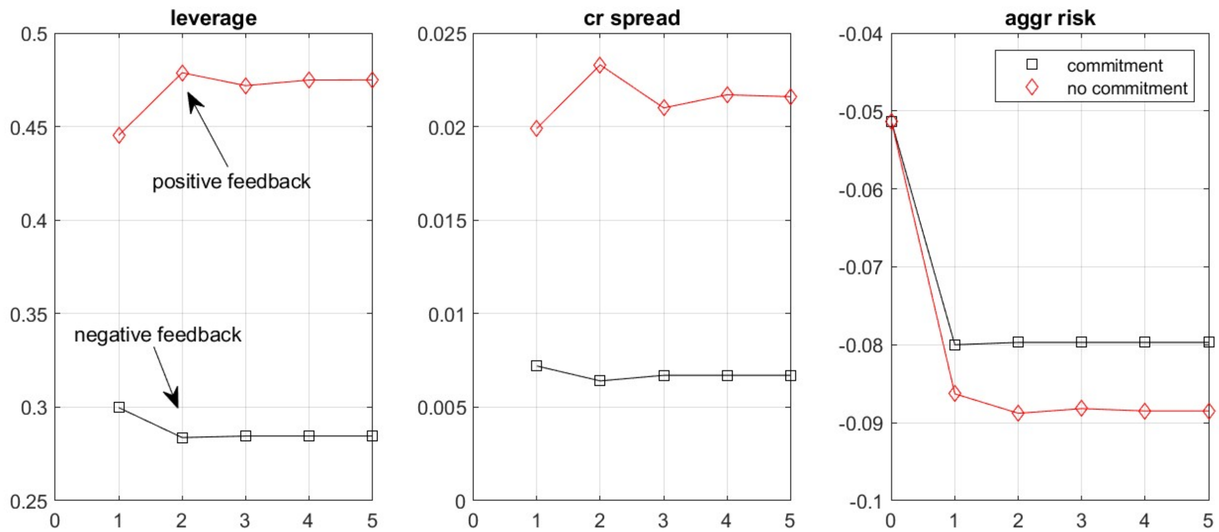
cyclicality in the sense that bad-time/good-time ratios are too small, whereas they are mainly too large in NC2. These patterns make sense in that the real world is likely best described as a middle case, with neither perfect commitment nor its total absence. Put differently, incorporating some degree of equity maximization in debt policy can improve the empirical performance of a standard trade-off model in explaining cyclical features of leverage, credit spreads, and defaults.

Before turning to welfare analysis, it is useful to illustrate the feedback effects that come into play in general equilibrium. Recall from Section 3 that if we view the general equilibrium solution as evolving iteratively from partial equilibrium decisions of managers,[26] then, starting from an initial pricing kernel, managers will first choose excessive leverage that then leads to increased aggregate risk, which then feeds back to a higher price to default risk, which subsequently lowers the firm's optimal debt. This process is shown in Figure 3 which plots the firm and aggregate outcomes at successive optimization steps for the C1 and NC1 models. What is noteworthy is that the negative feedback effect (seen in step 2 for the C1 model) does not operate, or is overridden, in NC1. Instead, in the no-commitment economy, the initially excessive debt leads to higher aggregate risk that lowers firm value, which increases expropriation incentives. This creates a positive feedback effect that increases debt further. In NC1, the effect is quantitatively significant, increasing optimal leverage by two to three percentage points, compared to a dampening effect of the same magnitude for C1.

Table 6 shows how the differences in capital structure policies across economies translate into social losses. According to Proposition 3, the form of households' value function is $j(s)\ Y^{1-\gamma}/(1-\gamma)$. So the change in value between two economies can be expressed in terms of equivalent fractions of permanent income via the change in the certainty equivalent $j(s)^{\frac{1}{1-\gamma}}$. The number can also be interpreted as the amount of wealth lost in moving from a firm value maximizing economy to an equity maximizing one. From

[26]The iteration described here is, in fact, the solution algorithm we implement.

Figure 3: Feedback Effects



For the calibrated models C1 and NC1, the table plots outcomes at successive optimization steps in the general equilibrium solution algorithm. The left panel shows the firm's optimal choice of leverage, the middle panel shows the resulting price of its debt (credit spread), and the right panel shows the level of aggregate risk defined as $\log(d)$ where d is the drop in the economy's output on a down jump.

the first column in the table, in the model with time-varying business conditions, the representative agent would lose an amount greater than 25 percent of permanent income from such a transition.[27]

The table contrasts this loss with the general equilibrium version of the private costs, as defined in Section 3, namely the change in firm value per unit output, v. In this case, that loss is much less: only 4 percent of firm value. (Note that the columns are expressed in comparable units, i.e., losses of value as a fraction of income.) One reason why the private cost is small is that the equity maximizing economy has a lower riskless rate r, due to higher precautionary savings, and this raises all asset values.[28] This conclusion,

[27]There are actually two values for this number, depending on the current state of the economy when the comparison is made. The numbers reported in the table take the average of the two social costs using the unconditional regime probabilities.

[28]If we compute private costs holding the pricing kernel fixed (as in Section 3) the private cost is higher, at 15.03% or 15.42% using the no commitment and commitment kernels respectively. Note that these costs are still much smaller than the social cost.

indicates that social incentives to restrict excess leverage exceed private incentives. Hence that there is a potential policy motivation to enhance capital structure contractibility.

We have emphasized two distortions due to lack of commitment: more default, and more cyclical default. The third column quantifies this by computing the social cost in the single-regime economies whose exogenous risk and growth rate equal the unconditional average of the two-regime economies. Here the social cost is still very large at over 13 percent of permanent income. However it is only half as bad as the social cost with business cycles. This is a consequence of the procyclical leverage dynamics without commitment, which increase default more in bad times. This is a key result of the paper.

Finally the right-most column reports that the private costs in the single regime economy is actually negative, reinforcing the conclusion that private incentives can strongly differ from social ones. This finding is again due to the effect of the lower interest rate in the equity maximizing economy.[29] It may seem paradoxical that the firm value maximizing economy could produce a lower firm value. However, this can happen precisely because managers do not take into account the general equilibrium implications of their policies.

The primary conclusions from the table are all robust to variation in the preference parameters. In unreported results,[30] we find that the social costs of equity maximization increase strongly with risk aversion (γ), are little changed by the EIS (ψ) and rise mildly with the rate of time preference (β). However, for all configurations we have examined, it remains the case that the social costs are substantially greater than the private cost, and increase strongly with business cycle risk. The computation assumes full loss of output from defaulted projects, which may seem extreme. However, raising the recovery parameter to, e.g., $\alpha = 0.50$, does not result in lower social costs because firms' optimal leverage scales up with α, worsening the externality and increasing default losses.

[29] If we compute private costs as in Section 3, the private cost is 7.54% or 3.56% using the no commitment and commitment kernels respectively.

[30] The results are available upon request.

Of course, there are still numerous caveats to the findings. The model does not account for any costs of achieving commitment, for example from monitoring and enforcement. These burdens would lower the private cost of noncommitment, but leave the social cost unaffected. Also, the model includes no special social *benefits* to debt, e.g., through resolving asymmetric information or moral hazard problems, and thus potentially increasing total investment. However, again, the comparisons we are drawing is between economies with and without commitment, where both include the same debt contract. It seems harder to argue that any social benefit to debt would *increase* when policy commitment is removed as a possibility. The model does not consider investment effects. In general equilibrium, there is another feedback channel from aggregate risk to optimal investment via the cost of capital. When the elasticity of substitution exceeds one, increased risk lowers investment lowering growth. Hence, the level and cyclicality effects of lack of commitment harm welfare more than in the cases presented here.

The analysis also neglects any welfare effects from taxation. If taxes fund welfare enhancing government fiscal policy, then lower – and more procyclical – tax receipts in the no-commitment economy would have another negative effect. The opposite would be true if taxes are distortionary.

Finally, as recognized above, it is true that the analysis here compares polar extremes: full commitment versus none. In real life, both protected and unprotected debt exist. Having noted this, it is also true that the analysis here considers only one dimension of contractibility, namely, the firm's leverage. In the real world, financial policy is multi-dimensional. Even debt policy encompasses numerous dimensions (including maturity, seniority, collateral, etc.) along which managerial choice could diverge from firm value maximization.[31] The degree to which our estimates overstate losses that are due to

[31] It is also worth noting that, in a model with repeated lending and stochastic commitment, Kovrijnykh (2013) finds that social costs are actually lower with intermediate levels of commitment than with none at all. Hence, it may be that our comparison does not exaggerate, but actually understates welfare effects.

partially incomplete contracting along many dimensions thus requires further analysis.

6. Conclusion

This paper contributes to the literature that seeks to analyze the real effects of financial policies, and to highlight the frictions that influence those policies. The inability to commit managers to firm-value maximizing, as opposed to equity-maximizing, debt policies imposes both private and social costs by increasing the risk of default and inefficient liquidation. Our setting allows us to quantify these costs in general equilibrium and with time-varying economic conditions. For a range of plausible parameters, the social cost significantly exceeds the costs borne by firms, implying that private incentives to solve the problem (e.g., by monitoring and enforcing covenants) may be insufficient from a policy standpoint.

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Table 5: Data and Model Moments

	DATA	MODELS			
	unconditional value				
	[bad /good]	C1	NC1	C2	NC2
bad time duration	1.10 [a] , 3.20 [b]			1.79	1.79
	[0.21 [a] , 0.42 [b]]			[0.25]	[0.25]
aggregate growth rate	2.03 [c] , 2.26 [d]	3.40	2.96	3.43	2.65
	[-0.67 [c] , -2.54 [d]]	—	—	[-0.52]	[-0.99]
aggregate volatility	3.99 [c] , 8.68 [d]	7.87	8.32	7.80	8.68
	[1.21 [c] , 1.18 [d]]	—	—	[1.22]	[1.39]
market leverage	43 [e] , 35 [f]	28.4	47.5	27.9	54.3
	[1.15 [g]]	—	—	[0.63]	[1.22]
default rate	1.52 [b] , 0.98 [f]	0.49	1.52	0.39	2.00
	[3.20 [b]]	—	—	[2.34]	[3.05]
credit spread	1.94 [e] , 0.95 [f]	0.67	2.16	0.67	4.30
	[1.65 [h]]	—	—	[1.00]	[2.00]
equity volatility, index	15.3 [c]	11.0	15.9	12.1	44.6
	[2.00 [c]]	—	—	[1.19]	[2.44]
equity volatility, firm	32.2 [i]	28.8	39.6	28.7	64.6
	[1.75 [i]]	—	—	[1.51]	[2.42]
equity PE	32.8 [c] , 16.9 [d]	33.0	58.6	26.7	40.1
	[0.69 [c] , 0.77 [d]]	—	—	[0.91]	[0.94]

The table shows data statistics and analogous quantities evaluated in calibrations of the general equilibrium model of Section 2.5. In the Data column for each statistic the first line shows estimates of unconditional means or volatilities, and the second line shows estimates of the ratio of the same quantities in bad times (recessions) to those in good times (expansions). Numbers in rows 2-8 are annual percentages. Sources/samples for the data moments are as follows. [a] NBER, 1926-2019 ; [b] Giesecke, Longstaff, Schaefer, and Strebulaev (2011), 1860-2008 ; [c] Shiller, S&P 500, real dividends, 1926-2019 ; [d] Shiller, S&P 500, real earnings, 1926-2019 ; [e] Huang and Huang (2012), Baa firms ; [f] Gomes and Schmid (2021) ; [g] Halling, Yu, and Zechner (2016). Compustat; [h] Moody's, 1919-2019 ; [i] Bekaert, Hodrick, and Zhang (2012). Models C2 and NC2 are regime-switching versions with and without capital structure commitment. Models C1 and NC1 are constant parameter versions with the same unconditional growth rate and volatilities of output and firm cash flow as the regime switching models. The regime switching models have parameters $\mu = [0.05, -0.025]$, $\sigma = [0.09, 0.135]$, $L = [2.3, 2.9]$, and $\omega = [0.14, 0.56]$. The single regime models use $\mu = 0.035$, $\sigma = 0.10$, and $L = 2.5$. All models have $\tau = 0.2$, $\alpha = 0$, and preference parameters $\gamma = 4$, $\beta = 0.05$, $\psi = 2$.

Table 6: Costs of Non-Commitment

WITH BUSINESS CYCLES		WITHOUT BUSINESS CYCLES	
social cost	private cost	social cost	private cost
25.78	4.41	13.46	-0.86

The table reports the percentage losses due to non-contractibility in financial policy in the economies described in Table 5. The private cost is the loss in value to owners of firms, expressed as a percentage of firm output. The social cost is the percentage reduction in the representative agent's value function between economies with and without commitment, expressed in terms of equivalent loss in aggregate output. The two left columns compare the regime switching economies C2 and NC2. The right-hand columns compare the single regime switching economies C1 and NC1.

Appendix

A. Proofs

This appendix provides the proofs of the results in Sections 2 and 3. We first establish the two lemmas on default policy. Then we derive the prices of assets in partial equilibrium. Then we give the general equilibrium solution.

Proof of Lemma 1.

The lemma asserts that the optimal default policy for equity holders is to abandon following a jump to Y_t if and only if the value of the firm is below the pre-jump value of debt, F_{t-}. To see this, if equity holders do not abandon, then their optimal debt policy at t is to adjust to the new quantity C_t whose value is F_t. If they do so, they repay the difference $F_{t-} - F_t > 0$ to debt holders, and their claim is then worth $V_t - F_t$. They will optimally do this if and only if the debt repayment is less than the value they receive:

$$V_t - F_t > F_{t-} - F_t \iff V_t > F_{t-}$$

as asserted.

From this observation, it follows that we can link the optimal leverage ratio with the critical default threshold. Default occurs iff $V_t \leq F_{t-}$. So, if equality holds, we have

$$e^{\underline{\varphi}} \equiv \frac{Y_t}{Y_{t-}} = \frac{V_t}{V_{t-}} = \frac{F_{t-}}{V_{t-}} = \frac{pc}{v}.$$

Here the middle equality uses the assumption that, prior to default, firm value is linear in output. The final equality uses the assumptions that, prior to default, C is linear in Y and F is linear in C. *QED*

Proof of Lemma 2. As above, equity holders will not abandon the firm upon a switch of states (from $s = 1$ to $s = 2$, say) if and only if the amount they owe to creditors is less than the net equity they will have upon payment, or $V(2) > F(1)$. Using the previous lemma, we can write $F(1) = e^{\underline{\varphi(1)}} V(1)$. The conclusion that default occurs if and only if $v(2)/v(1) < e^{\underline{\varphi(1)}}$ then follows given the linearity $V(s) = v(s)Y$ because Y is assumed independent of the state switching process. (The result for switches from 2 to 1 follows by symmetry.) *QED*

Next, since Proposition 1 is a special case of Proposition 2, it suffices to prove the latter. To facilitate understanding, we give the the key results in the single-regime case as well. The approach is to assume the form of the pricing kernel in (4) and then derive the equations satisfied by the bond and firm value. The proof conjectures and verifies that linear forms solve these equations.

The proofs use the notation $G^{\pm}(s)$ and $g^{\pm}(s)$ for the distribution and densities of up and down jumps in each state. It is useful to define the following definite integrals:

$$H(\underline{\varphi}) = \int_{-\infty}^{\underline{\varphi}} e^x \; g^-(|x|) \; dx \tag{17}$$

$$D(\underline{\varphi}) = \int_{\underline{\varphi}}^{0} e^x \; g^-(|x|) \; dx \; + \; \alpha H(\underline{\varphi}) \tag{18}$$

$$U = \int_{0}^{\infty} e^x \; g^+(x) \; dx. \tag{19}$$

Proof of Proposition 2.

First consider the debt claim whose value is F and whose coupon amount is C. The price of a claim to $1/C$ units of the debt is denoted p. The proposition assumes that we are given a default policy (pair) $\underline{\varphi}(s)$ determining the exit jump threshold, and that these are constants (not functions of Y). Under this assumption, it is reasonable to conjecture that, absent default, p is not a function of Y since jumps that do not trigger default leave the firm exactly as far from the threshold as before (and the jump size distribution is independent of Y).

Let T denote the sooner of the firm's default time or the repayment time of the claim. (The firm may retire debt at any time by repurchasing at the open market price, and we may assume that it does so *pro rata* across outstanding units.) Then on $[0, T)$, the price p obeys the canonical equation that requires that its instantaneous payout per unit time (in this case, 1) times the pricing kernel Λ equals minus the expected change of the product process $p\Lambda$. Using Itô's lemma for jumping processes to expand the expected change implies

$$\left(\eta + \tfrac{1}{2}\lambda\left(\left(u^{-\gamma}\mathrm{E_t}\left[\frac{p^+}{p}\right] - 1\right) + \left(d^{-\gamma}\mathrm{E_t}\left[\frac{p^-}{p}\right] - 1\right)\right)\right) p(s) + \omega(s)\left(\xi(s)p(s') - p(s)\right) = -1. \tag{20}$$

Here s and s' represent current and alternative regimes, respectively, and $\frac{p^+}{p}$ and $\frac{p^-}{p}$ denote the fractional changes in p conditional on an up and down jump, respectively.

(The notation suppresses the possible dependence of η, λ, α, u, and d on s.) Next, rewrite the left side using the fact that the expected growth rate of the pricing kernel is minus the riskless rate:

$$r(s) = -\eta(s) - \tfrac{1}{2}\lambda(s)\left(\left(u(s)^{-\gamma} - 1\right) + \left(d(s)^{-\gamma} - 1\right)\right) - \omega(s)\left(\xi(s) - 1\right) \tag{21}$$

to get

$$\left(-r - \tfrac{1}{2}\lambda d^{-\gamma}\left(1 - \mathrm{E_t}\left[\frac{p^-}{p}\right]\right)\right) p(s) + \omega(s)\xi(s)\left(p(s') - p(s)\right) = -1. \tag{22}$$

where we have used the conjecture that outside of default p is not affected by jumps to put $\frac{p^+}{p} = 1$. To evaluate the down-jump expected change in p, we integrate over the jump-size distribution. If the jump in output, $\frac{Y^-}{Y}$ is greater than $\exp(\underline{\varphi})$, then $\frac{p^-}{p} = 1$. For worse jumps, our assumption is that creditors gain the rights to α times the cash flow stream. This is worth $V(\alpha Y^-)$ per unit claim, so $V(\alpha Y^-)/C(Y)$ to holders of p. Hence the recovery fraction is $\frac{p^-}{p} = V(\alpha Y^-)/(pC) = V(\alpha Y^-)/F(Y) = e^{-\underline{\varphi}}\; V(\alpha Y^-)/V(Y)$, where the last equality uses Lemma 1. Using the linearity of V (to be verified below), the recovery fraction is $\alpha e^{x-\underline{\varphi}}$ where x is the jump size $\log(Y^-/Y)$. All together then, $\mathrm{E_t}\left[\frac{p^-}{p}\right]$ is,

$$\int_{\underline{\varphi}}^{0} g^-(|x|)\; dx + \alpha e^{-\underline{\varphi}} \int_{-\infty}^{\underline{\varphi}} e^x\; g^-(|x|)\; dx,$$

or

$$= G^-(|\underline{\varphi}|) + \alpha\; H(\underline{\varphi}) e^{-\underline{\varphi}}.$$

Hence, (22) becomes

$$\left(r + \tfrac{1}{2}\lambda d^{-\gamma}\left(1 - \left(G^-(|\underline{\varphi}|) + \alpha\; H(\underline{\varphi}) e^{-\underline{\varphi}}\right)\right)\right) p(s) - \omega(s)\xi(s)\left(p(s') - p(s)\right) = 1. \tag{23}$$

This is a linear system in p that can be written as $K_p\; p = 1_2$ where $1_2 = \{1, 1\}'$ and the coefficient matrix is

$$K_p = \left\{ \begin{matrix} r(1)+\tilde{o}(1)+\tilde{\ell}_d(1)(1-G^-(|\underline{\varphi}(1)|)-\alpha(1)H(1)e^{-\underline{\varphi}(1)}) & -\tilde{o}(1) \\ -\tilde{o}(2) & r(2)+\tilde{o}(2)+\tilde{\ell}_d(2)(1-G^-(|\underline{\varphi}(2)|)-\alpha(2)H(2)e^{-\underline{\varphi}(2)}) \end{matrix} \right\}$$

with $\tilde{o}(s) = \omega(s)\; \xi(s)$ and $\tilde{\ell}_d = \frac{1}{2}\lambda\; d^{-\gamma}$ being the risk neutral intensities of regime switches and down jumps, respectively.

The regularity condition referenced in the proposition is that K_p be positive definite. If the system has a strictly positive solution, this verifies that p is not a function of C and therefore $F = pC$. Similarly the solution is not a function of Y, verifying that conjecture.

In the one-regime case, the solution is

$$p(\underline{\varphi}) = 1./\left(r + \tilde{\ell}_d \left(1 - G^-(|\underline{\varphi}|) - \alpha\ H(\underline{\varphi})e^{-\underline{\varphi}}\right)\right) \tag{24}$$

For a perpetuity, the yield to maturity is $1/p$, so the credit spread is

$$\tilde{\ell}_d(1 - G^-(|\underline{\varphi}|) - \alpha\ H(\underline{\varphi})e^{-\underline{\varphi}}).$$

The first term is the risk neutral probability of default. The second is the risk neutral expected recovery. With zero recovery and uniform jumps Y'/Y, the second term is gone and first term is just $\tilde{\ell}_d\ e^{\underline{\varphi}}$.

Next, consider the valuation of the whole firm. Again, we are taking as given the default policy $\underline{\varphi}$. By Lemma 1, this gives us $e^{\underline{\varphi}} = F/V$ prior to default. And by the result just shown that $F = pC$, it follow that taking the default threshold as given is equivalent to taking the debt amount as a known function of V: $C = Ve^{\underline{\varphi}}/p$.

The after-tax cash flow stream to the firm prior to default is $(1-\tau)Y + \tau C$. Proceeding as above, we equate this quantity times Λ to minus the drift of $V\Lambda$, which yields

$$-\eta V - \mu Y\frac{\partial V}{\partial Y} - \tfrac{1}{2}\lambda\left(\left(u^{-\gamma}\mathrm{E_t}\left[V^+\right] - V\right) + \left(d^{-\gamma}\mathrm{E_t}\left[V^-\right] - V\right)\right) - \omega\left(\xi V' - V\right) = (1-\tau)Y + \tau C. \tag{25}$$

(This expression suppresses the dependence on the current state s and denotes the value of V in the other state as V'.) We now look for a linear solution $V = v(s)Y$. In that case, cancelling a factor Y, the right side can be written

$$v\ \left(-\eta - \mu - \tfrac{1}{2}\lambda\left((u^{-\gamma}U - 1) + (d^{-\gamma}D - 1)\right)\right) - \omega\left(\xi v' - v\right)$$

where we have defined $U(s)$ and $D(s)$ as $\mathrm{E}[Y^+/Y]$ and $\mathrm{E}[Y^-/Y]$, respectively. Using $C = Ve^{\underline{\varphi}}/p$, the left side is now

$$(1-\tau) + \tau v\ e^{\underline{\varphi}}/p.$$

Next, plugging in the expression for r in (21) the right side becomes

$$v \ \left(+r - \tilde{\mu} - \tfrac{1}{2}\lambda\left(u^{-\gamma}(U-u) + d^{-\gamma}(D-d)\right)\right) - \omega\xi\,(v'-v)$$

where we have defined $\tilde{\mu} = \mu + \frac{1}{2}\lambda(u^{-\gamma}(u-1) + d^{-\gamma}(d-1))$. Bringing the $e^{\underline{\varphi}}$ term to the right side, the above expression is equivalent to the linear system $K_v \ v = 1_2$ where the coefficient matrix is

$$K_v = \left\{ \begin{matrix} r(1)-\tilde{\mu}(1)+\tilde{o}(1)-\tilde{\ell}_d(1)(D(1)-d(1))-\tilde{\ell}_u(1)(U(1)-u(1))-\tau e^{\underline{\varphi}(1)}/p(1) & -\tilde{o}(1) \\ -\tilde{o}(2) & r(2)-\tilde{\mu}(2)+\tilde{o}(2)-\tilde{\ell}_d(2)(D(2)-d(2))-\tilde{\ell}_u(2)(U(2)-u(2))-\tau e^{\underline{\varphi}(2)}/p(2) \end{matrix} \right\}.$$

The second regularity condition is that K_v be positive definite. If so, then there is a unique positive solution to the system, verifying the linearity conjecture.

In the one-regime case, the pricing function is

$$v = (1-\tau)./\left(r - \tilde{\mu} - \tilde{\ell}_d\ (D-d) - \tilde{\ell}_u\ (U-u) - \tau e^{\underline{\varphi}}/p\right). \tag{26}$$

Despite some unfamiliar looking terms, the expression is just the tax-adjusted "Gordon growth" formula. The presence of the p term in the denominator means that the expression does not simplify greatly even in the case of uniform jumps with zero recovery. However it is worth remarking that, in general equilibrium, we will have $U = u$ and $D = d$ eliminating two of the terms.

QED

Turning to general equilibrium, the notation Y now denotes aggregate output, whose dynamics are derived in the text.

Proof of Proposition 3.

Given the aggregator function $f(C, J)$, the Bellman equation for J tells us that $\mathrm{E}[dJ] + f(C, J)\ dt = 0$. Under the conjectured form for $J = J(s, Y)$, we have $\mathrm{E}[dJ]/J =$

$$(1-\gamma)\left(\mu(s) + \tfrac{1}{2}\lambda\left((u(s)^{1-\gamma} - 1) + (d(s)^{1-\gamma} - 1)\right)\right) + \omega(s)\left(\frac{j(s')}{j(s)} - 1\right)$$

Dividing $f(C, J)$ by J and using $Y = C$, we get the two terms[32]

$$\beta\theta j(s)^{-\frac{1}{\theta}} - \beta\theta.$$

[32] Recall $f(C, J) \equiv \frac{\beta C^{\rho}/\rho}{((1-\gamma)\ J)^{1/\theta - 1}} - \beta\theta J$.

Adding these to the $\mathrm{E}[dJ]/J$ terms and multiplying by j gives:

$$\beta\theta j(s)^{1-\frac{1}{\theta}}-\beta\theta j(s)+(1-\gamma)\left(\mu j(s)+\tfrac{1}{2}\lambda\left((u(s)^{1-\gamma}-1)+(d(s)^{1-\gamma}-1)\right)j(s)\right)+\omega(s)(j(s')-j(s))=0.$$

This is the algebraic system referred to in the proposition, whose solution gives the constants $j(1), j(2)$.

Given solutions for J and C, Duffie and Skiadas (1994) show that the pricing kernel under stochastic differential utility is

$$\Lambda_t = e^{\int_0^t \; f_J(C_u,J_u)\; du}\; f_C(C_t, J_t).$$

Here, using $C = Y$ and the solution for J, we get $f_C(C, J) = \beta\; j(s)^{1-\frac{1}{\theta}}\; Y^{-\gamma}$, and

$$f_J(C, J) = \beta\theta \left((1-\frac{1}{\theta})\; j(s)^{-\frac{1}{\theta}} - 1\right).$$

The proposition then just evaluates the dynamics $d\Lambda/\Lambda$ from these expressions.

The integral term contributes an f_J term to the drift. To this we add df_C/f_C, which is

$$-\gamma\mu_Y\; dt + d\left(\sum_{j=1}^{\mathcal{J}_t}\left((u^{-\gamma}-1)1_{\{j,+\}}+(d^{-\gamma}-1)1_{\{j,-\}}\right)\right)$$
$$+d\left(\sum_{i=1}^{\mathcal{I}_t}\left(\left(\left(\frac{j(s')}{j(s)}\right)^{1-\frac{1}{\theta}}-1\right)1_{\{i,s'\}}+\left(\left(\frac{j(s')}{j(s)}\right)^{1-\frac{1}{\theta}}-1\right)1_{\{i,s\}}\right)\right).$$

The expression for η in the proposition is f_J plus the drift contribution from the previous expression. The ratios in the last term, which represent the fractional changes of f_C on a change in state, are are the quantities denoted $\xi(s)$. The expression for riskless rate is minus the expected change of $d\Lambda/\Lambda$.

QED

Returning to the single-regime case, we next prove Corollary 2.1.

Proof. The corollary considers the case $\alpha = 0$. So, using the expressions in Proposition 1, take the derivative of v with respect to $\underline{\varphi}$, and set it equal to the derivative of $\bar{c}\; p$. Bringing all multiplicative factors to the right side, and letting $y = 1/p$, left side is simply

$$(1-\tau)\tilde{\ell}_d\; g^{-}(|\underline{\varphi}|) - \tau y.$$

This represents the net marginal cost of debt: the first term corresponds to the marginal default cost and the second to minus the marginal tax benefit. Under commitment, the firm looks for a zero of this function to find the optimal $\underline{\varphi}$. To ensure one exists, and is unique, we impose conditions sufficient for the function to be monotonically increasing, and negative at $\underline{\varphi} = -\infty$ and positive at $\underline{\varphi} = 0$. Here $y = r + \tilde{\ell}_d \ (1 - G^-)$. So differentiating again, monotonicity is equivalent to the first condition assumed in the statement of the corollary: $\frac{1}{g^-(x)} \frac{dg^-(x)}{dx} \leq -\frac{\tau}{1-\tau}$. For the limit at infinity, g^- goes to zero, and y goes to r so the expression's limit is $-\tau r$ which is negative if r is positive. At $\underline{\varphi} = 0$, G^- is zero, so positivity is equivalent the second assumption $g^-(0) > \frac{\tau}{1-\tau} \frac{r+\tilde{\ell}_d}{\tilde{\ell}_d}$.

(In the numerical work, we assume G^- is a gamma distribution with mean σ and variance $L^2\sigma^2$, with $L \geq 1$. For this parameterization, the assumed conditions are satisfied when $L\sigma < \frac{1-\tau}{\tau}$.)

Next we turn to the right side of the first order condition. After some rearrangement, this is

$$(1 - \tau)\bar{c} \ \tilde{\ell}_d \ g^-(|\underline{\varphi}|) \ (p/v)^2 \ e^{-\underline{\varphi}}$$

which is positive for any $\bar{c} > 0$. Thus, if there is a $\bar{c}$ that satisfies the condition of a no-commitment equilibrium, the left side must intersect the right side and do so at a positive value of both. Since the left side has been shown to be monotonically increasing, it follows that the intersection is to the right of the firm-value maximizing point, which is the zero of the left side. A higher (less negative) value of $\underline{\varphi}$ corresponds to higher market leverage because of the optimal default condition that equates $e^{\underline{\varphi}}$ to $F/V = cp/v$.

We also note that, if $\bar{c}$ is an equilibrium solution, then we may use the latter condition, $\bar{c} = e^{\underline{\varphi}} \ v/p$ to substitute it out in the right-side expression above to obtain simply

$$(1 - \tau) \ \tilde{\ell}_d \ g^-(|\underline{\varphi}|) \ p/v.$$

This is the expression referred to in the text as the expropriation incentive.

□

It remains to prove Corollary 2.2.

Proof. To prove the existence and uniqueness of the no-commitment equilibrium, we need to find a unique fixed point c^* such that $c^* = \bar{c}^*$. Consider the marginal incentive function $m(\bar{c}) = \frac{dv}{dc} - \bar{c} \ \frac{dp}{dc}$. , it is the marginal incentives to alter debt when we start off with debt $c = \overline{c}$. If there is a unique c^* such that $c^* = \overline{c}^*$, then the unique no-commitment equilibrium exists.

If $\overline{c}$ is large enough, the objective function $v - \overline{c}p$ is always negative. Denote $\overline{c}_h$ as the highest $\overline{c}$ with nonnegative equity value. The domain of the fixed point $\overline{c}^*$ is $(0, \overline{c}_h)$.

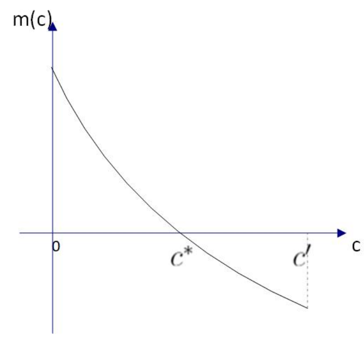
Here are three conditions we must establish to get the unique NC equilibrium.

(i) $m(c) > 0$ when $c \to 0$,

(ii) $m(c')$ is negative for some high value $c' < \overline{c}_h$,

(iii) $m(c)$ is monotonically decreasing in c, where c $\in (0, c')$

Figure 4



The figure illustrates the three conditions we need to establish to get the unique NC equilibrium.

In the following discussion, we list all the assumptions needed to establish each condition. For the sake of readability, we treat the case with $\alpha = 0$ and $\sigma = L = 0$.

Condition (i)

Given the assumptions in Corollary 2.1, we have condition (i). Intuitively, when the firm starts off with $\overline{c} = c = 0$, the marginal incentive being positive means it has the incentive to increase debt.

Condition (ii)

Assumptions needed to establish condition (ii) are the following:

Assumption 1.

$$\frac{dc}{d\underline{\varphi}} > 0$$

Assumption 2.

$$\frac{dp}{d\underline{\varphi}} > 0$$

Assumption 3.

$$\tilde{\ell}_d e^{\underline{\varphi}'} - \frac{\tau}{p'} > 0$$

First, we simplify the expression of $H(\underline{\varphi})$ and $D(\underline{\varphi})$.

$$\begin{aligned}
H(\underline{\varphi}) &= \int_{-\infty}^{\underline{\varphi}} \ e^x \ g^-(|x|) \ dx \\
&= \int_{-\infty}^{\underline{\varphi}} \ e^x \ g(-x) \ dx \\
&= \int_{-\underline{\varphi}}^{\infty} \ e^{-t} \ g(t) \ dt \\
&= \int_{-\underline{\varphi}}^{\infty} \ e^{-t} \frac{1}{\Gamma(a) b^a} t^{a-1} e^{-\frac{t}{b}} dt \\
&= \left(\frac{1}{b+1}\right)^a \int_{-\underline{\varphi}}^{\infty} \ \frac{1}{\Gamma(a)\left(\frac{b}{b+1}\right)^a} t^{a-1} e^{-\frac{t}{\frac{b}{b+1}}} dt \\
&= \left(\frac{1}{b+1}\right)^a - \left(\frac{1}{b+1}\right)^a \int_{-\infty}^{-\underline{\varphi}} \ \frac{1}{\Gamma(a)\left(\frac{b}{b+1}\right)^a} t^{a-1} e^{-\frac{t}{\frac{b}{b+1}}} dt \\
&= \left(\frac{1}{b+1}\right)^a - \left(\frac{1}{b+1}\right)^a G^-\big(|\underline{\varphi}| \ ; \ a, \frac{b}{b+1}\big) \\
&= \frac{1}{2} + \frac{1}{2} e^{2\underline{\varphi}},
\end{aligned}$$

where $G\big(\ x \ ; a, \frac{b}{b+1}\big)$ is the cumulative distribution function of the Gamma distribution with a and $\frac{b}{b+1}$ as parameters.

$$
\begin{aligned}
D(\underline{\varphi}) &= \int_{\underline{\varphi}}^{0} e^{x}\ g^{-}(|x|)\ dx\ +\ \alpha H(\underline{\varphi}) \\
&= -\int_{-\underline{\varphi}}^{0} e^{-t}\ g(t)\ dt\ +\ \alpha H(\underline{\varphi}) \\
&= -\int_{-\underline{\varphi}}^{0} e^{-t}\frac{1}{\Gamma(a)b^{a}}t^{a-1}e^{-\frac{t}{b}}dt +\ \alpha H(\underline{\varphi}) \\
&= -\left(\frac{1}{b+1}\right)^{a}\int_{-\underline{\varphi}}^{0} \frac{1}{\Gamma(a)\left(\frac{b}{b+1}\right)^{a}}t^{a-1}e^{-\frac{t}{\frac{b}{b+1}}}dt + \alpha H(\underline{\varphi}) \\
&= -\left(\frac{1}{b+1}\right)^{a}\left[\int_{-\underline{\varphi}}^{\infty} \frac{1}{\Gamma(a)\left(\frac{b}{b+1}\right)^{a}}t^{a-1}e^{-\frac{t}{\frac{b}{b+1}}}dt - 1\right] + \alpha H(\underline{\varphi}) \\
&=\left(\frac{1}{b+1}\right)^{a} - (1-\alpha)H(\underline{\varphi}) \\
&= -\frac{1}{2}e^{2\underline{\varphi}}.
\end{aligned}
$$

Therefore, $\frac{dH}{d\underline{\varphi}} = e^{2\underline{\varphi}}$, and $\frac{dD}{d\underline{\varphi}} = -e^{2\underline{\varphi}} < 0$.

Our goal is to prove that the marginal incentive $m(c') = \frac{dv}{dc'} - \bar{c}\frac{dp}{dc'} < 0$. As $\frac{dc}{d\underline{\varphi}} > 0$, it is the same as proving $\frac{dv}{d\underline{\varphi}}\Big|_{\underline{\varphi}=\underline{\varphi}'} - \bar{c}\frac{dp}{d\underline{\varphi}}\Big|_{\underline{\varphi}=\underline{\varphi}'} < 0$. Consider that

$$
\begin{aligned}
v &= \frac{1-\tau}{A} \\
A &= r - \tilde{\mu} - \tilde{\ell}_{d}\ (D-d) - \tilde{\ell}_{u}\ (U-u) - \tau e^{\underline{\varphi}}/p \\
p &= \frac{1}{B} \\
B &= r + \tilde{\ell}_{d}\big[1 - G^{-}(|\underline{\varphi}|) - \alpha\ H(\underline{\varphi})e^{-\underline{\varphi}}\big]
\end{aligned}
$$

We have

$$
\begin{aligned}
\frac{dv}{d\underline{\varphi}} &= -\frac{v^2}{1-\tau}\frac{dA}{d\underline{\varphi}}, \\
\frac{dp}{d\underline{\varphi}} &= -p^2\frac{dB}{d\underline{\varphi}}, \\
\frac{dA}{d\underline{\varphi}} &= -\tilde{\ell}_d\frac{dD}{d\underline{\varphi}} - \tau\frac{e^{\underline{\varphi}}}{p} + \tau\frac{e^{\underline{\varphi}}}{p^2}\frac{dp}{d\underline{\varphi}}, \\
\frac{dB}{d\underline{\varphi}} &= \tilde{\ell}_d g^-(|\underline{\varphi}|) \\
\frac{dp}{d\underline{\varphi}} &= -p^2\tilde{\ell}_d e^{\underline{\varphi}}
\end{aligned}
$$

Thus, we can simplify $\frac{dv}{d\underline{\varphi}}\Big|_{\underline{\varphi}=\underline{\varphi}'} - \bar{c}\frac{dp}{d\underline{\varphi}}\Big|_{\underline{\varphi}=\underline{\varphi}'}$ as

$$
\begin{aligned}
&\frac{dv}{d\underline{\varphi}}\Big|_{\underline{\varphi}=\underline{\varphi}'} - \bar{c}\frac{dp}{d\underline{\varphi}}\Big|_{\underline{\varphi}=\underline{\varphi}'} \\
=& -\frac{v'^2}{1-\tau}e^{\underline{\varphi}'}\Bigg[\underbrace{\tilde{\ell}_d e^{\underline{\varphi}'} - \frac{\tau}{p'}}_{>0}\Bigg] - \left[\frac{\tau}{1-\tau}e^{\underline{\varphi}'}\frac{v'^2}{p'^2} + \bar{c}\right]\underbrace{\frac{dp}{d\underline{\varphi}}}_{>0} < 0
\end{aligned}
$$

The economic intuition of condition (ii) is that for some high value $c' < \bar{c}_h$, the firm has the incentive to reduce its debt. From (i) (ii) and continuity of $m(c)$ we conclude there exists a c^* where $m(c^*) = 0$.

Condition (iii)

Assumptions needed to establish condition (iii) are the following:

Assumption 4.

$$\frac{d^2c}{d\underline{\varphi}^2} > 0$$

Assumption 5.

$$p\tilde{l}_d e^{\underline{\varphi}} > \frac{1}{2}$$

Assumption 6.

$$\frac{2}{v}\left(\frac{dv}{d\underline{\varphi}}\right)^2 - \frac{v^2}{1-\tau}\frac{d^2A}{d\underline{\varphi}^2} < 0$$

The sign of $\frac{dm}{dc}$ is the same as the one of $\frac{dm}{d\underline{\varphi}}$.

$$\frac{dm}{d\underline{\varphi}} \\ = \frac{d^2 v}{d\underline{\varphi}^2} - \bar{c}\frac{d^2 p}{d\underline{\varphi}^2} < 0$$

where

$$\begin{aligned}
\frac{d^2 p}{d\underline{\varphi}^2} &= \frac{2}{p}\left(\frac{dp}{d\underline{\varphi}}\right)^2 - \tilde{l}_d p^2 e^{\underline{\varphi}} \\
&= p^2 e^{\underline{\varphi}} \tilde{l}_d [2p\tilde{l}_d e^{\underline{\varphi}} - 1] > 0 \\
\frac{d^2 A}{d\underline{\varphi}^2} &= 2(1-\tau)\tilde{l}_d e^{2\underline{\varphi}} - \tau\frac{e^{\underline{\varphi}}}{p} - \tau e^{2\underline{\varphi}}\tilde{l}_d \\
\frac{d^2 v}{d\underline{\varphi}^2} &= \frac{2}{v}\left(\frac{dv}{d\underline{\varphi}}\right)^2 - \frac{v^2}{1-\tau}\frac{d^2 A}{d\underline{\varphi}^2} \\
&= \frac{2}{v}\left(\frac{dv}{d\underline{\varphi}}\right)^2 - \frac{v^2}{1-\tau}\left[2\tilde{l}_d e^{2\underline{\varphi}} - \tau\frac{e^{\underline{\varphi}}}{p} + 2\tau e^{\underline{\varphi}}\frac{1}{p^2}\frac{dp}{d\underline{\varphi}} - 2\tau e^{\underline{\varphi}}\frac{1}{p^3}(\frac{dp}{d\underline{\varphi}})^2 + \tau e^{\underline{\varphi}}\frac{1}{p^2}\frac{d^2 p}{d\underline{\varphi}^2}\right] < 0
\end{aligned}$$

Monotonicity of $m(c)$ ensures the uniqueness of the fixed point $\bar{c}^*$. □

A.1 Analysis of Sufficient Conditions

We first examine the conditions that lead to debt reduction incentives.

A necessary condition for existence of a fixed-point in the capital structure problem is that the marginal incentive function $\frac{dv}{dc} - c\frac{dp}{dc}$ (viewed as a function of c) must switch sign. In our solutions, it can be the case that, when existing debt is high enough, after a differential reduction $-dc$ the net gain to equity through reduced default risk can be positive. Or, equivalently, the fraction of the firm value increase that accrues to debt holders (which is $c\frac{dp}{dc}/\frac{dv}{dc}$) can be less than one. Referring to equations (26)-(24) and setting $\alpha = 0, \tilde{\ell}_d = 1$ to simplify, it is convenient to differentiate p and v with respect to the default threshold, $\underline{\varphi}$ (and then $\frac{d\underline{\varphi}}{dc} > 0$ appears in both expressions). [33] Then the two terms work out to be

$$c\frac{dp}{d\underline{\varphi}} = -p\ v\ e^{\underline{\varphi}}\ g^-(|\underline{\varphi}|)$$

[33]The default threshold affects the denominator of each expression, that is, we can write $p = 1/y_p(\underline{\varphi})$ and $v = (1-\tau)/y_v(\underline{\varphi})$.

(which uses $c = e^{\underline{\varphi}} v/p$), and

$$\frac{dv}{d\underline{\varphi}} = -v^2 \; e^{\underline{\varphi}} \left(g^{-}(|\underline{\varphi}|) - \frac{\tau}{1-\tau} \frac{1}{p} \right).$$

Cancelling a common $v \; e^{\underline{\varphi}}$, the change in firm value scales like v times the marginal change in default probability, $g^{-}(|\underline{\varphi}|)$, whereas the change in the debt price scales like p times that probability. Typically, $v > p$, and this is the basis for the observation that firm value can increases more than the value gain to existing creditors upon a debt reduction. The remaining term in the second equation $\frac{\tau}{1-\tau} \frac{v}{p}$ is always positive: this is the contribution of an increase dc to tax shields. But the incentive function is negative if

$$(v - p) \; g^{-}(|\underline{\varphi}|) > \frac{\tau}{1-\tau} \frac{v}{p}.$$

When c is small, $\underline{\varphi}$ is very negative and the marginal change in default probability is negligible. In this case the tax shield term dominates and the inequality goes the other direction. As c increases and $\underline{\varphi}$ rises towards zero, the left side can dominate (depending on the shape of the jump density) leading equity to prefer reduced debt. This may fail for many parameter configurations, and the model will reproduce the ratchet effect. However, for the cases we examine in a which a unique linear equilibrium exists, it *must* be true that equity holders' local incentives always lean in the direction of a debt reduction whenever $c > c^{\star}$, as happens following any down-jump in Y (unless it induces default).

B. Fixed Point Policies as an Equilibrium Outcome

This note formalizes our fixed-point capital structure policy as an equilibrium between the firm and the market in the following sense (that we adapt from Aguiar and Amador (2020)).

Definition B.1. *An equilibrium consists of a market-conjectured firm debt and default policies* $\{\tilde{C}, \tilde{t}_B\}$*, and the firm's actual policy* $\{C^*, t_B^*\}$ *, and a price schedule* $p(C)$ *at which the firm can buy or sell debt,*[34] *and a post-issuance/repurchase equity valuation function* $S(C)$*, having the following properties:*

(i) *The conjectured and actual policies are Markov processes in the state* (Y_t, C_{t-})*. The debt price schedule and equity valuation are functions of* (Y_t, C_{t-})*, that is,* $p(C) = p(C; Y_t, C_{t-}), S(C) = S(C; Y_t, C_{t-})$

(ii) *The market "breaks even" in the sense that* $p(C)$ *and* $S(C)$ *evaluated at the firm's policies are equal to the discounted expected cash flows to debt and equity, respectively, under the market's risk neutral measure.*

(iii) *Given the schedule* $p(C)$ *and equity valuation* $S(C)$*, at time* t *non-defaulting firms choose* C_t *to maximize the pre-adjustment market value of their equity. Firms default at* t *if and only if they anticipate that the maximal value is negative.*

(iv) *The firm's policy and the market's conjectured policy coincide.*

Note that the firm's maximization in *(iii)* has as its objective the pre-adjustment value $S(C) + p(C)[C - C']$ where C' denotes the pre-adjustment debt quantity.

Furthermore, from condition *(ii)* the post-adjustment equity value at t must satisfy

$$S(C) = \mathrm{E}_t^Q \left\{ \int_t^{\tilde{t}_B} e^{-r(s-t)} \, [(1-\tau)(Y_s - C_s)\, ds + p(C)\, dC_s] \right\}.$$

In Section 2 in the text, we focused on pricing linear debt strategies. Here, we impose that restriction in terms of a requirement for instantaneously homogeneous *changes* to debt policy.

[34] To clarify notation, the schedule defined by a function $p(C)$ means that, if the current debt is C' the firm can adjust to C'' by buying/selling $C'' - C'$ for unit price $p(C'')$.

Definition B.2. *The* **linear adjustment requirement** *for an issuance strategy $\hat{C}(Y_t, C_{t-})$ is that over each interval dt, the firm either defaults or issues/repurchases $dC = \frac{\hat{C}}{Y_t}\, dY$ at the market price $p(\hat{C} + dC)$.*

Formally, the requirement could be imposed over finite time intervals. That is, following Hugonnier, Malamud, and Morellec (2015), we could have the market be "open", i.e., provide price schedules, at a discrete set of times, $\{t_n\}$, and require that the firm issue $(C_{t_n}/Y_{t_n}) \int_{t_n}^{t} dY$ on (t_n, t_{n+1}). Maintaining the differential form of the requirement, the timing implict in point *(iii)* is as shown in the diagram below.

t $\qquad\qquad\qquad\qquad \overbrace{\qquad\qquad\qquad\qquad\qquad}^{t+dt}$

market offers $\{p(C), S(C)\}$ $\qquad$ dY realized $\qquad$ market offers $\{p(C), S(C)\}$

$\nearrow$ firm issues $(C_t/Y_t)\ dY$ $\qquad$ firm chooses C_{t+dt}

firm chooses C_t

$\searrow$ firm defaults

Next, we describe the fixed-point strategies and then verify that the above definition is satisfied.

Let $p1(c)$ and $v1(c)$ be the pricing functions defined in Proposition 1 in the text[35], and let $s1(c) = v1(c) - p1(c)c$. The proposition below verifies that the market can use these functions to define state-invariant price schedules, that is

$$p(C; Y_t, C_{t-}) = p1(C/Y_t) \quad \text{and} \quad S(C; Y_t, C_{t-}) = s1(C/Y_t)\ Y_t.$$

Proposition 4. *Assume there is exactly one fixed point $c^\star$ of the mapping $c(\bar{c}) = \arg\max_c [v1(c) - p1(c)\bar{c}$. Assume that at time $t, C_{t-} = c^\star Y_{t-}$ Then under the linear adjustment requirement, the following combination of prices and policies is an equilibrium in the sense of Definition B.1.*

[35]The proposition gives $p1$ and $v1$ in terms of the default threshhold $\underline{\varphi}$. Using Lemma 1, define $c = f(\underline{\varphi}) = \exp(\underline{\varphi})\ v1(\underline{\varphi})/p1(\underline{\varphi})$. Then let $p1(c) = p1(f^{-1}(c))$ and likewise for $v1(c)$.

- *The firm defaults at t if and only if $Y_t/Y_{t-} < c^\star p1(c^\star)/v1(c^\star)$.*

- *At any time t, for non-defaulting firms, the market offers $p1(c)$ and values the firm's equity as $s1(c)Y_t$.*

- *The firm does not issue or repurchase additional debt.*

Proof. The market conjectures that the firm will never deviate from $c = c^\star$, and that default will occur the first time that $Y_t/Y_{t-} < c^\star p1(c^\star)/v1(c^\star)$.

Assume we are at t and the firm has not defaulted. Then its current debt is $C = c^\star Y_t$ by the linear adjustment requirement. The firm's objective is $v1(c) - p1(c)c + p1(c)[c - c^\star] = v1(c) - p1(c)c^\star$ (times Y_t). By the fixed-point condition, this is maximized at $c = c^\star$. So not deviating is the policy that maximizes equity value. This verifies the first condition in *(iii)* of the definition.

Prior to this, the firm faces the decision of whether to default. Given the above outcome, not defaulting leads to the value $[v1(c^\star) - p1(c^\star)c^\star]Y_t$. The cost of attaining this is only positive when $Y_t < Y_{t-}$, i.e., at a down jump, at which point, instituting the extant policy costs $p1(c^\star)c^\star[Y_t - Y_{t-}]$. Since the payoff to defaulting is zero, equity maximization implies implementing the policy whenever

$$[v1(c^\star) - p1(c^\star)c^\star]Y_t - p1(c^\star)c^\star[Y_t - Y_{t-}] > 0,$$

which is equivalent to the second condition in *(iii)*.

Clearly the policy and the price schedules are Markov in (Y_t, C_{t-}), verifying *(i)* of the definition.

The firm optimal policy coincides with the market's conjecture, verifying *(iv)*.

For linear policies, the functions $p1(c)$ and $v1(c)$ are discounted risk neutral expectations by Proposition 1. Thus the transaction prices $p1(c^\star)$ and $s1(c^\star)$ are as well. Since the firm does follow the conjectured linear policy, the "break even" condition *(ii)* is satisfied.

$\square$

The upshot of the proposition is that at the fixed point, the market can offer prices with respect to which the manager maximizes, but which render no benefit to deviation. The policies and prices in the outcome described by the proposition are Markovian and define a stationary equilibrium.

The linear adjustment requirement is imposed so that the market never has to offer prices for firms away from the fixed point. It is not obvious how to formulate such prices and maintain an equilibrium while imposing the break-even condition. Alternatively, we could define the debt contract such that not issuing $(C_t/Y_t)\,dY_t$ triggers a technical default at $t + dt$.

If there are multiple fixed points, the specification of the strategy is simply that the market offers any of them to the firm, the firm chooses the one with highest equity payoff, and never deviates.

Extending the equilibrium to the two-regime case and proving the analogous result for the fixed-point pairs defined in Section 3 is straightforward.

B.1 Relation to Aguiar and Amador (2020)

In a recent paper, Aguiar and Amador (2020) develops a sovereign debt model in which the government cannot commit to a specific debt policy and can default at any time. They have shown that both a "borrowing equilibrium" and a "saving equilibrium" can emerge under a wide range of model parameters. Like our model, their model hasa dead-weight loss in bankruptcy. However, the two models have several notable differences: First, in Aguiar and Amador (2020), the government discount rate (ρ) is strictly greater than the investors' discount rate (r). In our model, the discount rate is determined in the stochastic discount factor of the economy, and is the same for both the firm and the investors (households). Second, in their model, the government debt has a random maturity structure with a constant hazard rate δ. At each instant, a fraction δ of the government bonds mature. All bonds that yet to mature have the same expected maturity going forward. We abstract bond maturity from our model. This corresponds to $\delta = 0$. Third, in their model, the government flow endowment is a constant Y. In our model, Y is a pure-jump process with a constant drift term μ. Fourth, in their model, the government's outside option to default follows an exogenous Poisson process. At any time, the government can default and walk away from its debt obligations with a value $\underline{V}$. At Poisson times (with a constant intensity λ), the outside option value to default jumps from $\underline{V}$ to $\bar{V}$. We model the allocation of bankruptcy value between shareholders and creditors in a standard way from the dynamic capital structure literature (e.g., Goldstein, Ju, and Leland, 2001): Upon default, the shareholders receive nothing and the creditors receive a fraction of the unlevered firm value going forward from the default time. The remaining fraction is the dead-weight cost of bankruptcy.

Their paper studies Markov perfect equilibria. Their equilibrium concept is similar to what we have above. Both equilibrium definitions require: (i) Lenders break even in expectation at their price schedule, given the borrower's debt policy and default policy; (ii) Given lenders' price schedule, at any instant, the borrower chooses the optimal debt and default policies to maximize its value.[36]

Aguiar and Amador (2020) develop two possible Markov perfect equilibria: the "borrowing" equilibrium and the "saving" equilibrium. In the "borrowing" equilibrium, the government issues debt and consumes the maximal possible consumption $\bar{C}$ until it reaches the endogenous debt limit (face value of the debt equal to) $\bar{b}_B$. At $\bar{b}_B$, the government consumes the amount that keeps its value function at $\underline{V}$, rolls over the debt to stay put at $\bar{b}_B$, and defaults the first time the option value to default jumps to $\bar{V}$.[37] In the "saving" equilibrium, if the government's continuation value $v \geq \bar{V}$, or equivalently, its debt level $b \leq \underline{b}_S$, which is called "safe zone" in their paper. The government stays inside the safe zone forever. Thus, it borrows sufficiently to consume $\bar{C}$ when it is strictly inside the safe zone. At the boundary $\underline{b}_S$, the government rolls over its debt to keep its debt level at $\underline{b}_S$, and consumes the difference between the flow endowment and the interest payment, to keep its continuation value at $\bar{V}$. If the debt level is mildly high that the government its in "crisis zone" but not far away from the safe zone boundary $\underline{b}_S$, i.e., $b \in \left(\underline{b}_S, b^I\right]$, the government will actively save until its value reaches $\bar{V}$ and its debt level is reduced to $\underline{b}_S$, i.e., the safe zone boundary. If the debt level is sufficiently high that $b > b^I$, the government's consumption and default policies are the same as those in the "borrowing" equilibrium. Both equilibria can be supported by the creditors believing the firm will follow the strategies in respective equilibria and price the debt using discounted risk-neutral expectations.

The co-existence of these two equilibria is because of the following two elements: (i) $\rho > r$, i.e., government is relatively impatient compared with the creditors, which favors borrowing and front-loading consumption, and (ii) time-invariant maturity structure, parameterized by δ, which favors saving to enjoy a higher debt rollover price. The two elements are emphasized throughout their paper. For example, Proposition 7 of their paper gives condition of model parameters to support the multiple equilibria. To

[36]In Aguiar and Amador (2020), the government's control is the consumption policy. This is equivalent to choosing the debt policy (face value of the debt) due to the accounting identity of equation (16) of their paper.The admissible consumption level is a closed interval $[\underline{C}, \bar{C}]$.

[37]$\bar{b}_B$ is determined endogenously by the government's indifference between following the aforementioned strategy and defaulting immediately.

illustrate, according to equation (26), if government is equally patient as the creditors, $\rho = r$, then the "borrowing " equilibrium will not exist. Meanwhile, according to equation (24) and the equation immediately after it, if the government debt maturity is perpetual, i.e., $\delta = 0$, the "saving" equilibrium will not exist as long as $\rho > r$. The constant flow endowment, together with the Poisson process of option value to default, facilitates the demarcation of debt space into "safe zone" ($V \geq \bar{V}$) and "crisis zone" ($V < \bar{V}$). Since the marginal benefit of additional debt is $\rho - r$, it justifies whenever the government is in borrowing regime, its consumption is always at the highest boundary $\bar{C}$.

Our model has neither differences in time preferences between firms and the market nor the finite debt maturity. Instead, we focus on linear coupon strategies, which is also a common model solution in the literature. Consistently, in our paper, upon observing any $c = \frac{C}{Y}$, the market believes that the firm will stick to the same target c until it defaults, and the market prices the debt using standard techniques. When the debt restructuring cost is zero, the natural solution concept is the fixed point definition of Section 2.

C. Generalization of Demarzo-He Model

In this section, we generalize the model of DeMarzo and He (2020) to encompass our cash flow jumps and pricing kernel. We also extend the single-regime partial equilibrium model used in the text by including a Brownian diffusion shock process $(W_t)_{t\geq 0}$ to the cash flow process $(Y_t)_{t\geq 0}$, i.e., $\frac{dY_t}{Y_t} = \mu dt + \sigma_W dW_t + d\left[\sum_{j=1}^{J_t}\left(e^{\varphi_j^{(i)}} - 1\right)\right]$. We solve the firm"s capital structure decisions without commitment, following the steps outlined in Subsection C.5 in DeMarzo and He (2020). We omit firm indicator i in the rest of this section.

C.1 Risk-neutral pricing

With the diffusion process, the pricing kernel becomes

$$\frac{d\Lambda_t}{\Lambda_t} = \eta dt - \theta dW_t + d\left[\sum_{j=1}^{J_t}\left(\left(u^{-\gamma} - 1\right) 1_{\{j+\}} + \left(d^{-\gamma} - 1\right) 1_{\{j-\}}\right)\right] ,$$

where θ is the market price of the diffusion risk. Consistent with the main text, we assume equal intensity for upward and downward jumps if a jump occurs at time t. The risk-free rate is

$$r = -\eta - \frac{1}{2}\lambda\left[\left(u^{-\gamma} - 1\right) + \left(d^{-\gamma} - 1\right)\right] . \tag{27}$$

The instant change of $\ln \Lambda_t$ is

$$d\ln\Lambda_t = \left(\eta - \frac{1}{2}\theta^2 - \frac{\lambda}{2}\gamma\left(\ln u + \ln d\right)\right) dt - \theta dW_t - \gamma\left(\ln u \tilde{J}^{+}(dt) + \ln d \tilde{J}^{-}(dt)\right)$$

where $\tilde{J}^{+}(dt) = J^{+}(dt) - \frac{\lambda}{2}dt$ and $\tilde{J}^{-}(dt) = J^{-}(dt) - \frac{\lambda}{2}dt$ are compensated Poisson processes for upward and downward jumps, respectively.

For any time-T payoff A, we have

$$E_t^Q\left[e^{-r(T-t)}A_T\right] = E_t\left[\frac{\Lambda_T}{\Lambda_t}A_T\right]$$
$$= E_t\Bigg[e^{-r(T-t)}\exp\Bigg\{-\frac{1}{2}\theta^2(T-t) - \theta\int_t^T dW_s + \frac{\lambda}{2}\left[\left(1-u^{-\gamma}\right)+\ln u^{-\gamma}\right](T-t)$$
$$+\frac{\lambda}{2}\left[\left(1-d^{-\gamma}\right)+\ln d^{-\gamma}\right](T-t)\int_t^T\left(\ln u^{-\gamma}\tilde{J}^+(ds)+\ln d^{-\gamma}\tilde{J}^-(ds)\right)\Bigg\}A_T\Bigg] \tag{28}$$

By the jump version of Girsanov's theorem, under the risk-neutral measure Q where

$$\frac{dQ_T}{dP_T} = \exp\Bigg\{\frac{\lambda}{2}\left[\left(1-u^{-\gamma}\right)+\ln u^{-\gamma}\right](T-t)+\frac{\lambda}{2}\left[\left(1-d^{-\gamma}\right)+\ln d^{-\gamma}\right](T-t)$$
$$+\int_0^T\left(\ln u^{-\gamma}\tilde{J}^+(ds)+\ln d^{-\gamma}\tilde{J}^-(ds)\right)\Bigg\},$$

Under the risk-neutral measure, the compensated Poisson processes for the upward and downward jumps under risk-neutral measure jump process become

$$\begin{aligned}\tilde{J}_Q^+(dt) &= \left(1-u^{-\gamma}\right)\frac{\lambda}{2}dt+\tilde{J}^+(dt)\\ &= J^+(dt) - u^{-\gamma}\frac{\lambda}{2}dt\end{aligned} \tag{29}$$

and

$$\begin{aligned}\tilde{J}_Q^-(dt) &= \left(1-d^{-\gamma}\right)\frac{\lambda}{2}dt+\tilde{J}^-(dt)\\ &= J^-(dt) - d^{-\gamma}\frac{\lambda}{2}dt\end{aligned} \tag{30}$$

$dW_t^Q = dW_t + \theta dt$ is a Brownian process under the risk-neutral measure. Under the risk neutral measure, the cash flow process $(Y_t)_{t\geq 0}$ follows

$$\frac{dY_t}{Y_t} = \mu_Q dt + \sigma_W dW_t^Q + d\left[\sum_{j=1}^{J_t^Q}\left(e^{\varphi_j}-1\right)\right] \tag{31}$$

where $\mu_Q = \mu - \sigma_W\theta$ and $\left(J_t^Q\right)_{t\geq 0}$ is a Poisson process governed by (29) and (30).

Following DeMarzo and He (2020), we assume the firm follows a smooth debt adjustment policy. Denote $G_t dt$ the rate of coupon changes over $[t, t+dt]$. Consistent with DeMarzo and He (2020), the debt in the model is a coupon bond with an exponential maturity structure. More specifically, over $[t, t+dt]$, $\xi C_t dt$ units of debt will mature and command repayments of $\xi C_t dt$, corresponding to an average bond maturity $\frac{1}{\xi}$.[38] Then the evolution of C is $dC_t = (G_t - \xi C_t)\, dt$. Let $p(Y, c)$ is the market value of debt per unit of coupon. The firm pays taxes of $\tau \times (Y - C)$. We assume zero-recovery rates in this section, consistent with the benchmark model in DeMarzo and He (2020).

Under these assumptions, the HJB equation of the equity $S(Y, C)$ is

$$\begin{aligned}
rS(Y,C) = \max_G \left[Y - \tau(Y-C) - (1+\xi)C + Gp(Y,C) + (G - \xi C)S_C(Y,C)\right] \\
+ \mu_Q Y S_Y(Y,C) + \frac{1}{2}\sigma_W^2 Y^2 S_{YY}(Y,C) + \lambda_Q^+ \int_0^\infty \left[S(e^\varphi Y, C) - S(Y,C)\right] d\Gamma_Q^+(\varphi) \\
+ \lambda_Q^- \int_{-\infty}^0 \left[S(e^\varphi Y, C) - S(Y,C)\right] d\Gamma_Q^-(\varphi)
\end{aligned} \tag{32}$$

in which λ_Q^+ and λ_Q^- are Poisson intensity for upward and downward jumps under risk-neutral probability measure, respectively, whereas Γ_Q^+ and Γ_Q^- are risk-neutral cumulative distribution functions of upward and downward jump sizes, respectively.

The first order condition for optimal G is

$$p(Y,C) + S_C(Y,C) = 0 \tag{33}$$

Then (32) becomes

$$\begin{aligned}
rS(Y,C) = Y - \tau(Y-C) - (1+\xi)C - \xi C S_C(Y,C) + \mu_Q Y S_Y(Y,C) + \frac{1}{2}\sigma_W^2 Y^2 S_{YY}(Y,C) \\
+ \lambda_Q^+ \int_0^\infty \left[S(e^\varphi Y, C) - S(Y,C)\right] d\Gamma_Q^+(\varphi) \\
+ \lambda_Q^- \int_{-\infty}^0 \left[S(e^\varphi Y, C) - S(Y,C)\right] d\Gamma_Q^-(\varphi)
\end{aligned} \tag{34}$$

Notice that here the equity value is as if the firm commits to never adjust its coupon rate in the future, in other words, when we remove the commitment power of the firm regarding coupon policies, shareholder is indifferent regarding the debt adjustment policies. This is the key result from DeMarzo and He (2020).

[38] An equivalent and more natural way is to assume a coupon bond with a face value FV_t, $\frac{C_t}{FV_t} = m$, which is a constant, and an average bond maturity $\frac{1}{\tilde{\xi}}$. In this case, over $[t, t+dt]$, $\tilde{\xi} FV_t dt$ units of debt will mature and command repayments of $\tilde{\xi} FV_t dt = \xi C_t dt$, in which $\xi = \frac{\tilde{\xi}}{m}$.

Note that the market price of debt per coupon is

$$p(Y,C) = E_t^Q \left[\int_t^{\underline{T}} e^{-(r+\xi)(s-t)} (1+\xi)\, ds | Y_t = Y, C_t = C \right]$$

By Feynman-Kac formula,

$$\begin{aligned}(r+\xi)\, p(Y,C) = 1 + \xi + (G - \xi C)\, p_C(Y,C) + \mu_Q Y p_Y(Y,C) + \frac{1}{2}\sigma_W^2 Y^2 p_{YY}(Y,C) \\ + \lambda_Q^+ \int_0^\infty \left[p\left(e^\varphi Y, C\right) - p(Y,C) \right] d\Gamma_Q^+(\varphi) \\ + \lambda_Q^- \int_{-\infty}^0 \left[p\left(e^\varphi Y, C\right) - p(Y,C) \right] d\Gamma_Q^-(\varphi)\end{aligned} \tag{35}$$

Differentiate (34) with respect to C, and using (33), we have

$$\begin{aligned}-rp(Y,C) = \tau - (1+\xi) + \xi p(Y,C) + \xi C p_C(Y,C) - \mu_Q Y p_Y(Y,C) \\ - \frac{1}{2}\sigma_W^2 Y^2 p_{YY}(Y,C) - \lambda_Q^+ \int_0^\infty \left[p\left(e^\varphi Y, C\right) - p(Y,C) \right] d\Gamma_Q^+(\varphi) \\ - \lambda_Q^- \int_{-\infty}^0 \left[p\left(e^\varphi Y, C\right) - p(Y,C) \right] d\Gamma_Q^-(\varphi)\end{aligned} \tag{36}$$

Adding (35) and (36), we obtain the optimal debt issuance rate, G:

$$0 = \tau + G p_C(Y,C) \Rightarrow G = \frac{\tau'(Y-C)}{-p_C(Y,C)} = \frac{\tau}{S_{CC}(Y,C)} \tag{37}$$

We have characterized the optimal coupon adjustment policy without commitment, G, using the HJB approach, which is similar to DeMarzo and He (2020). The next section solves for the equity value, $S(Y,C)$, optimal bankruptcy threshold, $\underline{T}$, and issuance rate, G, explicitly.

C.2 Equity value without coupon adjustment

One can follow the following steps outlined in Subsection C.5 in DeMarzo and He (2020) to solve the no-commitment equilibrium. First, solve the equity value $S(Y,C)$ as if the firm commits to never adjust coupon in the future. Then determine the debt price $p(Y,C)$ by (33). Afterwards check the global optimality condition by verifying that $p(Y,C)$ is weakly decreasing in C, or, equivalently, the equity value $S(Y,C)$ is convex in C. Finally, given $p(Y,C)$, determine the optimal coupon adjustment policy G using (37).

The downward jump complicates the calculation of equity value $S(Y,C)$. Intuitively, whenever the cash flow Y_t hits the bankruptcy threshold $\underline{Y}$ through a downward jump, it will "overshoot" the bankruptcy boundary $\underline{Y}$. To obtain the closed-form solution, we need to assume that the distribution of the downward jump size is independent of the hitting time $\underline{T}$ and $\underline{Y}$. Correspondingly, we assume that the distribution of jump size φ follows a double exponential distribution. The double exponential distribution is a special gamma distribution with shape parameter equal to 1. Specifically, the probability density function of φ takes the following form:

$$f(\varphi) = \frac{1}{2}\eta_u e^{-\eta_u \varphi} 1_{\varphi \geq 0} + \frac{1}{2}\eta_d e^{-\eta_d \varphi} 1_{\varphi < 0}$$

where $\eta_u = \frac{\sigma+1}{\sigma} > 1$, $\eta_d = \frac{1}{\sigma} > 0$.

We use a probabilistic approach to calculate the the equity value $S(Y,C)$. First of all, we express the equity value as the expected discounted value of future cash flows accrued to equity holders, starting with $Y > \underline{Y}$ and C,

$$\begin{aligned} S(Y,C) &= E\left[\int_0^{\underline{T}} e^{-rt}\left[(1-\tau)Y_t - (1+\xi-\tau)C_t\right]dt | Y_0 = Y, C_0 = C\right] \\ &= E\left[\int_0^{\infty} e^{-rt}\left[(1-\tau)Y_t - (1+\xi-\tau)C_t\right]dt | Y_0 = Y, C_0 = C\right] \\ &\quad - E\left[\int_{\underline{T}}^{\infty} e^{-rt}\left[(1-\tau)Y_t - (1+\xi-\tau)C_t\right]dt | Y_0 = Y, C_0 = C\right] \end{aligned} \tag{38}$$

Due to the independence between the diffusion term and the jump term as well as between the jump size and its occurrence, together with the fact that $C_t = Ce^{-\xi t}$, the first term of (38) becomes,

$$\begin{aligned} &(1-\tau)E\left[\int_0^{\infty} e^{-rt} Y \exp\left\{\left(\mu_Q - \frac{1}{2}\sigma_W^2\right)t + \sigma_W W_t^Q\right\} \prod_{j=1}^{J_t^Q} e^{\varphi_j} dt\right] - \frac{1+\xi-\tau}{r+\xi}C \\ &= (1-\tau)Y\int_0^{\infty} e^{-\left(r-\mu_Q-(Z-1)\lambda_Q\right)t}dt - \frac{1+\xi-\tau}{r+\xi}C \\ &= (1-\tau)\frac{Y}{r-\mu_Q-(Z-1)\lambda_Q} - \frac{1+\xi-\tau}{r+\xi}C\ , \end{aligned} \tag{39}$$

where $Z = E\left[e^{\varphi}\right] = \frac{q_u^Q \eta_u}{\eta_u - 1} + \frac{q_d^Q \eta_d}{\eta_d + 1}$, which is the risk-neutral expectation of the exponential jump size e^{φ}. If a jump occurs at time t, q_u^Q and q_d^Q are risk-neutral probabilities that the signs of jump are positive and negative, respectively. $q_u^Q = \frac{\frac{\lambda}{2}u^{-\gamma}}{\lambda_Q}$ and $q_d^Q = \frac{\frac{\lambda}{2}d^{-\gamma}}{\lambda_Q}$. $\lambda_Q = \frac{\lambda}{2}\left(u^{-\gamma} + d^{-\gamma}\right)$.

Now, let us calculate the second term of (38). Consistent with the literature, we assume that the equity holders follow a first-passage-time strategy and declare the bankruptcy at $\underline{T}$, which is the first time Y_t hits some lower boundary $\underline{Y}$.[39] Define $Y_t' = Y_{\underline{T}+t}$ and $C_t' = C_{\underline{T}+t}$. Then the second term becomes,

$$
\begin{aligned}
& E\left[\int_{\underline{T}}^{\infty} e^{-rt}\left[(1-\tau)Y_t - (1+\xi-\tau)C_t\right]dt | Y_0 = Y, C_0 = C\right] \\
&= E\left[e^{-r\underline{T}} E_{\underline{T}}\left[\int_0^{\infty} e^{-rt}\left[(1-\tau)Y_t' - (1+\xi-\tau)C_t'\right]dt | Y_0' = Y_{\underline{T}}, C_0' = C_{\underline{T}}\right]\right] \\
&= \frac{(1-\tau)}{r - \mu_Q - (Z-1)\lambda_Q} E\left[e^{-r\underline{T}} Y_{\underline{T}}\right] - \frac{1+\xi-\tau}{r+\xi} C E\left[e^{-(r+\xi)\underline{T}}\right]
\end{aligned} \tag{40}
$$

where the first equality follows from the tower rule of iterated expectations and the second equality follows from the strong Markov property of $(Y_t)_{t\geq 0}$. The rest of this subsection is devoted to solve two quantities: $E\left[e^{-\alpha\underline{T}}Y_{\underline{T}}\right]$ and $E\left[e^{-\alpha\underline{T}}\right]$. Here we follow Kou and Wang (2003) and use their formulas to directly derive the two quantities.[40] First, we introduce the following function.

$$
K(x) = -\left(\mu_Q - \frac{1}{2}\sigma_W^2\right)x + \frac{1}{2}\sigma_W^2 x^2 + \lambda_Q\left[\frac{q_u^Q \eta_u}{\eta_u + x} + \frac{q_d^Q \eta_d}{\eta_d - x} - 1\right] \tag{41}
$$

Then, according to Kou and Wang (2003),

$$
E\left[e^{-r\underline{T}}Y_{\underline{T}}\right] = c_1 \underline{Y}\left(\frac{\underline{Y}}{Y}\right)^{\beta_1} + c_2 \underline{Y}\left(\frac{\underline{Y}}{Y}\right)^{\beta_2} \tag{42}
$$

where $c_1 = \frac{\eta_d - \beta_1}{\beta_2 - \beta_1}\frac{\beta_2 + 1}{\eta_d + 1}$ and $c_2 = \frac{\beta_2 - \eta_d}{\beta_2 - \beta_1}\frac{\beta_1 + 1}{\eta_d + 1}$. β_1 and β_2 are two positive roots of $K(x) = r$ with $0 < \beta_1 < \eta_d < \beta_2 < \infty$.

$$
E\left[e^{-(r+\xi)\underline{T}}\right] = h_1\left(\frac{\underline{Y}}{Y}\right)^{\gamma_1} + h_2\left(\frac{\underline{Y}}{Y}\right)^{\gamma_2} \tag{43}
$$

[39] Notice that unlike the pure diffusion case, here if the bankruptcy is triggered by a downward jump, $Y_{\underline{T}}$ will be smaller than $\underline{Y}$.

[40] An alternative approach is to solve an integro-ordinary differential equation, but the above two quantities are not differentiable at the boundary. For details, please refer to Kou and Wang (2003).

where $h_1 = \frac{\eta_d - \gamma_1}{\eta_d} \frac{\gamma_2}{\gamma_2 - \gamma_1}$ and $h_2 = \frac{\gamma_2 - \eta_d}{\eta_d} \frac{\gamma_1}{\gamma_2 - \gamma_1}$. γ_1 and γ_2 are two positive roots of $K(x) = r + \xi$ with $0 < \gamma_1 < \eta_d < \gamma_2 < \infty$. Bringing (42) and (43) to (40), we have

$$
\begin{aligned}
&\frac{(1-\tau)}{r - \mu_Q - (Z-1)\lambda_Q} E\left[e^{-r\underline{T}} Y_{\underline{T}}\right] - \frac{1+\xi-\tau}{r+\xi} C E\left[e^{-(r+\xi)\underline{T}}\right] \\
= &\frac{(1-\tau)}{r - \mu_Q - (Z-1)\lambda_Q} \left[c_1 \underline{Y} \left(\frac{Y}{\underline{Y}}\right)^{\beta_1} + c_2 \underline{Y} \left(\frac{Y}{\underline{Y}}\right)^{\beta_2}\right] \\
&- \frac{1+\xi-\tau}{r+\xi} C \left[h_1 \left(\frac{Y}{\underline{Y}}\right)^{\gamma_1} + h_2 \left(\frac{Y}{\underline{Y}}\right)^{\gamma_2}\right]
\end{aligned} \tag{44}
$$

Therefore, the equity value $S(Y, C)$ is

$$
\begin{aligned}
S(Y, C) = (1-\tau) &\frac{Y}{r - \mu_Q - (Z-1)\lambda_Q} - \frac{1+\xi-\tau}{r+\xi} C \\
&- \frac{1-\tau}{r - \mu_Q - (Z-1)\lambda_Q} \left[c_1 \underline{Y} \left(\frac{Y}{\underline{Y}}\right)^{\beta_1} + c_2 \underline{Y} \left(\frac{Y}{\underline{Y}}\right)^{\beta_2}\right] \\
&+ \frac{1+\xi-\tau}{r+\xi} C \left[h_1 \left(\frac{Y}{\underline{Y}}\right)^{\gamma_1} + h_2 \left(\frac{Y}{\underline{Y}}\right)^{\gamma_2}\right] \\
= &\frac{1-\tau}{r - \mu_Q - (Z-1)\lambda_Q} \left[Y - \underline{Y} \left[c_1 \left(\frac{Y}{\underline{Y}}\right)^{\beta_1} + c_2 \left(\frac{Y}{\underline{Y}}\right)^{\beta_2}\right]\right] \\
&- \frac{1+\xi-\tau}{r+\xi} C \left[1 - h_1 \left(\frac{Y}{\underline{Y}}\right)^{\gamma_1} - h_2 \left(\frac{Y}{\underline{Y}}\right)^{\gamma_2}\right]
\end{aligned} \tag{45}
$$

C.2.1 Optimal bankruptcy boundary

The optimal bankruptcy boundary $\underline{Y}$ is such that $S_Y(Y, C)|_{Y=\underline{Y}} = 0$. Differentiating (45) with respect to Y and setting it to zero, we have

$$
\underline{Y} = \frac{r - \mu_Q - (Z-1)\lambda_Q}{1 + c_1\beta_1 + c_2\beta_2} \frac{1+\xi-\tau}{1-\tau} \frac{C}{r+\xi} (h_1\gamma_1 + h_2\gamma_2) \tag{46}
$$

We could prove the following:[41]

(i) At $Y = \underline{Y}$, the equity value is equal to zero: $S(\underline{Y}, C; \underline{Y}) = 0$.

(ii) The bankruptcy threshold must be weakly greater than $\underline{Y}$ to ensure the limited liability constraint: $\forall Y \geq \underline{Y}$, $S(Y, C; \underline{Y}) \geq 0$.

[41] The proofs are tedious algebra manipulations and are omitted here. They are available upon request.

(iii) (Local convexity) $\forall Y \geq \underline{Y}$, $S_Y(Y, C) > 0$.

(iv) For any other bankruptcy threshold $\underline{\hat{Y}} \geq \underline{Y}$, $S_{\underline{\hat{Y}}}\left(Y, C; \underline{\hat{Y}}\right) \leq 0$.

Therefore, $\underline{Y}$ is the optimal bankruptcy threshold. The derivative of $\underline{Y}$ with respect to C is:

$$\frac{\partial \underline{Y}}{\partial C} = \frac{r - \mu_Q - (Z-1)\lambda_Q}{1 + c_1\beta_1 + c_2\beta_2} \frac{1+\xi-\tau}{1-\tau} \frac{h_1\gamma_1 + h_2\gamma_2}{r+\xi} \tag{47}$$

C.3 Debt price and optimal coupon adjustment policy

From (33) and (47), the debt price is

$$\begin{aligned} p(Y, C) &= -S_C(Y, C) \\ &= \frac{1+\xi-\tau}{r+\xi} \frac{h_1\gamma_1 + h_2\gamma_2}{1 + c_1\beta_1 + c_2\beta_2} \left[c_1(1+\beta_1)\left(\frac{\underline{Y}}{Y}\right)^{\beta_1} + c_2(1+\beta_2)\left(\frac{\underline{Y}}{Y}\right)^{\beta_2} \right] \\ &\quad + \frac{1+\xi-\tau}{r+\xi} \left[1 - h_1(1+\gamma_1)\left(\frac{\underline{Y}}{Y}\right)^{\gamma_1} - h_2(1+\gamma_2)\left(\frac{\underline{Y}}{Y}\right)^{\gamma_2} \right] \end{aligned} \tag{48}$$

One can check that $p(Y, C) > 0$. From (37), the optimal coupon adjustment policy $G(Y, C) = \frac{\tau}{-p_C(Y,C)} = \frac{\tau}{S_{CC}(Y,C)}$, where

$$\begin{aligned} S_{CC}(Y, C) = \frac{1+\xi-\tau}{(r+\xi)C} \Bigg\{ &\left[h_1(1+\gamma_1)\gamma_1\left(\frac{\underline{Y}}{Y}\right)^{\gamma_1} + h_2(1+\gamma_2)\gamma_2\left(\frac{\underline{Y}}{Y}\right)^{\gamma_2} \right] \\ &- \frac{h_1\gamma_1 + h_2\gamma_2}{1 + c_1\beta_1 + c_2\beta_2} \left[c_1(1+\beta_1)\beta_1\left(\frac{\underline{Y}}{Y}\right)^{\beta_1} + c_2(1+\beta_2)\beta_2\left(\frac{\underline{Y}}{Y}\right)^{\beta_2} \right] \Bigg\}. \end{aligned} \tag{49}$$

D. Calibration Data Sources

This appendix provides details of the sources and samples for which empirical moments are listed in the Data column of Table 5.

For the duration of "bad times" and "good times", the first number uses the set of NBER recession and expansion dates from 1926 through 2019. The second number uses the years identified as being in default clusters by Giesecke, Longstaff, Schaefer, and Strebulaev (2011) from 1860 to 2008.

For average growth rates and volatilities of aggregate cash flow, and for the equity price-earnings ratio and equity index volatility, we use data on S&P 500 companies from Robert Shiller's website `http://www.econ.yale.edu//~shiller/data.htm` for the years 1926-2019. For the cyclicality ratios, we use NBER recession dates.

Average market leverage numbers are from Gomes and Schmid (2021) Table 2 and Huang and Huang (2012) Table 1, where values are reported by rating. We take the average reported for Baa firms. For the cyclicality of leverage, we utilize the estimates of target leverage ratios in Halling, Yu, and Zechner (2016) who utilize all Compustat firms and NBER recession dates.

For average default rates, we again utilize the sample of Giesecke, Longstaff, Schaefer, and Strebulaev (2011) who report value weighted annual default rates. We use these rates along with that paper's classification of default clusters to compute the cyclicality ratio. Also reported in the table is 1/3 of the 3-year average rate reported by Gomes and Schmid (2021).

For average credit spreads, we again utilize Gomes and Schmid (2021) Table 2 and Huang and Huang (2012), Table 1. For the cyclicality ratio we use the Moody's Baa-Aaa spread from FRED, and NBER recession dates from 1919 to 2019.

For average firm-level volatility and its cyclicality ratio we utilize the estimates from the regime-switching model reported Bekaert, Hodrick, and Zhang (2012) for idiosyncratic volatility of U.S. firms. We add back the average systematic volatility from the previous row (assuming a market beta of one for the average firm).

E. Empirical Evidence

A key prediction from the model is that, in the absence of commitment to capital structure, firms will exhibit a greater degree of countercyclicality of leverage policy. That is, they will tend to increase leverage more in bad times (or decrease it less) than they would under complete contracting. This countercyclicality contributes an extra component to the general equilibrium effects documented in the previous section, on top of the effect of unconditionally higher leverage from lack of commitment. We now bring this prediction to the data.

We test the idea using cross-sectional variation in debt protection. Before examining the results, it is important to first address the endogeneity of that protection. We argued in Section 4 that firms that choose unprotected debt will be the ones for which the surplus loss from lack of commitment to value-maximizing policies is smaller. For the current section to provide valid tests of the model prediction, we need to verify that, when the protection choice is driven by the *unconditional* surplus gain from commitment, this does not affect the *conditional* cyclicality differential across firms with and without protection.

Indeed, for a range of numerical examples like those in Section 4, this is the case. For example, contrasting a low-growth firm ($\mu = [0.03, 0.00]$ in the two states) with commitment, to a high-growth one ($\mu = [0.06, 0.03]$) without commitment, the former optimally reduces its outstanding debt (C) by 21% in the bad state while the latter reduces only by 15%, in line with our hypothesis.[42] While this is not to dismiss all concerns regarding endogenous debt protection, it does show that, at least within the model, the prediction we wish to test is robust to this possibility.

Next, the main empirical challenge is to find a measure for firms' degree of commitment. A firm with outstanding syndicated loans typically has a set of financial covenants that restrict the firm's accounting ratios and financial quantities within a specific range. A firm with stringent covenants therefore has less scope to adjust its financial policies to exploit existing creditors, simply because doing so would risk relinquishing control rights to those creditors. Prior research (e.g., Chava and Roberts (2008)) has demonstrated that the presence of such covenants and the associated risk of transfer of corporate control following covenant violation does, in fact, restrict the borrowing firm's investment, payout and debt financing policies *ex ante*. In this sense, covenant protection is a natural proxy for the degree of capital structure commitment.

[42] Other parameters for this calculation are as given in the caption to Table 3.

Following the existing literature, we use two measures for covenant strictness of a loan contract based on the LPC-Dealscan database. DealScan reports contract details from syndicated and bilateral loans collected by staff reporters from lead arrangers and SEC filings starting from 1981. The first measure is simply the (log) number of covenants of each loan package.[43]

The second measure, first proposed by Murfin (2012), is the estimated *ex ante* probability of covenant violation when the loan contract is initiated. In practice, there is a wide range of covenant strictness both within and across loan packages. The calculation assumes that the changes in financial ratios follow a multivariate normal distribution with mean zero and a variance-covariance matrix that varies across industries and over time. From this distribution, we then compute the probability that firm's own ratio will fall outside the restricted range during the life of the loan.[44] We apply this estimation to our sample, and extend the sample period to the present. Following Murfin (2012), we exclude loan contracts containing covenants that appear to be violated at the beginning of the contract. However, our results are robust if we assume that the violation probability for those contracts are 1.0.

For each firm-quarter, we calculate the two measures of protection for all the active loan contracts that the firm has, and take the maximum of each as our measures of firm covenant strictness for that particular quarter. The covenant strictness measure requires more non-missing financial observations resulting in a smaller sample size. (In particular, it is not computable for any of the loan observations in the 1980s). We note that our measures do not take into account potential covenants in other debt instruments of the sample firms (e.g., public bond issues) that are not covered in Dealscan. The sample also does not include any observations of firms that do not have outstanding syndicated loans. For a detailed description of the selection effects of the LPC-DealScan database, see Chava and Roberts (2008).

Table A1 presents the summary statistics of the two samples in our study, corresponding to the periods in which each of the commitment proxies can be computed. Despite losing roughly 40 percent of the raw covenant observations, the characteristics of the Murfin firm-quarters are very close to those of the larger sample.

[43]Loan transactions are reported in both facility-level and package-level in LPC-Dealscan database, where a package is a collection of facilities. We calculate our covenant strictness measures at package level, since covenants are only reported at package level. The tests below restrict the sample to firm-quarters in which there is at least one covenant. Results are not affected by including loans without covenants and using the raw number (i.e., not the log).

[44]For details about the construction of this measure, please refer to Murfin (2012).

We are interested in the relationship between a firm's debt policy and its covenant strictness. We use the quarterly COMPUSTAT database to compute measures of financing activities and other control variables.[45] For the debt financing measure, we use changes in total financial debt ($dlcq + lttq$) scaled by lagged total assets (atq). Welch (2011) points out that the above ratio is problematic in the sense that it treats non-financial liabilities as equity instead of debt. He proposes two alternative denominators for leverage ratios: (i) lagged sum of financial debt and book value of equity ($seqq$); and (ii) the lagged sum of financial debt and market value of equity, defined as the product of common share price and common share outstanding ($prccq \times cshoq$). We use these two alternative scalings as robustness checks.

Other control variables follow respective measures in the literature (e.g., Covas and den Haan (2011)). We control for the following firm-level variables: logarithmic total assets ($\log(atq)$); Tobin's Q, the sum of market value of equity, capital value of preferred stock (pstkq), dividend paid on preferred stock ($dvpq$) and total liabilities (ltq), over total assets; cashflow, defined as the sum of income before extraordinary items (ibq) and depreciation and amortization (dpq) scaled by lagged total assets; asset tangibility, the net value of property plant and equipment ($ppentq$) over total assets.[46]

The key prediction of the model is that compared with firms without commitment, firms that can commit to their capital structures reduce their debt more in bad times. Following the recent asset pricing literature, literature (e.g., Joslin, Priebsch, and Singleton (2014); Giglio, Kelly, and Pruitt (2016)), we first use two proxies for the aggregate state, based on the Chicago Fed National Activity Index (CFNAI), a weighted average of 85 monthly indicators of US economic activity. The first measure is the composite index of all indicators. Then, as a second measure, we use the sub-index (the Production and Income series) that focuses on the conditions of the corporate sector. For both indexes, negative values indicate below-average growth (in standard deviation units); positive values indicate above-average growth. To facilitate the interpretation of the results, we define our bad-times variable (*recession*) as the negative of the three-month moving average of these two monthly series, so a high value means the economic growth rate is lower.

[45] The COMPUSTAT variable codes are given in italics.

[46] Also following the literature, we exclude financial firms and regulated utilities. We also exclude firm quarters with $Q > 10$ or $Q < 0$.

The regression specification for our analysis, then, is the following

$$\begin{aligned} Y_{i,t} = \alpha + \beta\ commitment_{i,t-1} &+ \delta\ recession_{i,t} + \gamma\ commitment_{i,t-1} \cdot recession_{i,t} \\ &+ B\ controls_{i,t-1} + \zeta_i + \xi_t + \epsilon_{i,t} \end{aligned} \tag{50}$$

where the outcome variable is $Y_{i,t}$ is the change in debt from $t-1$ to t, scaled as described above.[47] The unit observation is a firm-quarter. Previous studies show that unobserved firm-level time-invariant factors explain a majority of variations in leverage ratios Lemmon, Roberts, and Zender (2008). Hence, we include firm fixed effects. We also include fiscal-year fixed effects. Standard errors are clustered at the firm level and are robust to heteroscedasticity.

The hypothesis of interest is that $\gamma < 0$: the difference in debt issuance between firms with and without commitments is more negative when the aggregate growth rate of the economy is lower. Table A2 presents the results. The top panel uses the composite CFNAI recession measure; the bottom panel uses the Production and Income sub-index.

In both panels, with either covenant measure, and using each of the three scalings for debt changes, the coefficient on the interaction term is negative and statistically significant at the 5% or 1% level. Consistent with prior literature, covenant restrictions are unconditionally associated with greater reductions in debt. Consistent with Halling, Yu, and Zechner (2016), debt issuance is also overall countercyclical, i.e., more positive in recessions. However, firms with greater debt protection follow the opposite pattern, reducing debt in bad times.

Caption to Table A2. The table reports results of panel regressions of quarterly debt changes on measures of debt protection and economic conditions. The first three columns use the Murfin (2012) measure of covenant strictness. The next three columns use the log number of covenants. The covenant measures are lagged by one quarter. For each measure, three standardizations of debt changes are used as the dependent variable, following Welch (2011). In Panel A, the economic conditions variable (Recession) is the negative of the three-month moving average of the Chicago Fed National Activity Index (CFNAI). In Panel B, the variable is constructed likewise from the CFNAI Production and Income sub-series. Control variables are as defined in the text. Robust standard errors are clustered at firm level and are shown in parentheses. Asterisks (*,**,***) denote significance at the 10%, 5%, and 1% level, respectively.

[47] Under our model, firms adjust their debt continuously as their own output varies. Since output will be correlated with the state of economy, we also run versions of the regression that control for contemporaneous (or lagged) changes in firm sales. Inferences are unaffected and the point estimate of γ are similar. These specifications are omitted for brevity.

Regarding the economic significance of the interaction effect, consider the change in implied debt issuance when the economy goes from one standard deviation above trend to one standard deviation below trend. Using the numbers from Table A1, this corresponds to an increase of 1.528 in the main recession measure. From the point estimates in column (1) of Panel A, a firm with well protected debt (strictness one standard deviation above the mean) would be expected to decrease debt by .00086 times assets, whereas a firm with unprotected debt (strictness one standard deviation below the mean) would increase debt by .00149 times assets. The difference in responses is 0.235% of assets per quarter or almost 1% per year. Viewed as an aggregate effect, this magnitude is economically meaningful. During the recent financial crisis, for example, in roughly two years from trough to peak back to trough, the economy's net debt to total book assets moved up and then down by approximately 4 percentage points,[48] while the CFNAI index dropped and then recovered by approximately four standard deviations. Thus, the effects estimated in the regression suggest that the contracting friction studied here could potentially play a significant role in aggregate financial cycles.

As a robustness test, Table A3 repeats the regressions replacing the measure of economic activity with measures of aggregate firm valuations. These tests directly confront the model's implication that expropriation incentives scale inversely with valuation multiples. Hence here the "recession" or "bad times" measure is the reciprocal of economy-wide estimates of Tobin's Q. The top panel computes the value-weighted average of firm-level Q for each COMPUSTAT firm each quarter. The results again strongly support the hypothesis of a negative interaction coefficient. The bottom panel uses the "bond-market Q" measure of Philippon (2009), which uses aggregate bond valuation data to impute Q from a structural model. This series only goes through 2007, reducing the power of the test, but also affirming that the findings here are not driven by the subsequent financial crisis. Using the Murfin proxy for commitment, the sample period only encompasses 1990-2007 and there is very little variation in the Q measure during this time. Still, all the point estimates are all negative, and two cases attain statistical significance at the 10% level. With the log covenant proxy for commitment, the number of observations almost doubles. Now, although the point estimates are actually reduced, they are highly statistically significant.

[48] The calculation uses the Federal Reserve Z.1 data for the U.S. nonfinancial corporate sector.

Caption to Table A3. The table reports results of panel regressions of quarterly debt changes on measures of debt protection and economic conditions. The first three columns use the Murfin (2012) measure of covenant strictness. The next three columns use the log number of covenants. The covenant measures are lagged by one quarter. For each measure, three standardizations of debt changes are used as the dependent variable, following Welch (2011). In Panel A, the economic conditions variable (Recession) is the inverse of the value-weighted average of firm-level Tobin's Q estimate. In Panel B, the variable is the inverse of the "bond market Q" measure of Philippon (2009). Control variables are as defined in the text. Robust standard errors are clustered at firm level and are shown in parentheses. Asterisks (*,**,***) denote significance at the 10%, 5%, and 1% level, respectively.

A potential concern with findings here is the role of supply-side shocks. If firms with stricter covenants are also more dependent on bank debt, then their leverage could appear more pro-cyclical if banks shrink their credit supply in bad times, and they are unable to switch to other sources of funding. Chava and Purnanandam (2011) have documented such real effects, classifying firms as bank-independent in a given year if they have both non-zero debt and a credit rating (specifically, an S&P domestic long-term issuer rating) at the end of the previous fiscal year. Using their definition, Table A4 repeats our previous tests on the bank independent subsample. Panels A and B replicate the specifications from Tables A2 Panel A and A3 Panel A, respectively. (Results for the panel B specifications are similar and are omitted for brevity.) Despite the smaller sample, the interaction coefficient magnitudes are similar or larger, suggesting that bank-dependence and supply-side shocks are not driving our findings. Similarly, in the full sample, including an interaction of the recession variable with a dummy indicator for a bank-dependent firm year, the interaction between the recession and covenant variables remains significantly negative.

Caption to Table A4. The table repeats the specifications in Table A2 Panel A and Table A2 Panel A using the subsample of firms classified as being not bank-dependent using the criterion of Chava and Purnanandam (2011). Robust standard errors are clustered at firm level and are shown in parentheses. Asterisks (*,**,***) denote significance at the 10%, 5%, and 1% level, respectively.

Summarizing, the results here confirm a novel prediction of our model, and constitute an addition to the empirical literature on covenant use. We document that firms with less protected debt tend to issue more debt in bad times. This is consistent with the interpretation that incentives to exploit existing creditors are higher in those times, resulting in a countercyclical component to the leverage of firms without commitment.

Table A1: Summary Statistics

VARIABLES	N	Mean	Median	Std dev	N	Mean	Median	Std dev
	Sample 1: June 1981 - April 2017				Sample 2: February 1990 - March 2017			
Δ financial debt/total assets	96,920	0.007	0.000	0.063	57,889	0.007	0.000	0.060
Δ financial debt/fin debt+B equity	96,920	0.009	0.000	0.096	57,889	0.010	0.000	0.091
Δ financial debt/fin debt+M equity	96,920	0.004	0.000	0.058	57,889	0.004	0.000	0.054
Log(Total Assets)	96,920	6.409	6.448	1.886	57,889	6.770	6.829	1.857
Q	96,920	1.710	1.414	0.976	57,889	1.752	1.467	0.962
Cash Flow	96,920	0.016	0.021	0.040	57,889	0.019	0.023	0.037
Asset Tangibility	96,920	0.302	0.227	0.240	57,889	0.303	0.228	0.237
Commitment	96,920	0.866	1.099	0.473	57,889	0.196	0.143	0.192
-Chicago Fed National Activity Index	431	0.089	-0.037	0.764	326	0.152	0.043	0.726
-Production and Income Index	431	0.013	-0.027	0.285	326	0.022	-0.027	0.278

The table reports summary statistics for firm-quarters for which each commitment proxy can be computed. In Sample 1, Commitment is the maximum of the log number of covenants across all active syndicated loan packages. In Sample 2, Commitment is the maximum of the Murfin (2012) measure of covenant strictness across all active syndicated loan packages.

Table A2: Debt Protection and Issuance

Panel A: Composite CFNAI Recession Measure

	(1) $\frac{\Delta\text{financial debt}}{\text{total assets}}$	(2) $\frac{\Delta\text{financial debt}}{\text{fin. debt+B equity}}$	(3) $\frac{\Delta\text{financial debt}}{\text{fin. debt+M equity}}$	(4) $\frac{\Delta\text{financial debt}}{\text{total assets}}$	(5) $\frac{\Delta\text{financial debt}}{\text{fin. debt+B equity}}$	(6) $\frac{\Delta\text{financial debt}}{\text{fin. debt+M equity}}$
Covenant Strictness	-0.022***	-0.033***	-0.017***			
	(0.003)	(0.004)	(0.002)			
Strictness × Recession	-0.004**	-0.006**	-0.004**			
	(0.002)	(0.003)	(0.002)			
Log(#Covenants)				-0.002**	-0.003**	-0.002***
				(0.001)	(0.001)	(0.001)
Log(#Covenants)×Recession				-0.002***	-0.003***	-0.003***
				(0.001)	(0.001)	(0.001)
Recession	0.001	0.002*	0.001*	0.002***	0.002**	0.002***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
Log(Total Assets)	-0.012***	-0.017***	-0.010***	-0.010***	-0.015***	-0.009***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
Q	0.007***	0.010***	0.003***	0.006***	0.009***	0.003***
	(0.001)	(0.001)	(0.000)	(0.001)	(0.001)	(0.000)
Cash Flow	0.028*	0.028	0.031**	0.023**	0.035**	0.025**
	(0.014)	(0.023)	(0.013)	(0.010)	(0.017)	(0.010)
Asset Tangibility	0.035***	0.053***	0.033***	0.034***	0.049***	0.033***
	(0.007)	(0.010)	(0.006)	(0.005)	(0.008)	(0.005)
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Observations	55,009	55,009	55,009	91,173	91,173	91,173
R-squared	0.083	0.077	0.075	0.080	0.075	0.071

Panel B: Production & Income Recession Measure

	(1) $\frac{\Delta\text{financial debt}}{\text{total assets}}$	(2) $\frac{\Delta\text{financial debt}}{\text{fin. debt+B equity}}$	(3) $\frac{\Delta\text{financial debt}}{\text{fin. debt+M equity}}$	(4) $\frac{\Delta\text{financial debt}}{\text{total assets}}$	(5) $\frac{\Delta\text{financial debt}}{\text{fin. debt+B equity}}$	(6) $\frac{\Delta\text{financial debt}}{\text{fin. debt+M equity}}$
Covenant Strictness	-0.023***	-0.034***	-0.018***			
	(0.003)	(0.004)	(0.002)			
Strictness × Recession	-0.010**	-0.016**	-0.010**			
	(0.004)	(0.007)	(0.004)			
Log(#Covenants)				-0.002***	-0.003**	-0.003***
				(0.001)	(0.001)	(0.001)
Log(#Covenants)×Recession				-0.004**	-0.006**	-0.005***
				(0.002)	(0.002)	(0.002)
Recession	0.003**	0.006**	0.003***	0.004**	0.006**	0.005***
	(0.001)	(0.002)	(0.001)	(0.002)	(0.003)	(0.002)
Log(Total Assets)	-0.012***	-0.017***	-0.010***	-0.010***	-0.015***	-0.009***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
Q	0.007***	0.010***	0.003***	0.006***	0.009***	0.003***
	(0.001)	(0.001)	(0.000)	(0.001)	(0.001)	(0.000)
Cash Flow	0.028**	0.028	0.031**	0.023**	0.036**	0.026***
	(0.014)	(0.023)	(0.013)	(0.010)	(0.017)	(0.010)
Asset Tangibility	0.035***	0.053***	0.033***	0.034***	0.049***	0.033***
	(0.007)	(0.010)	(0.006)	(0.005)	(0.008)	(0.005)
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Observations	55,009	55,009	55,009	91,173	91,173	91,173
R-squared	0.083	0.077	0.075	0.080	0.075	0.071

Table A3: Debt Protection and Issuance

Panel A: Aggregate Q

	(1) $\frac{\Delta \text{financial debt}}{\text{total assets}}$	(2) $\frac{\Delta \text{financial debt}}{\text{fin. debt+B equity}}$	(3) $\frac{\Delta \text{financial debt}}{\text{fin. debt+M equity}}$	(4) $\frac{\Delta \text{financial debt}}{\text{total assets}}$	(5) $\frac{\Delta \text{financial debt}}{\text{fin. debt+B equity}}$	(6) $\frac{\Delta \text{financial debt}}{\text{fin. debt+M equity}}$
Covenant Strictness	-0.001	-0.004	0.004			
	(0.011)	(0.017)	(0.010)			
Strictness × Recession	-0.045**	-0.064*	-0.047**			
	(0.022)	(0.033)	(0.020)			
Log(#Covenants)				0.008**	0.009	0.007**
				(0.004)	(0.006)	(0.003)
Log(#Covenants) × Recession				-0.022***	-0.027**	-0.020***
				(0.007)	(0.011)	(0.007)
Recession	0.014	0.029*	0.007	0.016*	0.025*	0.011
	(0.010)	(0.015)	(0.010)	(0.010)	(0.015)	(0.009)
Log(Total Assets)	-0.012***	-0.017***	-0.010***	-0.010***	-0.015***	-0.009***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
Q	0.007***	0.010***	0.003***	0.006***	0.009***	0.003***
	(0.001)	(0.001)	(0.000)	(0.001)	(0.001)	(0.000)
Cash Flow	0.028*	0.028	0.031**	0.023**	0.035**	0.025***
	(0.014)	(0.023)	(0.013)	(0.010)	(0.017)	(0.010)
Asset Tangibility	0.036***	0.053***	0.033***	0.034***	0.049***	0.033***
	(0.007)	(0.010)	(0.006)	(0.005)	(0.008)	(0.005)
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Observations	55,009	55,009	55,009	91,173	91,173	91,173
R-squared	0.083	0.077	0.075	0.080	0.075	0.071

Panel B: Bond Market Q

	(1) $\frac{\Delta \text{financial debt}}{\text{total assets}}$	(2) $\frac{\Delta \text{financial debt}}{\text{fin. debt+B equity}}$	(3) $\frac{\Delta \text{financial debt}}{\text{fin. debt+M equity}}$	(4) $\frac{\Delta \text{financial debt}}{\text{total assets}}$	(5) $\frac{\Delta \text{financial debt}}{\text{fin. debt+B equity}}$	(6) $\frac{\Delta \text{financial debt}}{\text{fin. debt+M equity}}$
Covenant Strictness	0.031	0.035	0.041			
	(0.035)	(0.052)	(0.033)			
Strictness × Recession	-0.088*	-0.115	-0.092*			
	(0.052)	(0.077)	(0.049)			
Log(#Covenants)				0.038***	0.046***	0.036***
				(0.011)	(0.016)	(0.010)
Log(#Covenants) × Recession				-0.061***	-0.075***	-0.059***
				(0.016)	(0.024)	(0.015)
Recession	-0.012	-0.023	-0.025	0.045**	0.049	0.041*
	(0.023)	(0.034)	(0.022)	(0.022)	(0.035)	(0.022)
Log(Total Assets)	-0.019***	-0.028***	-0.016***	-0.016***	-0.023***	-0.013***
	(0.002)	(0.003)	(0.002)	(0.001)	(0.002)	(0.001)
Q	0.007***	0.009***	0.003***	0.006***	0.008***	0.003***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.000)
Cash Flow	0.046**	0.058**	0.042**	0.027**	0.051**	0.030**
	(0.018)	(0.030)	(0.017)	(0.013)	(0.022)	(0.013)
Asset Tangibility	0.057***	0.082***	0.048***	0.049***	0.066***	0.045***
	(0.011)	(0.016)	(0.010)	(0.008)	(0.012)	(0.007)
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Observations	30,364	30,364	30,364	55,246	55,246	55,246
R-squared	0.114	0.106	0.101	0.105	0.097	0.092

Table A4: Debt Protection and Issuance – Non bank-dependent firms

Panel A: CFNAI Recession Measure

	(1) $\frac{\Delta\text{financial debt}}{\text{total assets}}$	(2) $\frac{\Delta\text{financial debt}}{\text{fin. debt+B equity}}$	(3) $\frac{\Delta\text{financial debt}}{\text{fin. debt+M equity}}$	(4) $\frac{\Delta\text{financial debt}}{\text{total assets}}$	(5) $\frac{\Delta\text{financial debt}}{\text{fin. debt+B equity}}$	(6) $\frac{\Delta\text{financial debt}}{\text{fin. debt+M equity}}$
Covenant Strictness(Murfin 2012)	-0.022***	-0.033***	-0.018***			
	(0.004)	(0.005)	(0.003)			
Strictness(Murfin 2012) × Recession	-0.006***	-0.008**	-0.005**			
	(0.002)	(0.003)	(0.002)			
Log(#Covenants)				-0.002*	-0.003	-0.003**
				(0.001)	(0.002)	(0.001)
Log(#Covenants) × Recession				-0.002***	-0.003***	-0.002***
				(0.001)	(0.001)	(0.001)
Recession	0.002**	0.002	0.001	0.001	0.002	0.001
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
Log(Total Assets)	-0.014***	-0.021***	-0.012***	-0.011***	-0.017***	-0.010***
	(0.002)	(0.002)	(0.001)	(0.001)	(0.002)	(0.001)
Q	0.009***	0.013***	0.003***	0.009***	0.013***	0.004***
	(0.001)	(0.002)	(0.001)	(0.001)	(0.002)	(0.001)
Cash Flow	0.036	0.043	0.047*	0.069***	0.091***	0.068***
	(0.032)	(0.048)	(0.026)	(0.022)	(0.034)	(0.020)
Asset Tangibility	0.053***	0.082***	0.047***	0.044***	0.063***	0.037***
	(0.010)	(0.015)	(0.009)	(0.008)	(0.013)	(0.008)
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Observations	24,828	24,828	24,828	35,845	35,845	35,845
R-squared	0.079	0.072	0.072	0.078	0.071	0.072

Panel B: Aggregate Q

	(1) $\frac{\Delta\text{financial debt}}{\text{total assets}}$	(2) $\frac{\Delta\text{financial debt}}{\text{fin. debt+B equity}}$	(3) $\frac{\Delta\text{financial debt}}{\text{fin. debt+M equity}}$	(4) $\frac{\Delta\text{financial debt}}{\text{total assets}}$	(5) $\frac{\Delta\text{financial debt}}{\text{fin. debt+B equity}}$	(6) $\frac{\Delta\text{financial debt}}{\text{fin. debt+M equity}}$
Covenant Strictness(Murfin 2012)	0.004	0.005	0.007			
	(0.015)	(0.023)	(0.013)			
Strictness(Murfin 2012) × Recession	-0.057**	-0.081*	-0.053**			
	(0.028)	(0.044)	(0.026)			
Log(#Covenants)				0.010*	0.013*	0.008*
				(0.005)	(0.008)	(0.004)
Log(#Covenants) × Recession				-0.026***	-0.034**	-0.023***
				(0.010)	(0.015)	(0.009)
Recession	0.017	0.025	0.010	0.011	0.019	0.004
	(0.013)	(0.020)	(0.012)	(0.013)	(0.020)	(0.011)
Log(Total Assets)	-0.014***	-0.021***	-0.012***	-0.011***	-0.017***	-0.010***
	(0.002)	(0.002)	(0.001)	(0.001)	(0.002)	(0.001)
Q	0.009***	0.013***	0.003***	0.009***	0.013***	0.004***
	(0.001)	(0.002)	(0.001)	(0.001)	(0.002)	(0.001)
Cash Flow	0.036	0.043	0.047*	0.070***	0.092***	0.069***
	(0.032)	(0.047)	(0.026)	(0.022)	(0.034)	(0.020)
Asset Tangibility	0.054***	0.083***	0.047***	0.045***	0.064***	0.038***
	(0.010)	(0.015)	(0.009)	(0.008)	(0.013)	(0.008)
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Observations	24,828	24,828	24,828	35,845	35,845	35,845
R-squared	0.079	0.072	0.072	0.078	0.071	0.072