Economic Uncertainty, Aggregate Debt, and the Real Effects of Corporate Finance

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Abstract

This paper develops a tractable general equilibrium with endogenous firm capital structure decisions driven by changes in economic uncertainty. The model enables a critical assessment of standard paradigms of corporate finance in order to highlight empirically important directions for improvement, and help understand potential real effects. The standard trade-off version of the model implies that debt incentives contract with risk. Yet, surprisingly, aggregate and firm-level evidence shows that leverage increases with uncertainty. This effect is driven by debt quantities, and is not due to the leverage denominator. It is also not explained by precautionary cash hoarding, binding restructuring constraints, or capital supply frictions. The analysis thus points towards alternative formulations in which debt incentives increase with risk. A version of the model with moral hazard via default insurance can account for the joint dynamics of uncertainty, credit spreads and debt. In this version, unlike the trade-off case, the real effects of debt can become severely negative.

1. Introduction

This paper addresses two important questions concerning the role of corporate debt in the macroeconomy. The first question is how well we understand the evolution of the prices and quantities of debt at the aggregate level. The second is whether the distortions giving rise to debt are likely to have large real effects.

Both topics are motivated, in part, by the widespread view that, following the financial crisis of the last decade, "standard" macroeconomic and finance models had been discredited by their failure to capture the real risks associated with (household and corporate) credit. To understand such failures, however, requires starting from the critical evaluation of some such standard model that includes endogenous financial policy. To that end, I develop a tractable general equilibrium framework that can encompass benchmark formulations of corporate debt.

The most prominent model in the corporate finance literature for analyzing debt dynamics has been the classic trade-off theory of capital structure choice.[1] The starting point for the current analysis is, therefore, also a trade-off model. Empirical assessment of this benchmark will be used to identify dimensions for improvement.

Uncertainty (the risk of costly default) is one of the two prongs of the trade-off decision (the other being tax benefits), and is thus a key driver of leverage choice. The model economy therefore incorporates exogenous shocks to uncertainty. In general equilibrium, exogenous risk shocks will be intertwined with endogenous credit risk, which depends on debt policy. That policy, in turn, will reflect the endogenous discount rates associated with default losses.[2]

To make analysis of the model as transparent as possible, it contains no supply-side frictions in the capital markets, or capital structure adjustment costs for firms. As a result, the solution delivers explicit characterizations of optimal capital structure and

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[1]Important advances in partial equilibrium include Hackbarth, Miao, and Morellec (2006), Bhamra, Kuehn, and Strebulaev (2010), and Chen (2010).

[2]Recent macroeconomic models, including Arellano, Bai, Kehoe, et al. (2012), Gourio (2013), Gilchrist, Sim, and Zakrajšek (2014), Christiano, Motto, and Rostagno (2014), have highlighted the interaction of uncertainty shocks with financial frictions.

default policies. Explicit expressions are also derived for aggregate default losses, default probabilities, credit spreads and credit risk premia. Consumption, marginal utility, firm value, and welfare, are all also directly obtainable as solutions to ordinary differential and algebraic equations.

To be clear, financial frictions are undoubtedly important for many firms. They are omitted from the basic set-up only to provide a clean and tractable baseline whose implications can be assessed relative to potential generalizations incorporating more complexity. Empirical assessment of the model enables us to identify features of debt dynamics (both quantities and prices) that frictions may help us understand.

Fitting the model to U.S. aggregate real and financial data over the last 40 years does, in fact, yield some surprising results.

First, despite its lack of frictions, the trade-off model actually provides a reasonable description of leverage over the business cycle: periodic increases in uncertainty lead to both higher default probabilities and higher discount rates, causing falls in asset prices and rises in volatility and credit spreads. Moreover, debt quantities contract substantially, and the credit contractions can be followed by persistent recessions. When uncertainty spikes are followed by negative output realizations, the result is a wave of default and inefficient liquidation.

However, the benchmark frictionless model has a fundamental problem in matching the time-series of aggregate credit spreads and leverage. In the data, these two quantities are reliably positively correlated, whereas the model implies that they should be almost perfectly negatively correlated. On further investigation, the problem is with debt quantities. The inherent logic of the trade-off set up implies that debt becomes less attractive as economic risk increases. Indeed, in the data and the model credit spreads are strongly positively associated with uncertainty. However, it turns out that the empirical association between uncertainty and debt is *positive*.

This finding would not be noteworthy if it simply reflected the denominator of leverage ratios (asset values or operating earnings) contracting with uncertainty, with the numerator adjusting slowly. However, this is not the case. Rather, the positive relation-

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ship is driven by the numerator: debt issuance increases when aggregate risk rises.[3] The result holds both in the aggregate data and at the firm level. The effect is robust and economically significant.

Digging deeper into the data, the leverage-volatility relation is driven by debt build-ups that occur at the onset of recessions, as output flattens (e.g., starting in 2007). This is also when risk measures are typically spiking. By the time debt starts to contract, recessions are well under way, and uncertainty is declining (e.g., after the fourth quarter of 2008). From the point of view of capital-structure theory, the feature of the data that demands explanation is not the credit contractions during economic downturns, but rather the expansions of credit that precede them.

Interestingly, there is evidence that the debt increases at the onsets of recessions are voluntary policy decisions of firms, and not a consequence of supply-side frictions or adjustment costs. Debt increases in absolute as well as relative terms, so restructuring constraints are obviously not binding. Precautionary borrowing (liquidity hoarding) is also not the full story: debt increases even after netting out increases in corporate cash holdings. Perhaps most revealing, debt does not increase because of capital market supply constraints: net equity issuance is *negative* during these episodes. Firms could have avoided debt increases (without cutting investment) just by reducing discretionary payouts to equity.

In short, the trade-off model's first-order empirical problem is not the absence of financial frictions. If anything, improving its depiction of corporate debt dynamics would seem to call for some kind of <u>negative</u> "financial shocks."

Following the paper's road map, the next question is how to improve on the baseline model to reconcile it with the empirical evidence. What could lead firms to intentionally build up debt even as its risk and cost increase? I develop an alternative version of the

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[3]The finding would also be unsurprising if uncertainty was measured by the (endogenous) volatility of *equity*. Anything that lowered equity values would mechanically raise market leverage and increase stock volatility. This is the well-known "leverage" effect in standard structural models of credit risk, where debt is fixed. The tests in the paper use uncertainty proxies built from nonfinancial data (although the VIX index is included for robustness). So reverse causality is unlikely.

model in which creditors receive a degree of default protection from the government in the event of a systematic jump.[4] The subsidy means that, in effect, the market misprices default risk due to moral hazard, and firms respond to this incentive. The value of the subsidy (to the firm) endogenously increases with volatility. Hence, this formulation can account for the positive relation between leverage and uncertainty.

Finally, returning to the topic of real effects, using fitted parameters for the uncertainty process, I compute the impact of debt policies on aggregate risk and welfare. The unconditional welfare loss, under both formulations of debt, is on the order of 2-5 percent of permanent consumption. However, unlike the trade-off case, the default-insurance version of the model implies that the welfare losses can become very large – 10-20 percent of permanent consumption – when uncertainty is high. These finding are shown to be robust to a range of plausible preference parameters. The welfare results have parallel implications for asset pricing. Under default insurance, ironically, investors are much worse off in high volatility states in terms of marginal utility and would pay much higher prices for insurance against them.

To summarize, the paper contributes to the literature a thorough critique of a standard model of endogenous firm financial policy, set in general equilibrium. The model helps us identify an important, and previously undocumented, shortcoming in the trade-off theory's depiction of observed debt dynamics, and points us in a possible direction for reconciling its predictions to the evidence. The moral-hazard alternative formulation of the model implies that debt can play a major role in amplifying real risks. The results suggest that our understanding of debt incentives may be missing a crucial element.

1.1 Related literature

There is long history in macroeconomics of investigating the role of capital supply frictions (usually modeled via a one-period debt contract) in general equilibrium.[5] Recent

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[4]This can be interpreted as a reduced form depiction of deposit insurance in a model with a competitive banking sector. The mechanism is discussed further in Section 3.4.

[5]See the large literature following Kiyotaki, Moore, et al. (1997), Carlstrom and Fuerst (1997) and Bernanke, Gertler, and Gilchrist (1999).

work, including Arellano, Bai, Kehoe, et al. (2012), Gilchrist, Sim, and Zakrajšek (2014), Christiano, Motto, and Rostagno (2014) and Chugh (2016), has highlighted the interaction of uncertainty shocks with financial frictions and shown that the combination can provide a quantitatively good description of the 2007-2009 experience.

For the most part, these models do not speak to the chief issues of finance: the pricing of debt and equity, and the endogenous choice of capital structure. Financial research has seen only a few attempts to date to tackle capital structure determination in a general equilibrium context.[6] Two related works are Miao and Wang (2010) and Gomes and Schmid (2012). Both papers solve business cycle models with trade-off formulations of capital structure choice, subject to adjustment frictions. The former authors use habit preferences with endogenous labor supply and idiosyncratic liquidity shocks. The latter use recursive preferences and allow for cross-section heterogeneity in productivity and leverage. Both settings are richer than the one used here, although each presents substantial computational challenges. Both papers present calibrations that do a reasonable job matching a broad set of real and financial moments.

Also closely related are the works of Gourio (2013) and Bianchi, Ilut, and Schneider (2017). Both papers endogenize debt and equity with time-varying second moments. In Gourio (2013), single-period firms face fluctuations in disaster risk, which generate large fluctuations in employment and investment through the risk that the capital stock will be destroyed in a crisis. In contrast to my model, Gourio assumes that default losses do not entail real loss of resources (they are rebated to consumers). The real consequences of debt then stem from the overinvestment of firms in response to tax shields. In Bianchi, Ilut, and Schneider (2017), second-moment shocks to productivity and fixed costs interact with time-varying ambiguity, to which the representative agent is averse. Their focus is on assessing the importance of each type of uncertainty in matching real quantities as well as equity valuations. The model includes only riskless one-period corporate debt, and thus does not speak to credit risk or real distortions.

Like the baseline model here, the models in the papers above consistent imply that

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[6]Examples include Levy and Hennessy (2007) and Gale and Gottardi (2015).

corporate debt declines with increases in uncertainty. As succinctly summarized by Christiano, Motto, and Rostagno (2014) (p50) "The economic intuition underlying the response of the model to a jump in the risk shock is simple. With a rise in risk, the probability of a [bad state] increases, and banks raise the interest rate on loans...to cover the resulting costs. Entrepreneurs respond by borrowing less, so credit drops." It is perhaps surprising that this influential body of literature has not critically examined the evidence for (or against) this basic implication.

The distinct focus of the present paper is on critically assessing the dynamic implications of the standard trade-off framework for the price and quantity of defaultable debt. I highlight a previously unappreciated empirical shortcoming and illustrate that addressing it may have important implications for understanding the welfare consequences of corporate leverage.

The outline of the paper is as follows. Section 2 describes the model, shows how to solve for its equilibrium, and characterizes the key real and financial dynamics. Section 3 fits the trade-off version to the U.S. data, and highlights the positive empirical relation between uncertainty and leverage. The alternative formulation of debt is also fitted to the data and is shown to resolve the puzzle. Section 4 assesses the real effects of debt on aggregate risk and welfare, and contrasts the implications of the two versions of the model. A final section summarizes and concludes.

2. Model

This section describes an economy in which corporate financial policies are driven by exogenous changes in uncertainty. Debt policy affects aggregate risk through costly default. The default risk in turn affects marginal utility, and hence feeds back into discount rates, the cost of credit, and hence debt decisions.

2.1 Firms, Shocks, and Aggregate Output

The model is set in continuous time on an infinite horizon. There is a single consumption good, and a single class of agents. The economy is endowed with a continuum of productive projects, whose measure is denoted M, each of which produces a non-negative stream of goods until termination. Following Gomes and Schmid (2012), a project is the model's depiction of a firm: each firm owns a single project.

The projects are all stochastically identical. Let $Y^{(i)}$ denote the instantaneous output flow of project i. I assume $Y^{(i)}$ follows the pure-jump stochastic process

$$\frac{dY_t^{(i)}}{Y_t^{(i)}} = \mu \, dt + d \left[\sum_{j=1}^{\mathcal{J}_t} \left(e^{\varphi_j^{(i)}} - 1 \right) \right].$$

Here $\mathcal{J}_t$ is a regular Poisson process with intensity λ, and the percentage jump sizes $\varphi_j^{(i)}$ are assumed drawn from a distribution, $\mathcal{F}^\varphi(t)$, that will depend on the aggregate state. The jump process itself, $\mathcal{J}_t$, is common across firms. Thus a jump is a systematic event. If a jump occurs at time t, the sign of the jump is a Bernoulli random variable (with both outcomes having equal probability) that is also common across firms. However, conditional on the sign, the individual jump incidences are assumed to be *i.i.d.* across firms. Specifically, I will take the $\varphi^{(i)}$ to be drawn from particular gamma distributions defined over the positive or negative real line, depending on the sign of the jump. (I will also impose that the density functions are monotonic.) The distribution $\mathcal{F}^\varphi(t)$ is thus a gamma-binomial convolution. Intuitively, firms differ in their exposure to a systematic event, although this is only revealed *ex post* and does not carry over from one event to the next.

The common scale of the jumps, denoted σ_t is assumed to vary exogenously as

$$d\sigma_t = m(\sigma_t) \, dt + s(\sigma_t) \, dW_t.$$

Here W is a standard Brownian motion. The drift and diffusion functions are assumed to

be such that σ_t is stationary on a finite interval, $[\sigma_l, \sigma_u]$. Technically, given the sign of the jump, σ will determine the mean of the jump size distribution (hence $\mathcal{F}^\varphi(t) = \mathcal{F}^\varphi(\sigma_t)$).

Integrating over firms, let Y denote aggregate output. Ignoring entry and exit for the moment, aggregate dynamics are

$$dY_t = \mu \; Y_t \; dt + d \int_0^{M_t} Y_t^{(i)} \left[\sum_{j=1}^{\mathcal{J}_t} \left(e^{\varphi_j^{(i)}} - 1 \right) \right] \; di$$

$$= \mu \; Y_t \; dt + Y_t \; d \left[\sum_{j=1}^{\mathcal{J}_t} \left(\mathrm{E_t} \left[e^{\varphi_j^{(i)}} | \varphi_j > 0 \right] 1_{\{j,+\}} + \mathrm{E_t} \left[e^{\varphi_j^{(i)}} | \varphi_j < 0 \right] 1_{\{j,-\}} - 1 \right) \right]. \quad (1)$$

where $1_{\{j,+\}}$ and $1_{\{j,-\}}$ are indicators for the sign of the jth jump. Applying a law of large numbers, the stochastic term is

$$Y_t \; d \left[\sum_{j=1}^{\mathcal{J}_t} (u(t) - 1) 1_{\{j,+\}} + (d_{nd}(t) - 1) 1_{\{j,-\}} \right]$$

where

$$u(t) = \mathrm{E}[e^\varphi | \text{up jump}, \sigma_t], \quad d_{nd}(t) = \mathrm{E}[e^\varphi | \text{down jump}, \sigma_t].$$

(The subscript nd – "no default" – indicates that this is the down jump size before taking into account the output losses due to exiting firms. These are handled below.) Thus aggregate output follows a binomial process. Conditional on σ_t and the sign of the jump, the size of aggregate shocks is not random.

Because the aggregate jump sizes scale with σ, this process controls overall economic uncertainty, and hence systematic risk. It is worth noting, however, that, because the jumps themselves are *i.i.d.* across firms, σ *also* controls the degree of firm-specific – or idiosyncratic – risk. In this sense, the model is distinct from other formulations of uncertainty-driven business cycles in which firm-level shocks do not typically scale with aggregate risk fluctuations.

The stochastic specification of the economy is obviously stylized. Aggregate output, and that of each project, is constant except on the occurrence of a jump. It is not

hard to generalize the model to include (i) firm-specific jump events, and (ii) a diffusion component to aggregate output. However, the simplicity of the set-up here still admits rich behavior while maintaining parsimony.

2.2 Debt

Each firm has access to debt financing in the form of a floating-rate line of credit, or, equivalently via issuing perpetual notes with a floating coupon rate. The firm faces no restrictions or transactions costs in altering its quantity of debt: it may freely draw down or repay any amount at any time. Equity finance is also assumed costless. The firm will thus re-optimize its capital structure continuously. Increases in debt are paid to equity holders; decreases are funded by equity holders. The firm retains no resources.[7]

While the assumption of costless restructuring of debt is made primarily for convenience, it is perhaps less unrealistic than the opposite extreme assumption often employed in the dynamic capital structure literature, that debt cannot be reduced at *any* cost.[8] In fact, credit lines – which can be paid down instantly and costlessly – constitute a large fraction of corporate debt.[9] So at least marginal changes in leverage are not constrained for many borrowers. The analysis in Section 3 will directly address the question of whether the conclusions are affected by the assumption of costless adjustment.

Another embedded assumption is that changes to debt and equity are off-setting (or that net external finance is small). This too is actually a reasonably good description of the aggregate U.S. data. Figure 1 shows net new debt and net new equity of the nonfinancial corporate sector.[10] The raw numbers in the left panel appear almost mirror

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[7]Technically, there is no physical capital in the model. As described below, household investment increases the mass of firms. It is straightforward to associate a differential unit of capital with each firm, so that the economy's capital stock is proportional to the mass. One could also view each firm's output process as coming from a linear (AK) production technology with shocks to capital.

[8]Costless capital structure adjustment is also assumed in Admati, DeMarzo, Hellwig, and Pfleiderer (2018).

[9]Over 70% of commercial bank loans are in the form of credit lines (Pennacchi (2006)).

[10]All data are from the Federal Reserve Z.1 reports. The debt series is the change in bonds and bank debt minus the change in cash items. The equity series is net new issues of equity minus dividends. The asset series is the book value of nonfinancial assets.

images of each other. The scaled, HP-filtered, series on the right have correlation -81%.

Figure 1: External Finance: U.S. Nonfinancial Corporate Sector
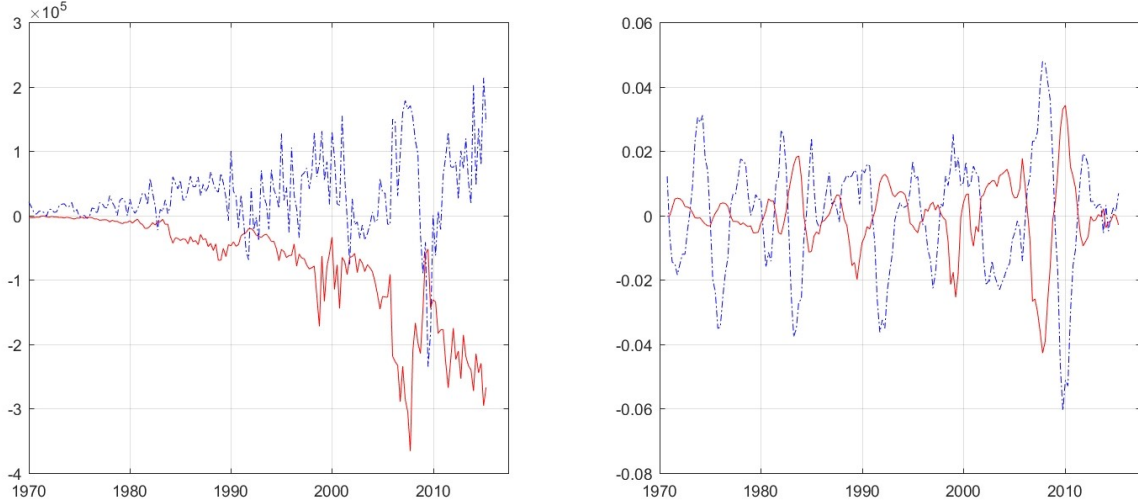
Description: Both plots show the time-series of net new debt (dashed line) and net new equity (solid line) of the U.S. nonfinancial corporate sector. The left panel is the raw quarterly data in millions of dollars. In the right panel, each series is scaled by assets, summed over trailing 4 quarters, and HP filtered. All data are from the Federal Reserve Z.1 reports. The debt series is the change in bonds and bank debt minus the change in cash items. The equity series is net new issues of equity minus dividends. The asset series is the book value of nonfinancial assets. **Interpretation:** Firms mostly issue debt to buy back equity and vice versa. Net external finance is small.

Denoting the asset value of the ith firm $V^{(i)}$ and its optimal debt $B^{(i)}$, it is intuitively clear (and will be verified below) that, because firms are stochastically identical, these values are both linear in firm output. So define the scaled lower-case variables via: $V^{(i)} = v(\sigma)Y^{(i)}$ and $B^{(i)} = b(\sigma)Y^{(i)}$.

The terms of the debt contract stipulate that the coupon rate is paid continuously at a rate that re-sets instantaneously in order to ensure that (outside of default) the market value of the debt is always equal to its face value. This is a convenient form of the contract, that is not unrealistic for a firm that is bank-financed or must roll over a significant proportion of its liabilities periodically.

Following the usual assumptions in trade-off models, I assume the firm receives a tax deduction for coupon interest paid, and that this deduction is realized continuously

as long as the firm is alive. On a pre-tax basis, this is equivalent to a subsidy paid continuously to the firm. I make one non-standard simplifying assumption regarding the tax shield on profits. I assume that it is proportional to the face value of debt times a fixed statutory interest rate, $\bar{r}$, rather than to the interest rate on the firm's debt. It is also straightforward to allow the rate to depend on the debt's credit spread. But what I preclude is dependence on the economy's real interest rate. This is one of the steps that permits the capital structure decision to be decoupled from the full equilibrium solution.[11]

While tax shields are the primary motivation for debt in the structural corporate finance literature, it is straightforward to incorporate some alternative formulations of the debt subsidy. In particular, I will also consider a reduced-form depiction of deposit insurance in which creditors of a firm that has defaulted receive a payment (a transfer from the government) of ΘB_{t-} where B_{t-} is the face value of debt prior to default. This formulation is discussed in more detail in Section 3.4.

As with all such models, the basic non-contractibility built into the set-up is that – even though households hold all the debt and equity claims of each firm – the firm management cannot commit in advance *not* to act in the interests of equity holders alone by defaulting when optimal (for them) to do so. For simplicity, I assume the project terminates upon abandonment. Default is always inefficient as it result in the destruction of strictly positive income streams. Because there is no physical capital debt recovery from the firm is zero. [12]

Firms will default following sufficiently negative jumps in output.[13] If it is optimal for a firm to exit upon a downward jump in $Y^{(i)}$, say of percentage $\varphi^\star$, then, by linearity, all firms with jumps of the same size or worse will also exit. Hence there is a

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[11]In practice, tax shields depend on nominal interest rates, not real ones. So linking the tax shield to the real riskless rate is unrealistic, especially because that rate may become negative.

[12]The zero-recovery assumption is the extreme case of inefficient liquidation or "fire sales" of assets. Clearly this represents an upper bound on the default losses. On the other hand, larger recovery values lead to higher debt prices, and hence higher leverage, in equilibrium.

[13]A familiar result in the partial-equilibrium capital structure literature is that floating rate debt is riskless when asset value follows a diffusion process. In the present context, this implies that diffusive changes in uncertainty σ_t will never trigger default.

largest (least negative) jump size such that default will occur with any jump below this level and not otherwise. Let $\varphi^\star(\sigma)$ denote this critical value. It will be derived below. However, even without knowing it, we can immediately deduce the effect of exit on the dynamics of aggregate output. In equation (1) above, we replace $E_t \left[e^{\varphi_j^{(i)}} | \varphi_j^{(i)} < 0 \right]$ with $E_t \left[e^{\varphi_j^{(i)}} | \varphi_t^\star < \varphi_j^{(i)} < 0 \right]$. That is, for downward jumps, we lose the mass of firms that experience jumps worse than the threshold. The effect of exit on output is simply to alter the downward aggregate jump size, d.

An immediate and useful result (established below) is that the optimal abandonment threshold $\varphi^\star$ is related to the firm's debt-to-output ratio and value-to-output ratio by the relation

$$ e^{\varphi^\star} = b(\sigma)/v(\sigma). \tag{2} $$

Hence, when the firm's owners determine b at time zero, they do so with the knowledge that it will induce the above default policy. Given this, under the trade-off model, the firm's problem can be expressed as

$$ \max_{b(\sigma)} \ E_0 \left\{ \int_0^T \Lambda_t \left[(1 - \tau)Y_t^{(i)} + \bar{r}\tau \ b(\sigma_t) \ Y_t^{(i)} \right] \ dt \right\} \tag{3} $$

where Λ_t is the economy's state price density, $\bar{r}\tau B_t$ is the benefit flow of the tax shield (and the expression uses $B_t = b \, Y_t^{(i)}$), and T denotes the firm's default time. The default time depends on the policy choice, b, which can be seen by defining T_j to be the jth jump time and writing

$$ T = \inf_{T_j} \varphi_j^{(i)} \leq \varphi_j^\star = \log \left(\frac{b(\sigma_{T_j})}{v(\sigma_{T_j})} \right). $$

In the alternative formulation of the model with default insurance, the expectation in (3) contains the additional term $\Theta B_T = \Theta b \, Y_T^{(i)}$ because the firm reaps the reward of all expected cash-flows to its claim-holders.

2.3 Households, Investment, and Consumption

There is a representative household characterized by preferences of the stochastic differential utility class (Duffie and Epstein (1992), Duffie and Skiadas (1994)), the continuous-time analog of Epstein and Zin (1989) preferences. Specifically, agents maximize the lifetime value function,

$$J_t = \mathrm{E_t} \left[\int_t^\infty f(C_s, J_s)\, ds \right].$$

where

$$f(C, J) = \frac{\beta C^\rho / \rho}{((1 - \gamma)\, J)^{1/\theta - 1}} - \beta \theta J.$$

Here β is the rate of time preference, γ is the coefficient of relative risk aversion, $\rho = 1 - 1/\psi$, where ψ is the elasticity of intertemporal substitution, and $\theta \equiv \frac{1-\gamma}{\rho}$. (I assume $\gamma \neq 1, \rho \neq 0$.)

Households' aggregate income is assumed equal to Y_t. A government sector is assumed to collect corporate taxes, net of tax shields, and rebate any surplus to consumers.

Real investment is determined at the household level. Households are endowed with a technology (R&D) for generating a flow of new projects. Specifically, if a fraction ι of aggregate output Y is expended, this is assumed to increase the mass of projects at the proportional rate $\zeta(\iota)$, where $\zeta()$ is an increasing, concave function. The flow of new projects shows up as an additional term, $\zeta(\iota)\, dt$ in the growth rate of aggregate output, dY/Y.

When new firms are created, they are distributed uniformly across households. Each household sells its firm(s) to all the others. Each firm then sells its initially optimal quantity of debt, the value of which passes to the equity holders. These financial transactions between households and themselves result in no net flow of real goods.[14]

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[14]There is an implicit assumption that households do not borrow against their valuable growth options. To the extent that tax shields are the incentive for debt, this is reasonable because *future* projects have no current profits to shield.

2.4 Solution

The model is tractable for two main reasons. First, the assumptions are sufficient to ensure that there is a single state variable (the level of uncertainty) that characterizes everything. In particular, the distribution of firm sizes does not enter into any aggregate quantities. Second, the components of the valuation problem can be effectively separated from each other and derived sequentially, as the following results demonstrate.

Proposition 1. *Let $V^{(i)}$ denote the value of the ith firm's project and $B^{(i)}$ the face value of its debt. Then, prior to default, $V^{(i)}$ and $B^{(i)}$ are linear in output $Y^{(i)}$: $V = v(\sigma)Y^{(i)}$ and $B = b(\sigma)Y^{(i)}$.*

The optimal default policy is for owners to abandon the firm on a jump of $V_t^{(i)}$ below $B_{t-}^{(i)}$ and the optimal market leverage ratio $B^{(i)}/V^{(i)} \equiv \ell$ is the same for all firms and is related to the abandonment threshold $\varphi^\star$ via

$$e^{\varphi^\star} = \ell(\sigma).$$

The first-order condition for the optimal quantity of debt is

$$\tfrac{1}{2}\lambda d^{-\gamma} \ f^{\varphi-}(\varphi^\star) = \bar{r}\tau \tag{4}$$

where $f^{\varphi-}$ is the density function of the negative jumps whose distribution is denoted $\mathcal{F}^{\varphi-}$. The aggregate output drop on a down jump is

$$d = \int_{\varphi^\star}^{0} e^{\varphi} \ d\mathcal{F}^{\varphi-}. \tag{5}$$

There is a unique solution of the preceding two equations for d and $\varphi^\star$. That solution implies $\ell \in (0,1)$. The optimal leverage policy is incentive compatible for equity holders.

Note: all proofs appear in Appendix A.

The proposition characterizes optimal leverage and default as a function of the state

σ in essentially closed form. The only required inputs are the distribution function of the negative jumps, their intensity, and the tax shield and risk aversion parameters. We do not need to solve for other features of the equilibrium (such as investment or the output growth rate). The debt equations also make no reference to the dynamic specification of the state variable, σ.

The first-order condition (4) follows from the differential form of the firm's Bellman equation whose left side is (3). It equates the marginal increase in the flow benefit of the tax shield, $\bar{r}\tau$, to the marginal increase in the risk-neutral probability of experiencing a jump that induces bankruptcy. (There is a factor of 1/2 because half of the jumps are up-jumps.)

The final statement in the proposition is also an important result in achieving tractability. Dynamic capital structure problems in continuous time generally embed a commitment problem and thus require careful treatment of the game between managers and the market (which prices the firm's claims conditional on policy beliefs). See DeMarzo and He (2016) for example. However, the solution here obviates this difficulty. Intuitively, this is a consequence of the stipulation that the unit price of the debt contract is always one, which implies that no policy can expropriate value from existing debt holders. As a result, managers cannot do better than maximizing firm value.

A second nice consequence of the assumed form of the debt contract is that it is easy to evaluate the credit spread. It is just the risk-neutral default intensity, which is the true intensity times a risk-aversion factor.

Corollary 2.1. *The credit spread on the firm's debt is*

$$\tfrac{1}{2}\lambda \, d(\sigma)^{-\gamma} \, (1 - \mathcal{F}^{-\varphi}(\varphi^{\star})).$$

The proof in the proposition is also readily modified to handle the case of default insurance as described above.

Corollary 2.2. *With default insurance, the optimal default policy is unchanged. The*

first-order condition for the optimal quantity of debt is

$$\tfrac{1}{2}\lambda\, d^{-\gamma}\,[(1-\Theta)\,f^{\varphi-}(\varphi^\star) - \Theta\,\mathcal{F}^{\varphi-}(\varphi^\star)] = \bar{r}\tau. \tag{6}$$

The aggregate output drop on a down-jump is again given by equation (5).

Intuitively, the marginal increase in default probability (the left side of (6)) is now tempered by the increased likelihood of the bailout subsidy. The subsidy appears in two terms because increasing B both increases the size of the recovery, ΘB, and increases the probability of a default-inducing jump by raising the optimal abandonment threshold.

Having solved for the optimal debt policies and the contribution of default to aggregate risk, the full aggregate dynamics are now determined.

Proposition 2. *The household's value function is $J = j(\sigma)\,Y^{1-\gamma}/(1-\gamma)$, and optimal consumption is $C = c(\sigma)Y$, where $j(\sigma)$ and $c(\sigma)$ are the solutions to (respectively) an ordinary differential equation and an algebraic equation given in the appendix.*

With the consumption process determined, the stochastic discount factor, the riskless rate, and the market price of risk are all immediately obtainable, and are also given in the appendix. Also the appendix shows that the differential equation defining j is guaranteed to have a unique solution, and is easy to solve numerically. The same is true of the equation for firm value in the following result.

Proposition 3. *The firm's price-output ratio $v(\sigma)$ is the unique solution to an ordinary differential equation given in the appendix. The optimal debt-output ratio is $b(\sigma) = v(\sigma)\ell(\sigma)$, where ℓ was determined above.*

Given the prices of the firm's claims, it is straightforward to derive their risk premia, expressions for which are also given in the appendix.

2.5 Parametric Assumptions

To take the model to data and to illustrate its properties numerically, requires specification of the jump distribution and the uncertainty process.

For the firm-specific output jumps, recall that the sign is the outcome of an independent Bernoulli draw. I assume the size of (log) down jumps is drawn from a two-parameter gamma distribution[15] whose mean is σ. The second distribution parameter, denoted $L \geq 1$, fixes the jump size standard deviation as $L\sigma$. Under this distribution, the expected decline conditional on a down jump is

$$d_{nd} = \mathrm{E}[e^{\varphi}|\text{down jump}] = \left[\frac{1}{1 + L^2\sigma}\right]^{L^2}.$$

Next, I assume jumps are symmetrical in the following sense:

$$u = \mathrm{E}[e^{\varphi}|\text{up jump}] = 1/d_{nd}.$$

Given the size of the up-jumps u, I assume these jumps are distributed exponentially, i.e., gamma distributed with $L = 1$. (The distribution of up jumps plays no role in the solution for optimal debt policy.)

For the uncertainty dynamics, a convenient choice is the Jacobi process, which is stationary and bounded. The specification is

$$d\sigma_t = \kappa(\bar{\sigma} - \sigma_t)\,dt \; + \; s_0\,\sqrt{(\sigma_u - \sigma_t)(\sigma_t - \sigma_l)}\,dW_t.$$

Besides the upper and lower limits, σ_u and σ_l, this process requires the choice of the unconditional mean, $\bar{\sigma}$, the mean-reversion speed, κ, and the volatility of volatility parameter, s_0. The stationary distribution is then in the beta class[16] on the open interval (σ_l, σ_u). The instantaneous variance of σ is a quadratic function, centered at the midpoint of the range. This will mean that *volatility risk* is itself increasing in σ most of the time, because, for reasonable calibrations, the mean of the distribution, $\bar{\sigma}$, will be closer to σ_l than to σ_u.

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[15]The pdf of the gamma distribution is usually written as $f(x; a, b) = x^{a-1}e^{-x/b}\,b^{-a}/\Gamma(a)$ with mean ab and standard deviation $\sqrt{a}b$. So in the notation used here $\sigma = ab, L = 1/\sqrt{a}$.

[16]See Gouriéroux and Valéry (2004).

For ease of interpretation, I take the jump intensity to be $\lambda = 1$ in annualized units, meaning that, on average, there is one output jump per year. Hence σ can be interpreted as the instantaneous scale of the annual percentage output shocks.

Finally, I will assume the investment technology is given by a simple functional form that guarantees a unique solution to the investment first-order condition is

$$\zeta(\iota) = \zeta_0 \, \iota^{\zeta_1} \tag{7}$$

with $\zeta_0 > 0$ and $1 > \zeta_1 > 0$.

2.6 Solution Properties

To understand the model's implications about debt quantities (leverage) and prices (credit spreads), consider the two equations (4) and (5) in Proposition 1. Each of these can be viewed as an equation for the output drop, d, as a function of leverage, ℓ. We can readily deduce the properties of each, and how each varies with σ.

Equation (5) is just accounting: it says output after a down-jump is zero for firms that exit. As leverage increases, the default threshold $\varphi^\star$ rises and more firms default, lowering d. Hence this line is downward sloping in ℓ.

The first order condition (4) equates the marginal benefit of an additional unit of debt to its marginal increase in default probability under the risk-neutral measure. Here risk aversion contributes the factor $d^{-\gamma}$ which is equal to the increase in marginal utility conditional on an output decline. This factor is greater than one ($d < 1$) meaning that default risk is systematic, which raises the marginal cost of debt. The true marginal default probability rises with leverage. Hence, for the left side to be constant, optimal leverage can be higher only if the marginal utility factor is lower (closer to one) or that d is higher. The equation thus describes an upward sloping relation.

Figure 2 plots the two equations for d as a function of ℓ using two values of σ.[17] The solid lines are for $\sigma = 0.05$ and they intersect at an optimal leverage of about 45

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[17]The numerical example in this section assume $\gamma = 7$, $L = 3$, $\bar{r} = 0.05$, and $\tau = 0.35$.

percent. The dashed lines show what happens when uncertainty rises to $\sigma = 0.20$. The accounting equation (red line) shifts down: for any given value of the default threshold, there is now more probability mass beyond it; more default means lower d. An increase in uncertainty also causes the first order condition line to shift strongly upward. For a given level of leverage, increasing risk raises the marginal default probability. Because the marginal benefit are constant, this can only happen if the marginal utility factor $d^{-\gamma}$ falls, meaning a rise in d.

Increased uncertainty thus causes optimal leverage to contract. At $\sigma = 0.20$ the two lines cross at a little over 10 percent. Indeed, the full function $\ell(\sigma)$ is everywhere monotonically decreasing. The first order condition is the driving force behind this. Intuitively, the inherent logic of the trade-off framework requires that increases in default risk make debt less attractive. The equilibrium effect that also sees increases in default risk amplified by larger drops in marginal utility reinforces the partial-equilibrium intuition.

Figure 2: Optimal Debt
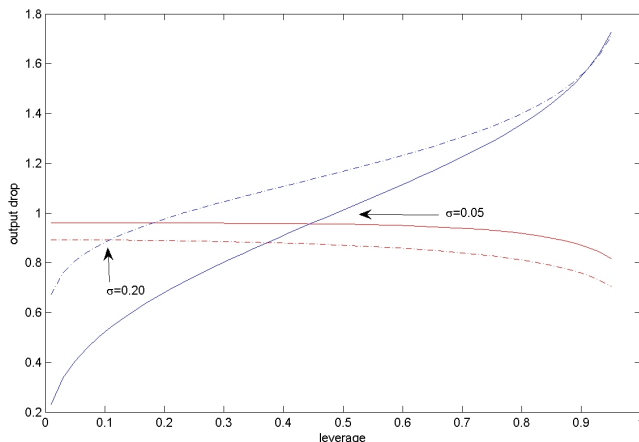
Description: The figure plots the two equations (4) and (5) that determine optimal leverage ℓ and equilibrium output drops d. The former are the upward sloping lines; the latter are the downward sloping ones. Solid lines use $\sigma = 0.05$. Dashed lines use $\sigma = 0.20$. The parameters are $\gamma = 7$, $\lambda = 1$, $L = 3$, $\hat{r} = 0.05$, $\tau = 0.35$.
Interpretation: Optimal debt declines with uncertainty in the general equilibrium trade-off model.

Next consider credit spreads. From Corollary 1, these are determined by both risk

and risk aversion. The risk aversion factor $d^{-\gamma}$ rises with σ because d falls. We have seen that optimal debt declines steeply as σ rises. Thus the effect on default risk is ambiguous. The right-hand panel of Figure 3 illustrates that the default probability may rise or fall depending on the fatness of the tail of the jump distribution, governed by the parameter L. However, the left panel of the figure shows that – even when the default probability falls with σ – the risk aversion effect dominates and credit spreads are still increasing. Thus the conclusion that the trade-off model implies a negative relation between credit spreads and the quantity of debt (or leverage) is robust to the shape of the jump distribution.[18]

Figure 3: Credit risk
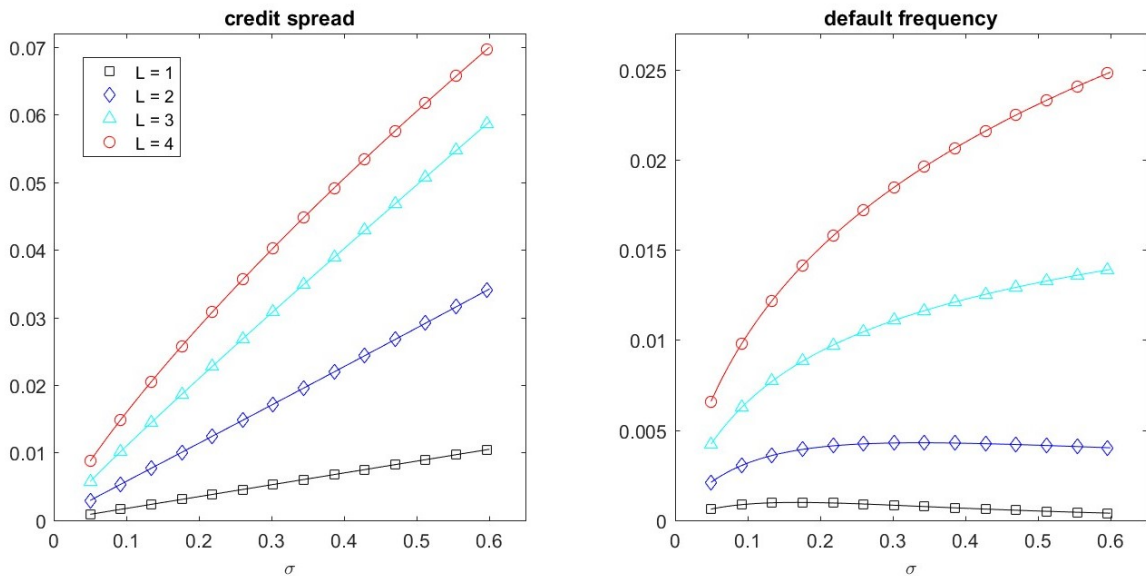
Description: The credit spread (left panel) and default frequency (right panel) are plotted using parameters $\gamma = 7$, $\lambda = 1$, $\hat{r} = 0.05$, $\tau = 0.35$ and values $L = 1, 2, 3, 4$ as shown in the legend.
Interpretation: Although the effect of uncertainty on the equilibrium default frequency may depend on the tail of the jump distribution, uncertainty unambiguously increases the credit spread.

In sum, although the model is quite stylized and omits many attributes credit markets, it does at least seem to capture important features of debt-driven crises. Rare excursions

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[18]This will cease to be true for low enough risk aversion and low tail-risk. In that case, the model will counterfactually imply that credit spreads decline with uncertainty.

into high σ states will see strong contractions in credit and spikes in credit spreads and credit risk premia (and hence discount rates). If a negative output jump does occur in such a state (which will not always happen), then there will be a wave of inefficient default imposing a real cost on households.

3. Empirical Evaluation

How well does the trade-off model do at explaining the observed behavior of debt quantities and prices? This is an open question. While it has been widely asserted that standard macro-finance models have been discredited for their descriptive failures, the literature has lacked a benchmark equilibrium model with endogenous firm financial decisions by which to judge such critiques.

This section first fits the trade-off version of the model to U.S. data, and examines the implications of the fitted specification. While the model can do a reasonable job matching many real and financial moments, it has an intriguing difficulty. Specifically, leverage and credit spreads are positively correlated in the data, and negatively correlated in the model.

Comparing the implied dynamic paths with actual experience, realized uncertainty shocks do an excellent job explaining the observed history of credit spreads. The problem is with leverage. In the data, leverage is positively associated with uncertainty. This is a new finding that is shown to hold across a variety of proxies and regression specifications.

This brings us to the original motivating question: what friction or distortion is the trade-off model missing? A closer look at corporate financing activity sheds some interesting light on the potential drivers of the volatility-leverage relation. The data suggest that the answer is not shocks to capital supply or costly adjustment of financing. Instead, it looks like firms face some positive incentive to increase leverage in the face of rising uncertainty.

As an example of such an incentive, I return to the version of the model with deposit-insurance type debt subsidy, introduced in Section 2. I describe in more detail the mech-

anism behind this subsidy, which the model captures in reduced form. An empirically estimated specification of this alternative formulation can resolve the central empirical shortcoming of the trade-off model, and hence lays the groundwork for the subsequent analysis of real effects in Section 4.

3.1 Estimation

The tractability of the model developed in Section 2 permits estimation of its parameters by the method of simulated moments. For any parameter values, the exact solutions to all quantities are numerically obtainable in a matter of seconds. Notably, no linearizations (or approximations of any order) are required. Also, the ergodic distribution of the state variable σ_t is available in closed-form, eliminating issues of convergence in computing population moments in simulations.

I fit the model to a collection of real and financial moments whose empirical counterparts correspond to the quantities that the model is being asked to speak to: the level and dynamic properties of credit spreads and leverage;[19] the distribution of output shocks; the levels of savings and firm valuation; and the risk premium in credit spreads. Choosing empirical quantities to correspond to quantities in the model necessarily requires some subjective judgements. The primary measure of credit spreads is the difference between Moody's Baa yield and the yield on 20-year Treasury bonds. Debt is the sum of debt securities and loans in the U.S. nonfinancial corporate sector from the Federal Reserve's Flow of Funds (Z.1) accounts. Output is the operating cash-flow of this sector measured as net operating surplus plus consumption of fixed capital, also from the Flow of Funds. Details of the other data choices are described in Appendix B.

Table 1 shows the resulting model fit in the right-hand column.[20] In terms of unconditional real and financial moments, the trade-off model can do a good job explaining many salient features of the data. However, the final line in the table shows that this de-

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[19]Here, following Jermann and Quadrini (2012), leverage is measured as the debt-to-output ratio. In the context of the model, debt scales linearly with output.

[20]Details of the procedure and parameter point estimates are given in the appendix.

scription is missing something important: namely, the trade-off model implies an almost perfectly negative correlation between leverage and credit spreads, while the correlation in the data is reliably positive. The positive correlation is a robust finding across choices of proxies and sample period, as Shown in Appendix C. The relation is shown visually in the scatter plot in Figure 4.

Table 1: Data and Model Moments

	Data	Model
output growth	0.0158	0.0170
output standard dev	0.0306	0.0300
output skewness	-0.3973	-0.3023
output kurtosis	5.5771	5.8204
investment rate	0.0780	0.0799
default rate	0.0087	0.0157
equity valuation	4.6342	2.7287
leverage	2.3360	2.4338
leverage standard dev	0.4578	0.4397
leverage change std dev	0.0741	0.1441
credit spread	0.0165	0.0186
credit spread standard dev	0.0068	0.0081
credit spread change std dev	0.0032	0.0027
leverage-credit spread correlation	0.5229	-0.9789

Description: The table shows statistics from the data used to estimate the trade-off model from Section 2, along with the implied moments from the fitted models. The first four rows show standardized moments of quarterly log changes in output measured as the cash-flow of U.S. nonfinancial corporations. The investment rate is the annual household savings rate. The default rate is the annual average number of bankruptcy filings divided by the number of firms. Equity valuation and leverage are the ratios, respectively, of the market value of equity and the net total debt of U.S. nonfinancial corporations scaled by the annualized output series. The credit spread is the difference in yields-to-maturity of the Moody's Baa benchmark and 20-year U.S. Treasury bonds. Further details of the data series and estimation are given in Appendix B.

The takeaway from the estimation exercise is that the trade-off model is misspecified

in a significant way. However, the results do not reveal whether the problem lies with the description of prices of debt (credit spreads) or quantities (leverage), or both. To investigate further, I examine the model-implied histories of these series.

Figure 4: Leverage and Credit Spreads
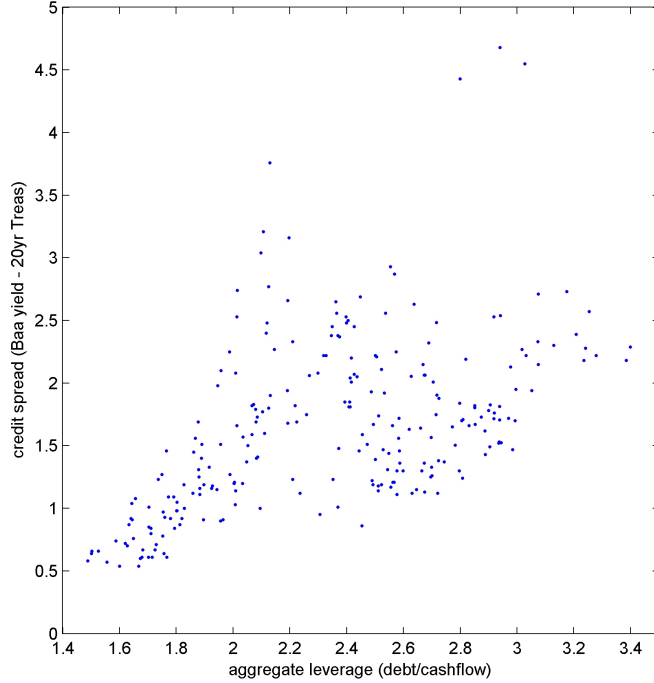
Description: The figure plots credit spreads against aggregate leverage in the U.S. data. Leverage is measured as the ratio of total debt to quarterly cashflow of the nonfinancial corporate sector, from the Flow of Funds accounts. The credit spread is the difference between the Moody's benchmark Baa yield-to-maturity and the interpolated yield-to-maturity on 20-year Treasury bonds.
Interpretation: The two series are positively related.

The model posits that uncertainty shocks drive capital structure decisions and bond prices. Recent advances in the empirical literature provide a number of potential measures of fundamental uncertainty that can be used to assess the model predictions. Here I employ the index constructed in Jurado, Ludvigson, and Ng (2015) (JLN) which averages the forecast standard deviations from time-series models of a large and diverse panel of economic and financial statistics. The series is exogenous with respect to credit market outcomes in the sense that, at each point in time, it is based on specifications fitted to

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rolling windows of backward-looking data.

In the upper panel of Figure 5, I plot the model-implied history of credit spreads when the JLN series is taken as the realization of the model's uncertainty state and fed into the estimated model.[21] The resulting series (dotted line) is shown along with its empirical counterpart (solid line). The model fit is actually remarkably good. This provides strong support for the premise that time-varying uncertainty explains credit spread fluctuations.

By contrast, the figure's lower panel shows that the fitted model – coupled with the time-series of uncertainty realizations – completely fails to describe the dynamics of debt quantities. In fact, the plot suggest an empirical regularity that, to my knowledge, has not previously been documented: that aggregate debt covaries *positively* with uncertainty. This finding is sufficiently unexpected and diverges so strongly from the theoretical prediction that it deserves further corroboration.

3.2   Uncertainty and Leverage

To gauge the strength of the evidence for a positive relationship between uncertainty and corporate leverage, this subsection presents both aggregate and firm-level regression results. I then examine a number of potential explanations in the following subsection.

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[21]The units of the JLN measure do not map directly to a corresponding model quantity. For this exercise, the series is rescaled to match the scale of the fitted σ process.

Figure 5: Model-implied Histories of Leverage and Credit Spreads
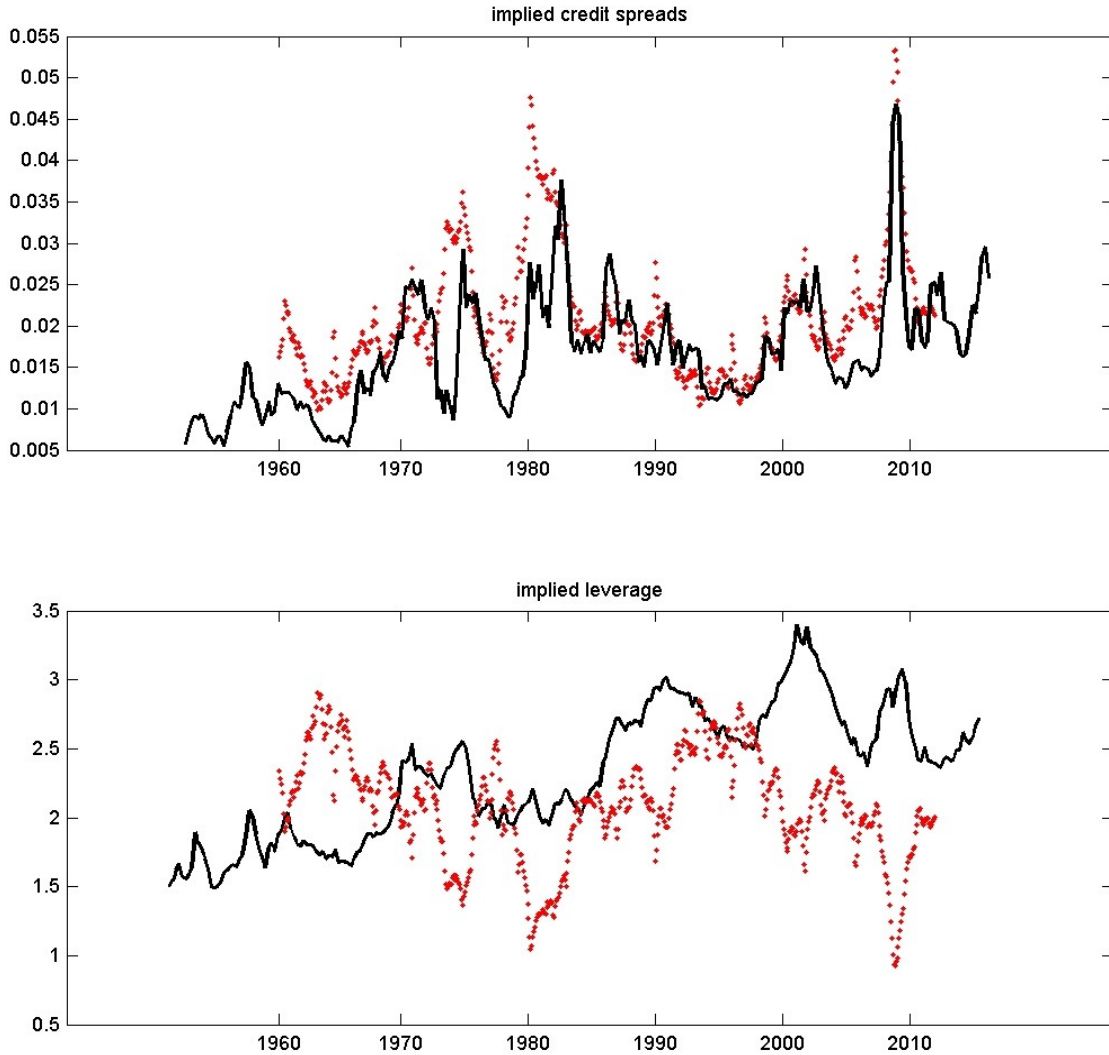
Description: Uncertainty shocks are fed in to the fitted trade-off model as described in the text. The uncertainty series is from Jurado, Ludvigson, and Ng (2015). The top panel plots model-implied credit spread series (dotted line) along with the actual realization from the data (solid line). The bottom panel does likewise for the model implied leverage. Leverage is measured as the ratio of total debt to quarterly cashflow of the nonfinancial corporate sector, from the Flow of Funds accounts. The credit spread is the difference between the Moody's benchmark Baa yield to maturity and the interpolated yield to maturity on 20-year Treasury bonds.

Interpretation: As the model would predict, uncertainty shocks are a primary driver of changes in credit spreads. In contradiction to the model, however, debt levels do not decrease with uncertainty. Indeed they appear to increase.

Table 2 shows the results of several time-series regression specifications using aggre-

gate data with different proxies for leverage and output. The table shows regressions of year-on-year changes in log leverage on year-on-year log changes in the JLN uncertainty series. In the first three columns leverage is net debt divided by income. In the next three columns leverage is net debt divided by assets. Columns 1,2,4, and 5 use data for the U.S. nonfinancial corporate sector. Columns 3 and 6 use broader measures for the private nonfinancial sector. (Full data details are provided in Appendix B.) The basic specifications in columns 1 and 4 control for the lagged level of leverage to account for mean reversion. The controls are augmented in the other columns with a broad set of lagged financial variables that could plausibly influence capital structure decisions, whose definitions are given in the table caption.[22]

The regressions establish that the positive relationship between uncertainty and aggregate debt is statistically strong and robust to measurement choices and specification. The coefficients are larger using the full set of controls, suggesting that omitted variables are unlikely to drive the result. Note that the use of *net* debt as the dependent variable means that precautionary cash hoarding is not a potential explanation. The finding is also not an artefact of the timing assumptions. The uncertainty coefficients remain significantly positive when lagging by one or two quarters.

Appendix C presents a number of additional specifications that verify the conclusion here. The findings are shown to be robust to using several other proxies for uncertainty, each constructed using distinct underlying data. Regressions that use debt changes as the dependent variable (rather than leverage changes) establish that the result is not due to a countercyclical leverage denominator. The positive relation with volatility changes is shown to be not masking a negative relation with volatility levels. Impulse response functions in quarterly vector autoregressions show that uncertainty shocks significantly positively forecast both debt changes and leverage levels.

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[22]Control variables, apart from the credit spread, are constructed from the monthly time series on Robert Shiller's website `http://www.econ.yale.edu/~shiller/data.htm`.

Table 2: Uncertainty and Aggregate Leverage

	$\Delta(D/Y)$			$\Delta(D/A)$		
Δv	0.4470	0.5466	0.3275	0.2135	0.2304	0.1957
	(4.74)	(4.94)	(7.30)	(2.79)	(4.13)	(4.08)
lag lvg	-0.0743	-0.0957	-0.0868	-0.1018	-0.1036	-0.0831
	(1.93)	(2.11)	(3.90)	(2.88)	(2.69)	(3.52)
R^{mkt}		-0.1745	-0.0988		-0.0426	0.0016
		(2.56)	(2.62)		(1.20)	(0.07)
SP_PE		-0.0068	-0.0176		-0.0213	-0.0159
		(0.23)	(1.62)		(1.70)	(1.90)
Y^{10yr}		0.0092	0.0031		0.0058	-0.0001
		(2.59)	(1.65)		(4.04)	(0.03)
CR_SP		0.0083	0.0103		-0.0091	-0.0041
		(0.59)	(1.21)		(1.39)	(0.49)
Δ CPI		-0.7868	-0.5322		-0.6481	-0.2217
		(2.21)	(3.24)		(2.42)	(1.22)
N	226	223	223	226	223	223

Description: The table reports time-series regressions of quarterly aggregate leverage on macroeconomic uncertainty, in log year-on-year changes. In the first three columns leverage is net debt divided by income. In the next three columns leverage is net debt divided by assets. Columns 1,2,4, and 5 use data for the U.S. Nonfinancial corporate sector. Columns 3 and 6 use data for the corporate and noncorporate nonfinancial sector. Details of all series are given in the appendix. Economic uncertainty is the 3-month-ahead average forecast dispersion of macroeconomic statistics constructed by Jurado, Ludvigson, and Ng (2015) and extended through 2017. Δv denotes year-on-year log differences in this series contemporaneous with the dependent variable. The regressions include the lagged level of the dependent variable as a control. Other controls are the lagged 3-month return on the S&P 500 index, the lagged log price-earnings ratio of the index, the yield-to-maturity on 10-year U.S. Treasury bonds; the yield spread of Moodys Baa index bonds minus the 20-year Treasury yield, and the log year-on-year change in the Consumer Price Index. The regressions are estimated quarterly. The sample period is 1961Q3-2017Q4 in columns 1 and 4, and 1961Q3-2017Q4 in the remaining columns. Numbers in parentheses are Newey and West (1987) T-statistics (absolute value) using 8 quarterly lags.
Interpretation: Aggregate leverage changes are positively related to uncertainty changes. The finding is not driven by the leverage denominator, the choice of aggregate debt series, or by omitted financial variables.

The positive response of debt to even lagged uncertainty casts doubt on the explanation that managers *want to* reduce debt in response to uncertainty shocks (consistent with the trade-off model) but are prevented from doing so by delays or other frictions. Also directly addressing this hypothesis, Halling, Yu, and Zechner (2016) estimate latent-variable models for firms' "target leverage" (i.e. the level that managers would like to adjust to in the absence of adjustment costs) and gauge how these vary over the business cycle. Consistent with the results here, they find that these levels *peak* during the first quarters of recessions (when uncertainty is rising). Below we will present additional evidence consistent with the idea that managers intend to increase leverage at these times.

Because the aggregate debt-uncertainty relation could be driven by the financing behavior of the largest firms (as suggested by Covas and den Haan (2011)), I also undertake panel data estimates in which each firm is weighted equally. Table 3 reports results of this firm-level analysis. The variable construction and sample restrictions are standard and follow the literature. Details are given in Appendix B. The regressions utilize observations from quarterly Compustat files. The dependent and independent variables are again overlapping 4-quarter differences. (Other controls are described in the table caption).

Leverage panel regressions are a mainstay of empirical corporate finance, and, following the logic of the trade-off model, specifications often include firm-level uncertainty measures as control variables. The results have been mixed. Frank and Goyal (2009) find that firms' stock volatility does not robustly enter as a core variable in their panels. Lemmon, Roberts, and Zender (2008) report that, while leverage has a negative relation with firms' earnings volatility in pooled OLS regressions, the coefficient actually switches signs in firm fixed-effect specifications. (See their Table V.) This implies that there is, if anything, a positive association of leverage with time-series variation in earnings risk within each firm.

Extending these findings, the top panel of Table 3 reports strong statistical significance for a positive effect of aggregate economic uncertainty on leverage. The estimated

coefficients are positive whether leverage is measured scaling debt by assets, sales, or book equity (reported in the columns from left to right), with the asset scaling producing especially precise estimates. The economic magnitude of the average effect is not large at the firm level: the annual standard deviation of changes to the JLN measure is 0.097, so the implied change in debt-to-assets of a one-standard-deviation increase is only about 0.01.[23] On the other hand, viewed as a systematic effect, the relationship is very economically significant. During the financial crisis, from trough to peak back to trough, the JLN measure spiked up and down by approximately 0.45. The estimated sensitivity in the table implies a spike in debt-to-assets of 5.2 percentage points across all firms, which is *larger* than the build up and contraction in aggregate leverage that actually took place: during the same period, net debt to total book assets in the Z.1 data moved up and then down by approximately 4 percentage points.

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[23]The median firm in the panel has a standard deviation of annual changes to debt-to-assets of 0.13.

Table 3: Firm Leverage and Uncertainty

	PANEL A: Leverage Changes		
	Dependent variable:		
	$\Delta \frac{D}{A}$	$\Delta \frac{D}{S}$	$\Delta \frac{D}{E}$
Δv	0.116	0.439	0.283
	(23.8)	(3.41)	(5.98)
N	192303	192303	192303

	PANEL B: Net External Financing	
	Dependent variable:	
	$\frac{\Delta D}{A}$	$\frac{\Delta(E-RE)}{A}$
Δv	0.168	-0.032
	(19.1)	(2.92)
N	192349	192349

Description: The table reports results of quarterly panel regressions from 1980 to 2011 of financing variables on contemporaneous changes in the macroeconomic uncertainty series of Jurado, Ludvigson, and Ng (2015). The dependent variables in the top panel are 4-quarter changes in three scaled measures of net debt: assets, sales, and book equity. The dependent variables in the bottom panel are 4-quarter changes in net debt (left column) and 4-quarter changes in book equity net of retained earnings. Both measures are scaled by the initial level of assets. All regressions include as controls one lag of the dependent variable, lagged values of Tobin's Q and cash flow, and lagged values of the level of scaled debt (or equity in the equity change regression). See Appendix B for a full description of the sample and variable construction. Numbers in parentheses are Newey and West (1987) T-statistics using 8 lags with firm-level clustering.

Interpretation: The positive leverage-uncertainty relation is also found at the firm level, and is therefore not driven by the dominance of large firms in the aggregate numbers. Moreover firms respond to uncertainty by actively adjusting both components of external finance.

The lower panel in Table 3 investigates how leverage changes are implemented by

individual firms in response to uncertainty shocks. The dependent variable in the left column is net new debt, scaled by initial assets. In the right column, it is the change in paid-in capital (book equity minus retained earnings), also scaled by assets.[24] The two specifications show that firms *both* increase debt and decrease net external equity when uncertainty rises.

The results again affirm that the leverage-uncertainty relation is not due to a counter-cyclical leverage denominator coupled with slowly adjusting debt. Instead, the findings are consistent with the view that uncertainty increases lead to active substitution of debt to equity, and they affirm that the effect is not confined to the largest borrowers.

3.3   Diagnostics

As described in the introduction, identifying empirical shortcomings in standard models of corporate debt was a primary goal of the study. Having highlighted one very significant deficiency in the frictionless trade-off model, the next step is to understand which simplifications of the set-up should be modified to address it. This section looks into the data to shed further light on specific hypotheses about the leverage-uncertainty relationship.

To start, it is worth clarifying how this relationship relates to the business cycle. It is well known that uncertainty is strongly countercyclical, while debt issuance is procyclical (Covas and den Haan (2011), Jermann and Quadrini (2012)). To reconcile these facts, Figure 6 plots the time series of aggregate debt and output separately. The series are in logs and the means have been aligned. (These quantities correspond to $\log Y$ and $\log B = \log b + \log Y$ in the model.)

From the top panel, the data show essentially three episodes in the last 40 years during which the two quantities diverge significantly. These correspond approximately to the last three U.S. recessions. Interestingly, in each episode, the recession starts with a *positive* excursion of debt from its stochastic trend (i.e., the output series). The lower panel magnifies the two series to focus on the Great Recession. For the non-financial

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[24]Both definitions follow Covas and den Haan (2011). The change in paid-in capital measures net sales of new equity.

corporate sector, the "credit crunch" starting in 2009 actually looks like the unwinding of the anomalous debt build-up at the end of the expansion in 2007 as output stagnated.[25]

Figure 6: Debt and Cashflow – U.S. Nonfinancial Corporations
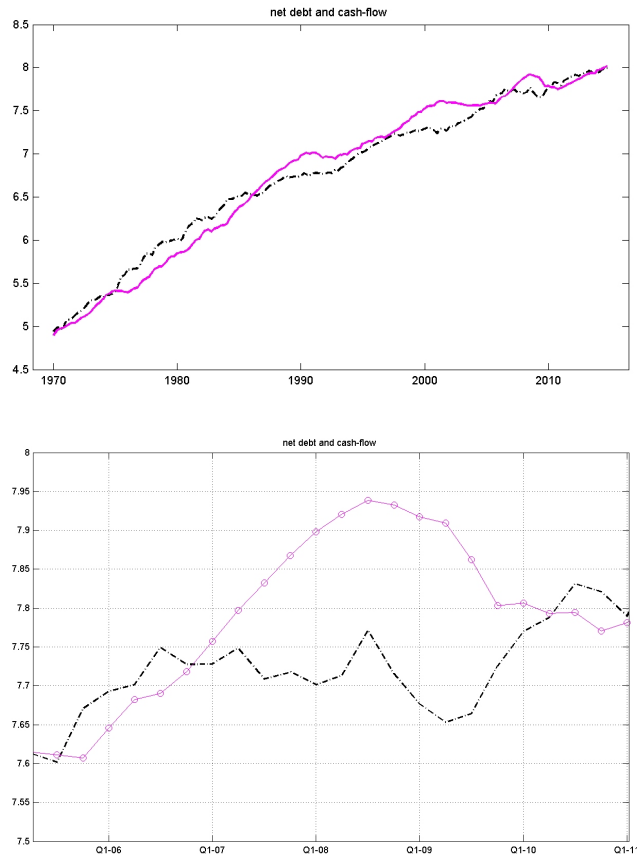
Description: Both panels plot the log of total debt (net of cash items) for nonfinancial U.S. corporations as a solid line, and the log total operating cashflow of this sector as a dashed line. The series have been aligned by subtracting a constant from log debt. The lower panel shows the same data for the 2007-2009 recession.

Interpretation: The deviation of aggregate debt from trend output is concentrated in three recessions. The episodes are not characterized by negative deviations (credit crunches) during the recessions, but rather by positive deviations (debt build-ups) at the ends of the preceeding expansions.

The positive leverage-uncertainty relation in the data arises from the fact that positive uncertainty shocks *also* happen early in recessions or around expansion peaks. The fact

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[25]The plot suggests that the level of output (the dashed line) appropriately captures the stochastic trend in debt. If, instead, the true trend is growing faster than output, then the 2009 contraction may have actually been a negative shock, rather than a reversion to the mean.

that debt rises much faster than output as output stalls at peaks also explains how countercyclical leverage can be consistent with procyclical debt issuance. [26]

Why would firms borrow *more* precisely when their operating cashflows are stalling and the economic environment poses increased risk? A closer look at the data presents some directly relevant features. Table 4 shows more detail on the sources and uses of corporate funds in the periods of debt build-ups going into the last three recessions. There are four immediate observations.

First, the debt increases are not due to precautionary borrowing in the face of uncertainty: as in the regressions above, debt here is measured *net of cash holdings*. Second, the increases are not due to debt's inability to adjust downward as output (or equity value) falls (as would happen in structural models which do not allow debt restructuring). In the data, leverage is increasing because debt is *increasing* in these periods, not merely staying fixed while the denominator contracts. Third, debt is not rising because of investment commitments: internal funds were more than sufficient to fund capital expenditures. Finally, and most tellingly, leverage is not reflecting capital supply frictions on the equity side. The table shows that in each of these build-up periods firms actively *reduced* their equity by buying back stock and continuing to pay dividends.[27]

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[26]The analysis here also reconciles seemingly contradictory results reported in Alfaro, Bloom, and Lin (2016). Also using aggregate U.S. data, those authors show that debt *growth rates* are negatively associated with the VIX index. Their specification relates changes to levels, whereas the analysis here relates levels to levels and changes to changes. The results are not inconsistent because debt contractions happen during recessions when uncertainty is still at a high level, although falling from its peak. Likewise, the debt expansions that occur at the on-sets of recessions (or the peaks of expansions) happen when uncertainty is rising, but from a low level.

[27]It is also worth noting that the table aggregate over each period merely for brevity. The patterns noted here held through out each period. In particular, debt changes and net payouts to equity were both positive for every individual quarter.

Table 4: U.S. Nonfinancial Corporate Sector: Sources and Uses of Funds (billions)

	1988:Q4-1990:Q2	1999:Q3-2001:Q1	2007:Q1-2008:Q3
cash flow from operations	1481.85	2529.69	3971.49
change in net debt	309.11	508.85	1037.37
physical investment	915.57	1945.43	2434.03
interest expense	266.57	312.26	437.57
income tax	177.31	287.42	482.88
change in net receivables	26.50	-47.24	209.89
dividends + equity repurchases	387.43	573.31	1766.27

Description: The table shows the main sources and uses of funds in the aggregate data during the three debt build-up episodes seen in Figure 6.
Interpretation: Firms voluntarily increased debt during these periods. Had they wished not to do so, they could have simply reduced discretionary payments to equity holders.

The numbers appear to establish that increases in leverage at the onsets of recessions were, in fact, the result of voluntary policy decisions by firms. They could have been entirely avoided by *not* borrowing *more*, and merely reducing discretionary payouts to equity holders.

Aggregates can be misleading. It is possible that the firms responsible for the increased borrowing were not the same ones that were repurchasing equity. Many small firms will have had neither strong cash-flow nor the ability to borrow during these periods. The point of the analysis here has been to ask what lessons the data hold towards the objective of improving the (representative-firm) model developed in Section 2. The answer is clear: incorporating costly external financing and/or debt adjustment costs will not help the model account for these important episodes (and thereby reverse the observed positive debt-uncertainty relation). Some other mechanism is required.

3.4 Alternative Model

The data suggest that the trade-off model is missing some positive incentive for firms to actively increase debt as risk rises. In the context of the model, such an incentive could be captured by a subsidy to debtholders, as in the version with default insurance analyzed in Section 2. In that formulation, the subsidy endogenously becomes more valuable as uncertainty rises.

Before developing this alternative further, it is worthwhile to explain its foundations in the intermediation literature. It is certainly not true that creditors of most corporations directly benefit from government default insurance. On the other hand, it is well accepted that moral hazard in the banking sector can feed through to lending behavior and potentially induce systematic risk effects. If, in the context of the current model, banks originate the risky lines of credit that fund corporations, and themselves are funded through insured deposits held by households,[28] then, assuming that intermediation is competitive, the value of the implicit subsidy in the deposit insurance would be (at least partly) passed on to borrowers in the form of cheaper loans. Hence the effect would be equivalent to the reduced form in the model.

The fact that the value of deposit insurance rises with systematic risk is reflected in research showing investor flows towards such deposits in times of stress (Pennacchi (2006)). Indeed, this response by investors would serve as an additional amplification mechanism to the (constant) level of moral hazard formulation of the model. Evidence that competitive lending behavior of insured institutions responds to deposit flows along both price and quantity dimensions is summarized in Calomiris and Jaremski (2016). Broader evidence that systemic risk increases in with deposit insurance appears, for example, in Demirgüç-Kunt and Huizinga (2004).

It thus seems at least plausible to hypothesize that the presence of default insurance at the household level affects the price of corporate credit. Recalling, the first-order condition for the default insurance model (equation (6)), the trade-off incentives for debt

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[28]The insurance also encompasses implicit backing, as in too-big-to-fail guarantees.

now augmented by a term that becomes more valuable as default becomes more likely. So debt becomes increasingly attractive to the firm, even though credit spreads are rising. Insurance means debt markets are sending firms the wrong signal by not raising borrowing costs *enough* as uncertainty rises, and managers are responding rationally by replacing equity with overpriced debt.

Figure 7 illustrates the optimal policy. The left panel varies, Θ, the fraction of the debt that is covered by insurance. The lowest line, $\Theta = 0.2$, shows that with little insurance the model preserves the implication of the trade-off model that leverage contracts with uncertainty. However, as the level of insurance increases, leverage now starts to *rise* with σ when uncertainty is high and the insurance is most valuable. Thus, the general implication of this case is a U-shape relation. The right hand plot shows that, as the tax deduction parameter $\bar{r}$ declines, the relation can become almost monotonically increasing.

Figure 7: Leverage with Default Insurance
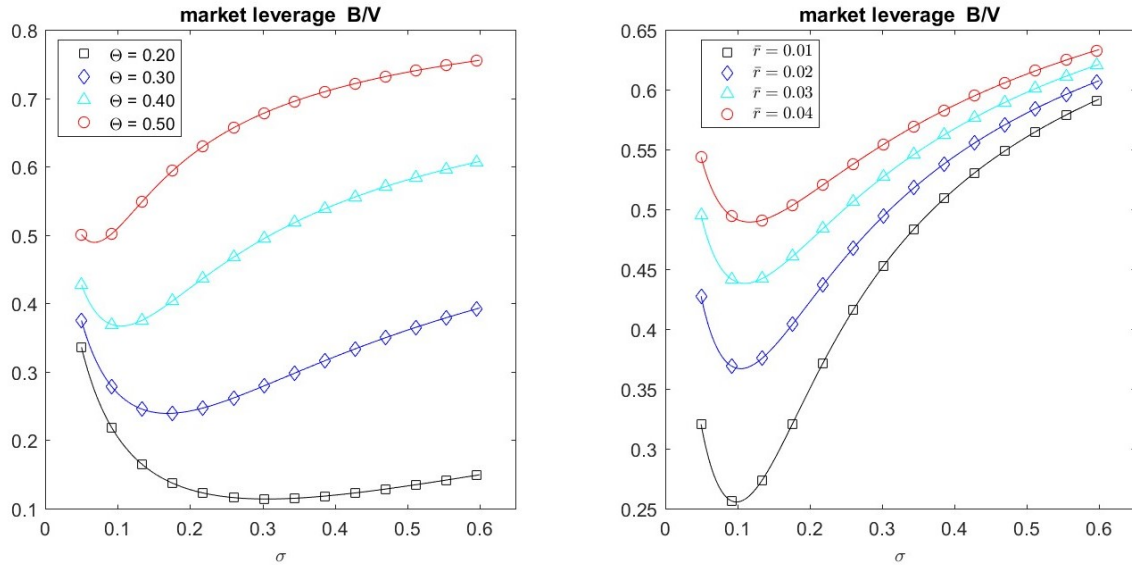
Description: The plots show the optimal leverage policy as a function of uncertainty under the default-insurance version of the model. The left plot varies Θ, the level of insurance (fixing $\bar{r} = 0.02$). The right-hand plot varies $\bar{r}$, the tax deduction parameter (fixing $\Theta = 0.4$). The other parameters are $\gamma = 7$, $\lambda = 1$, $\tau = 0.35$ and $L = 4$.

Interpretation: Now the model can deliver a leverage-uncertainty relation that is positive on average.

I fit this version of the model using the same target moments as used for fitting the trade-off model. The fit is shown in Table 5. This specification matches the observed positive correlation of leverage and credit spreads while also achieving a reasonable match on the other real and financial moments. Of the shortcomings that remain, the high unconditional equity valuation stems from a lower average riskless rate, due to high precautionary savings. Matching the economy's average leverage means having low leverage in normal times, which produces a low average credit spread and default frequency. Despite these issues, the SMM criterion function of minimized moment squared errors falls to 23.0 in this model from a value of 236.4 for the trade-off model.[29]

Table 5: Default Insurance Model Fit

	Data	Model
output growth	0.0158	0.0226
output standard dev	0.0306	0.0302
output skewness	-0.3973	-0.2743
output kurtosis	5.5771	5.6432
investment rate	0.0780	0.0782
default rate	0.0087	0.0101
equity valuation	4.6342	6.3098
leverage	2.3360	2.5633
leverage standard dev	0.4578	0.2513
leverage change std dev	0.0741	0.0738
credit spread	0.0165	0.0112
credit spread standard dev	0.0068	0.0104
credit spread change std dev	0.0032	0.0031
leverage-credit spread correlation	0.5229	0.5268

Description: The table shows the implied moments from a simulated method of moments estimation of the default-insurance version of the model, developed in Section 2. The estimation procedure is the same as that of the trade-off model described in the caption to Table 1.

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[29]Essentially all of this improvement is due to the leverage-credit spread correlation. The standard error of this correlation is approximately 0.10 and the TO model gets it wrong by $|0.52 + 0.98| = 1.5$ which contributes $15^2 = 225$ to the objective function.

Summarizing, the trade-off model is unable to account for the joint dynamics of credit spreads and leverage, which is driven by the positive association of *both* with economic uncertainty. Moreover, the data clearly establish that this association is not due to precautionary cash hoarding, restructuring constraints preventing decreasing debt, or supply restrictions on equity capital.

The default-insurance version of the model offers one potential resolution of the empirical problems. With this alternative in mind, the next section returns to the topic of the real effects of debt on the equilibrium. Some other possible mechanisms for explaining the uncertainty-leverage relation are also discussed below. Discriminating among them is beyond the scope of the present work. Rather, I will argue that the key point is that they are likely to share the major implications of the default insurance version for asset pricing, welfare, and policy.

4. The Real Effects of Debt

This work was motivated by the topic of quantifying the real distortions induced by corporate credit decisions. Here I exhibit the conclusions implied by the two models examined in the previous section. The framework permits precise quantification of the real costs, and illuminates the difference between the two versions.[30]

The analysis has three parts. First, I quantify how much corporate default increases aggregate risk. Second, I show the effect of this increase in output risk on investment, and growth. Third, I compute the total effect on welfare via the representative agent's value function. For each computation, the estimated economies are compared with an otherwise equal one in which the debt benefit functions have been set to zero and hence

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[30]Both versions of the model are, of course, stark simplifications of the macroeconomy, omitting many mechanisms – including labor market frictions, household and public sector credit, and financial constraints – that are important for understanding business cycles. Many of these features have been shown to produce amplification mechanisms for real shocks. The analysis below highlights the potential amplification effects of one formulation of corporate debt. Combining multiple amplification channels would likely reinforce the conclusions. Jermann and Quadrini (2012) argue, for example, that labor market effects significantly magnify the real impact of financial shocks.

there is no leverage.

For these calculations, in order to compare the trade-off and default-insurance models on the same footing, I use a single parameter set for both. It turns out that the best fitting specifications for the two produce quite significantly different preference parameters, which have major effects on consumption and welfare. There are also differences in the estimated dynamics of the σ process. In order to have a clean comparison, I adopt a specification, shown in Table 6, that roughly splits the difference between the two estimated sets of parameters. The debt parameters are chosen so that each model implies the same unconditional level of default frequency, which is close to the empirical counterpart shown in Table 1. Because the default frequency drives the real losses in both versions, they are thus calibrated equally for this comparison. The robustness of the conclusions to the preference parameters will be discussed below.

Table 6: Parameter values for evaluation of real effects

Risk aversion	γ	8.00
E.I.S.	ψ	1.00
Subjective discount rate	β	0.10
Interest deduction rate	$\bar{r}$	0.085/0.023
Corporate tax rate	τ	0.30
Debt recovery/insurance rate	θ	0.00/0.40
Production function scale	ζ_1	0.975
Production function curvature	ζ_0	0.105
Output growth constant	μ	0.065
Uncertainty mean	$\bar{\sigma}$	0.095
Uncertainty min/max	σ_l, σ_u	0.05/0.60
Uncertainty mean-reversion	κ	0.200
Uncertainty diffusion	s_0	0.220
Jump intensity	λ	1.0
Jump dispersion	L	4.0

Description: The table shows the parameters used for the comparision of real effects in the two versions of the model.

As we have seen, the key difference between the two versions of the model is that

the trade-off formulation implies that leverage contracts with uncertainty whereas the default-insurance formulation implies that it expands. The top left-hand panel of Figure 8 demonstrates that the former dynamic implies that the effect of defaults on output consequently shrinks as σ rises. Even though the default rate rises with σ, the rapid contraction in credit means that it takes increasingly large output jumps to trigger default. (Recall the default threshold is directly related to the firms leverage: $\ell = \exp(\varphi^\star)$.) Hence those firms that do default in high σ states are ones whose output would have been small anyway. By contrast, in the top right-hand panel we see expected aggregate default losses rise, rather than falling, with increases in risk under the alternative debt subsidy. Hence many more firms will default on a down-jump in high-σ states under this formulation, and the loss of their output is not negligible.

The two panels on the second row of Figure 8 show the effect of default on the second moment of the aggregate output process as compared to the no-debt equivalent economies. This is another way of quantifying the default effect: debt substantially increases aggregate risk in the default-insurance version (right panel) and has minor effects in the trade-off version (left). The two panels on the third row show the impact of these risk effects on aggregate investment. In the trade-off model, the real effect of debt is to lower the annual investment rate but only fractionally. For the deposit insurance case, the extra default losses inhibit investment substantially more, resulting in lower growth for the economy.

The net effect of uncertainty and growth on the well-being of the representative agent is quantified by her value function, which is readily obtainable in terms of current income, Y, and uncertainty, σ for both versions of the model. According to Proposition 2 of Section 2, the form of the function is $j(\sigma)\, Y^{1-\gamma}/(1-\gamma)$. So the difference between two economies with the same income level can be summarized in terms of percentage income by the log certainty equivalent function $\log(j)/(1-\gamma)$.[31] The bottom row of Figure 8 plots the value function in these units for each version. Reading left to right,

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[31]Note that $j > 0$, and the models are evaluated with $\gamma > 1$. Hence lower values of j imply higher (less negative) value functions.

the functions steeply decline with σ under both models, indicating that times of high uncertainty are truly bad states.

The dashed line in each panel shows the function in the analogous economies with no debt. Reading along the vertical axis, the difference between the two lines is the equivalent change in permanent income due to debt. In the left panel, the welfare loss in the trade-off formulation debt is roughly constant across states, and is of the order of three percent. The right panel shows that, in the default-insurance case, the loss is of a similar magnitude at the low end of the uncertainty spectrum. However, now the real effects rise dramatically with σ, exceeding 20 percent at the high end. In this model, we see debt playing a major amplification role in terms of welfare in recessions.[32]

How are these findings affected by the parameter choices? The overall real effect of debt can be summarized by integrating the certainty equivalent costs with respect to the ergodic distribution of the state variable. For the cases shown in the figure, this calculation yields expected losses of 3.61% and 5.98% for the left and right panels, respectively. It is straightforward to check the variation in these numbers with respect to the preference parameters.

First, the estimation used a relatively high subjective discount rate of 0.10. This has the effect of *lowering* the estimated welfare impact because the representative agent's effective horizon is short. Reducing it to $\beta = 0.05$ causes the welfare losses to rise to 7.08% for the trade-off model, and to explode to 15.16% for the deposit insurance model. Next, welfare losses also rise with the elasticity of intertemporal substitution. Lowering ψ to 0.5 causes the two models to have expected losses of 2.42% and 3.68%, and raising it to $\psi = 2.0$ yields expected losses of 4.97% and 8.73%, for trade-off and deposit-insurance versions, respectively.

Finally, the models have been evaluated with a relatively high risk aversion, $\gamma = 8$. It is not surprising that the estimation yields values in this range, given that asset pricing

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[32]While not the focus of the current analysis, the welfare results have parallel implications for marginal utility and asset pricing. Under default insurance, investors are much worse off in high volatility states and would pay much higher prices for insurance against them. Similarly, the conditional equity risk premium (even unlevered) becomes extremely sensitive to level of uncertainty.

moments were targeted. High risk aversion magnifies the impact of the increases in consumption volatility caused by debt. On the other hand, it also lowers the optimal degree of leverage.[33] This quantity effect dominates in the trade-off case, but not in the deposit-insurance case. Lowering γ to 4.0 produces higher expected welfare costs of 4.17% in the former case, and lower costs, 4.68%, in the latter case.

The choice of preference parameters thus causes the unconditional welfare cost of debt in the trade-off case to vary in the range of single-digit percentages (roughly 2% to 7%), with the default-insurance values ranging from marginally higher to more than twice as large. However, regardless of the parameter choices, the key *conditional* difference between the models is preserved. With all values of γ, β, and ψ the default-insurance version implies a strong increase in welfare losses as σ rises. The leverage build up induced by debt guarantees – and the associated increase in expected default losses – creates a truly financial element to the stress the economy experiences in times of high uncertainty.

While the default-insurance model did well empirically in Section 3, one can imagine other theories that could also be consistent with a positive volatility-leverage relationship. In the model of Levy and Hennessy (2007) countercyclical leverage arises from the need to maintain high managerial ownership in bad times to solve agency problems. Firms may also increase debt in anticipation of deteriorating operating performance to enhance bargaining power with labor (Liu (2017)). Managers may also face incentives to maintain commitment to equity payouts in bad times. Huang (2016) reports empirical evidence for such commitment in the period after 2007. Another prominent possibility is conflicting incentives between creditors and owners. Although the form of the debt contract in the current paper rules out expropriation of existing creditors, if, instead firms issued long-term debt without any obligation to limit future issuance, as in Johnson, Liu, and Yu (2018), then increases in risk could raise incentives to exploit expropriation opportunities by increasing leverage.

While there are undoubtedly significant differences in the predictions of these different

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[33]Note that neither β nor ψ affect the leverage choice. This follows immediately from Proposition 1.

stories, from the standpoint of welfare analysis, their implications are likely to align with the default-insurance model. When leverage increases with (exogenous) risk, for whatever reason, this will likely imply higher expected default losses in high-risk states, thus amplifying uncertainty shocks and increasing marginal utility.

Figure 8: Real Effects of Debt
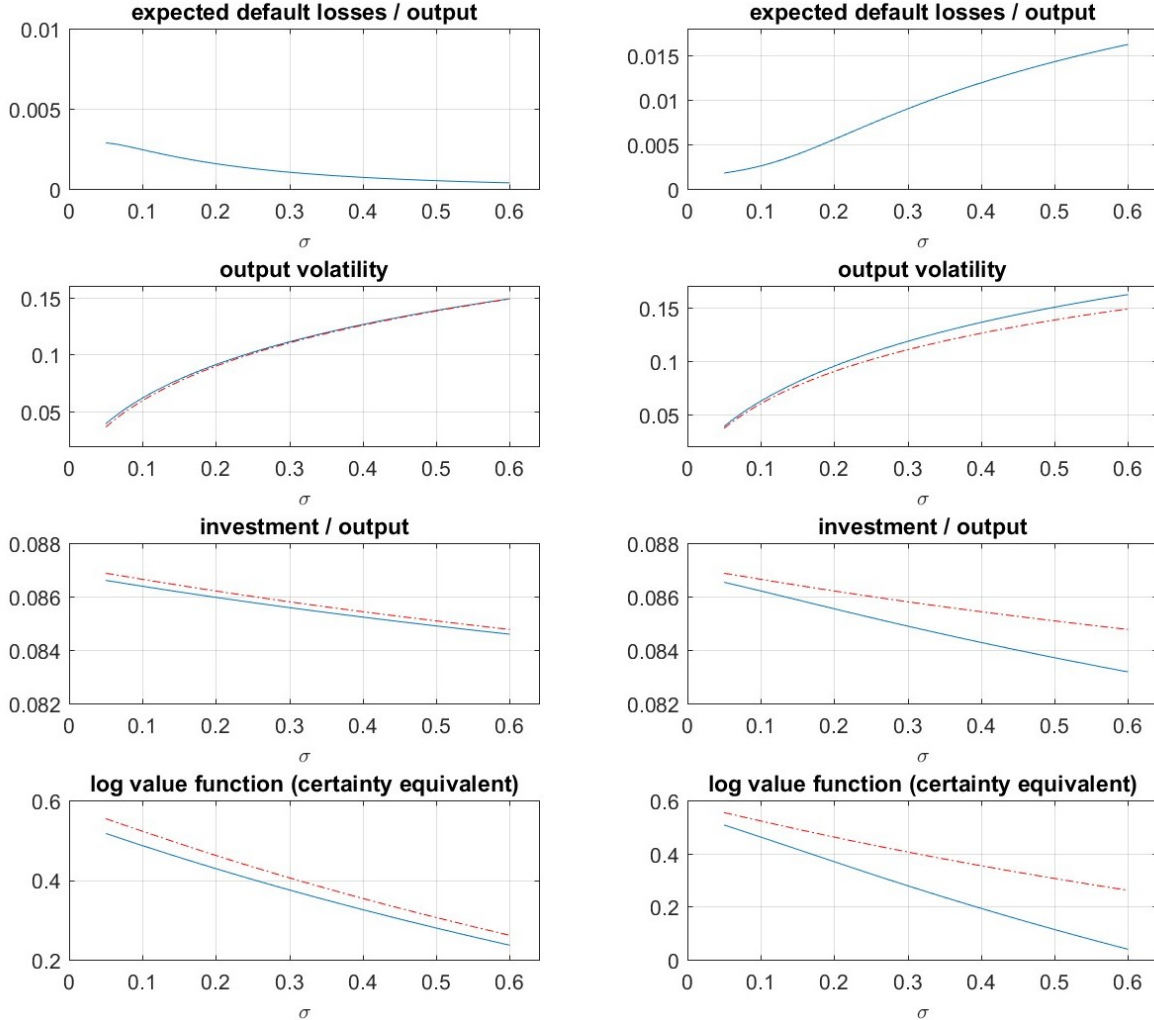
Description: The figure shows properties of the two versions of the model evaluated with parameters given in Table 6. In each row, the left panel shows the result for the trade-off model. The right panel shows the result for the default-insurance model. The top two panels show the aggregate percentage default losses conditional on a systematic negative jump. In the remaining plots, each economy is compared to an equivalent one without debt, plotted as dashed lines. The second row shows output volatility. The third row shows optimal investment. The final row plots the representative agent's welfare in certainty equivalent units.

Interpretation: When debt increases with uncertainty, as in the default-insurance model, there can be a large amplification of the effects of uncertainty shocks, resulting in steep welfare losses. This does not happen in the trade-off model.

5. Conclusion

This paper has analyzed a tractable general equilibrium model that includes optimal capital structure decisions by firms. The model serves two related goals. First, it permits a critical assessment of the empirical performance of a standard, benchmark theory – the trade-off framework – in terms of its ability to explain the price and quantity dynamics of debt. Second, it permits direct quantification of the real effects of corporate debt on the economy.

In fact, even without any financial frictions, the trade-off model does provide a reasonable description of the cyclical evolution of corporate finance. Periodic increases in uncertainty lead to spikes in credit spreads and discount rates. Substantial contractions in credit can be followed by waves of default and inefficient liquidation.

Unexpectedly, the primary empirical difficulty of this model is that the debt of U.S. nonfinancial firms actually increases with uncertainty rather than declining. This is a new finding, which holds both in the aggregate data and at the firm level. The effect is driven by apparently intentional increases in leverage at the peaks of business cycles, which see corporations add debt and repurchase equity even as output stalls and uncertainty increases. While much research has focused on the effect of financial frictions and supply constraints in amplifying real risk, if anything, the data seem to point towards a loosening of borrowing capacity in these episodes.[34]

The leverage-volatility relation has important implications for the paper's second objective: understanding the real effects of corporate credit. I illustrate this by developing an alternative debt formulation within the original modeling framework. In this version, the value of debt subsidies (to the firm) endogenously increase with risk, which can be viewed as a reduced-form depiction of moral hazard in intermediation, i.e., deposit insurance or too-big-to-fail subsidies. With this set-up, debt is effectively underpriced even as credit risk rises, and firms substitute away from equity finance, which can account for the

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[34]To be clear, I am not suggesting capital supply frictions are not present and important in the real world. I am only observing that such frictions will not resolve the particular puzzle highlighted here: the increases in leverage at the on-sets of recessions.

positive correlation between leverage and uncertainty. As a result of this substitution, uncertainty shocks lead to increases in default rates and large potential welfare losses. In high uncertainty states, the certainty equivalent loss, compared to an equivalent economy without debt, can be up to 20 percent of permanent income. Other capital structure theories that also embed incentives for firms to increase leverage when fundamental risks increase are likely to yield similar implications.

The findings point to the importance of understanding the true incentives driving the use of corporate debt, both for financial research (in asset pricing and corporate finance) and also more broadly for policy analysis that assesses the real effects of finance on macroeconomic risk and welfare.

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Appendix

A. Proofs

This appendix provides the proofs of the results in Section 2.

The first proposition solves the firm's capital structure problem before having found the pricing kernel. This is possible because we can deduce enough about aggregate dynamics in advance from the following lemmas.

The first lemma formalizes the dynamics of output deduced in the text.

Lemma 1. *Assume that firm value and optimal debt are linear linear in output –* $V^{(i)} = v(\sigma)Y^{(i)}$ *and* $B^{(i)} = b(\sigma)Y^{(i)}$ *– and that* $b < v$ *for all* σ.

Then aggregate output, including entry and exit effects, obeys the stochastic differential equation:

$$\frac{dY}{Y} = \mu_Y \ dt + \ d\left[\sum_{j=1}^{\mathcal{J}_t} \left((u_t - 1)1_{\{j,+\}} + (d_t - 1)1_{\{j,-\}}\right)\right]. \tag{17}$$

where $\mu_Y = (\mu + \zeta(I/Y))$ *includes the growth in the mass of firms due to aggregate investment, and* $1_{\{j,\pm\}}$ *are indicators for the sign of the* j*th jump, and* $u_t = u(\sigma_t)$ *is*

$$\mathrm{E_t}\left[e^{\varphi_j}|\varphi_j > 0\right]$$

and

$$d_t = \mathrm{E_t}\left[e^{\varphi_j}1_{\{\varphi_j > \varphi^\star\}}|\varphi_j < 0\right] = \int_{\varphi^\star}^{0} e^\varphi \ d\mathcal{F}^{\varphi-}(\sigma_t)$$

where $\varphi^\star = \varphi^\star(\sigma)$ *is the critical threshold for jumps below which firms default.*

Proof.   Formally, the probabilistic structure of the model assumes a set of *i.i.d.* firms indexed by $m \in [0, \bar{M}]$ where the interval is technically a dense limit of increasing countable subsets.[35] At time t, the set of firms that have come into existence is indexed by the subinterval

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[35]Formally, the "continuum" can be described as the limit of economies with countable, increasing index sets, each of which is endowed with a finitely additive measure of total mass M. Al-Najjar (2004)

$[0, M_t]$, where $M_t < \bar{M}$. For simplicity, we take M_t to be nondecreasing in t, meaning that the "mass" M_t counts firms with zero output (those that have exited). Also notice that the structure implicitly imposes that the distribution of output (or any other characteristic) of entering firms, i.e., those in $[M_t, M_t + dM_t)$, is the same as that of those that have previously entered by time t.

With this set-up, the dynamics of Y before considering entry and exit follow from a law of large numbers applied across firms at each point in time, as described in the text.

We deduce the existence of a single default-inducing jump threshold for all firms from the linearity assumption. (If no such threshold exists at t, take $\varphi^\star(t) = -\infty$.) The assumption $b < v$ implies that, absent a jump, there is no default due to changes in the aggregate state σ_t. The effect entry is to increase the mass of firms according to $dM/M = \zeta(I/Y)\ dt$ by assumption. Because the output distribution of the entering firms is the same as that of incumbents, their contribution to dY is just $Y\,dM$.   *QED*

The next lemma characterizes the form of the economy's value function, and deduces the dynamics of consumption and marginal utility.

Lemma 2. *Given an output process described by (17), the representative agent's value function is of the form $J = j(\sigma)\ Y^{1-\gamma}/(1-\gamma)$.*

The aggregate consumption process is $C = c(\sigma)Y$. The functions $j(\sigma)$ and $c(\sigma)$ are characterized (respectively) by an ordinary differential equation and an algebraic equation given in the proof.

Let Λ denote the pricing kernel. Its dynamics may be written

$$\frac{d\Lambda}{\Lambda} = \eta_0(\sigma)\ dt + \eta_1(\sigma)\ dW + d\left[\sum_{j=1}^{\mathcal{J}_t} \left((u(\sigma)^{-\gamma} - 1)1_{\{j,+\}} + (d(\sigma)^{-\gamma} - 1)1_{\{j,-\}} \right)\right].$$

Proof.   Given the aggregator function $f(C, J)$, the Bellman equation for J tells us that $\max_C\{\mathrm{E}[dJ] + f(C, J)\ dt\} = 0$. Under the conjectured form for $J = J(\sigma, Y)$, and using the

shows that integration of random variables in the limit economy is well defined and a strong law of large numbers applies.

known dynamics of σ, and Y, we have $\mathrm{E}[dJ]/J =$

$$\frac{j(\sigma)'}{j(\sigma)}\, m(\sigma) + (1-\gamma)\mu_Y(\sigma) + \tfrac{1}{2}s^2(\sigma)\,\frac{j(\sigma)''}{j(\sigma)} + \tfrac{1}{2}\lambda[(u(\sigma)^{1-\gamma}-1)+(d(\sigma)^{1-\gamma}-1)]$$

using the version of Itô's lemma for jumping processes. Dividing $f(C,J)$ by J and using the conjectured form of C, we get the two terms[36]

$$\beta\theta c(\sigma)^\rho j(\sigma)^{-\frac{1}{\theta}} - \beta\theta.$$

Adding these to the $\mathrm{E}[dJ]/J$ terms and multiplying by j gives the ODE:

$$\beta\theta c(\sigma)^\rho j(\sigma)^{1-\frac{1}{\theta}} - \beta\theta j(\sigma) + j(\sigma)'m(\sigma) + \tfrac{1}{2}s^2(\sigma)j(\sigma)'' + (1-\gamma)\mu_Y j(\sigma) + \tfrac{1}{2}\lambda[(u(\sigma)^{1-\gamma}-1)+(d(\sigma)^{1-\gamma}-1)]j = 0$$

or, more compactly,

$$\tfrac{1}{2}s^2 j'' + j'm + \beta\theta c^\rho j^{1-\frac{1}{\theta}} + \left((1-\gamma)\mu_Y + \tfrac{1}{2}\lambda[(u^{1-\gamma}-1)+(d^{1-\gamma}-1)] - \beta\theta\right)j = 0 \qquad (18)$$

which must hold at the optimal consumption policy. Recall that $\mu_Y = \mu + \zeta(1-c(\sigma))$. Hence, the FOC for consumption is simply

$$\beta c^{\rho-1} j^{-\frac{1}{\theta}} = \zeta'(1-c). \qquad (19)$$

Given any smooth function $c(\sigma)$, the ODE defining j is to be solved on the closed interval $[\underline{\sigma}, \bar{\sigma}]$, and coefficient $s(\sigma)$ on the second order term is zero at the endpoints. This is equivalent to two mixed boundary conditions (i.e., a relation $g(j', j) = 0$), which suffices for existence and uniqueness of a solution. (Baxley and Brown (1981).) Then (19) is just an algebraic equation for $c(\sigma)$ given $j(\sigma)$. Formally, the implicit solution can be inserted in the coefficients of the ODE. In practice, solving the two equations iteratively rapidly yields a convergent solution for the pair of functions. The existence of these solutions verifies the conjectured functional forms.

Given these J and C functions, Duffie and Skiadas (1994) show that the pricing kernel

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[36]Recall $f(C,J) = \frac{\beta C^\rho/\rho}{((1-\gamma)\,J)^{1/\theta-1}} - \beta\theta J.$

under stochastic differential utility is

$$\Lambda_t = e^{\int_0^t f_J(C_u, J_u)\ du}\ f_C(C_t, J_t).$$

Here $f_C(C, J) = \beta\ c(\sigma)^{\rho-1} j(\sigma)^{1-\frac{1}{\theta}}\ Y^{-\gamma}$

The drift and diffusion coefficient of Λ can be readily evaluated (as functions of σ) by Itô's lemma and straightforward algebra, and are not of immediate interest. What is important is that $d\Lambda/\Lambda$ inherits the jump structure of $Y^{-\gamma}$, which is equivalent to the conclusion of the lemma. *QED*

We now proceed to the proof of Proposition 1. While the preceding lemma would appear to have characterized aggregate dynamics, in fact, only the *form* of the pricing kernel has been determined. What has not been pinned down are the critical jump threshold $\varphi^\star(\sigma)$ and the size of the downward jump $d(\sigma)$.

Proof of Proposition 1.

To start, assume firm value is linear in output prior to default.

The proposition first asserts that the optimal default policy for equity holders is to abandon if and only if, following a jump to $Y_t^{(i)}$, the value of the firm is below the pre-jump level of optimal debt, $B_{t-}^{(i)}$. If equity holders do not abandon, then their optimal debt policy at t is to adjust to $B_t^{(i)}$. If they do so, they repay the difference $B_{t-}^{(i)} - B_t^{(i)} > 0$ to debt holders, and their claim is now worth $V_t^{(i)} - B_t^{(i)}$. Clearly they will do this if and only if the debt repayment is less than the value they receive:

$$V_t^{(i)} - B_t^{(i)} > B_{t-}^{(i)} - B_t^{(i)} \iff V_t^{(i)} > B_{t-}^{(i)}$$

as asserted.

From this observation, it follows that we can link the optimal leverage ratio prior to a jump with the critical default threshold. Default occurs iff $V_t^{(i)} \le B_{t-}^{(i)}$. So dividing by

$B_{t-}^{(i)}$, we have

$$e^{\varphi^\star} \equiv \frac{Y_t^{(i),\star}}{Y_{t-}^{(i)}} = \frac{V_t^{(i),\star}}{V_{t-}^{(i)}} = \frac{B_{t-}^{(i)}}{V_{t-}^{(i)}}$$

where the second inequality uses the conjectured linearity. Because the left side here is only a function of the aggregate state, the linearity of $V^{(i)}$ thus implies that of $B^{(i)}$. Denote the optimal market leverage ratio ℓ. Then we have characterized the optimal bankruptcy barrier given optimal leverage as

$$e^{\varphi^\star} = b(\sigma)/v(\sigma) = \ell(\sigma). \tag{20}$$

The value of the ith firm is characterized by the condition

$$\mathrm{E}[d\Lambda V^{(i)}]/(\Lambda V^{(i)}) = -((1 - \tau)Y^{(i)} + \bar{r}\tau B^{(i)}) \, dt/V^{(i)}.$$

The numerator on the right is the firm's after-tax earnings when interest deduction is permitted at the statutory rate $\bar{r}$, and the tax rate is τ.

Let us conjecture that, prior to default, $V^{(i)} = v(\sigma)(1 - \tau)Y^{(i)}$. Given the form of the pricing kernel, applying Itô's lemma to the left side of the above condition gives the equation

$$\tfrac{1}{2}s^2(\sigma)v'' + [m(\sigma) + \eta_1(\sigma)s(\sigma)]v' + \left(\eta_0 + \mu + \tfrac{1}{2}\lambda[(u^{1-\gamma} - 1) + (d^{1-\gamma} - 1) + \bar{r}\tau\ell(\sigma)]\right)v + 1 = 0. \tag{21}$$

As with the j equation above, existence and uniqueness of a solution to this equation will verify the linearity conjecture.

Now we consider the first order condition that maximizes v with respect to b, or, equivalently, with respect to ℓ.

Differentiating (21), there are contributions from the benefit flow term as well as from the down-jump term $d^{1-\gamma} - 1$. The latter term is the expectation of the percentage jump in the product $\Lambda V^{(i)}$. The ratios $V_t^{(i)}/V_{t-}^{(i)}$ and Λ_t/Λ_{t-} are independent given a jump, and the pricing kernel term is $d^{-\gamma}$. The firm takes this component as given and not

affected by its default decision. However, the jump in own-firm value is affected. Hence we differentiate

$$\int_{\varphi^\star}^{0} e^\varphi \ d\mathcal{F}^{\varphi-}$$

and multiply by $d^{-\gamma}$. From above, we know $\varphi^\star = \log(\ell)$. Differentiating this and using the chain rule gives the FOC as

$$\tfrac{1}{2}\lambda d^{-\gamma} \ f^{\varphi-}(\varphi^\star) = \bar{r}\tau$$

where $f^{\varphi-}$ is the density function of the negative jumps. And from Lemma 1,

$$d = \int_{\varphi^\star}^{0} e^\varphi \ d\mathcal{F}^{\varphi-}.$$

The preceding two equations form a system whose solutions are d and $\varphi^\star$. This closes the problem. It is easy to see that the first equation describe a locus of points d that is monotonically increasing from zero in $|\varphi^\star|$. The second describes a locus that monotonically decreases to zero as long as the density function does so, which has been assumed. Hence the system has a unique interior solution. (The fact that $\varphi^\star = 0$ is not a solution verifies the assertion that $b < v$ for all σ. That is, jumps alone can trigger default.)

So far, the derivation has assumed that leverage would be chosen to maximize the value of the firm. The proposition also asserts that resulting policy would also followed by managers who could not commit to maximizing firm value, and instead maximized the value of equity. Intuitively, this is a consequence of the stipulation that the price, p, of the debt contract per unit face value is always one, which implies that no policy can expropriate value from existing debt holders.

Formally, if the firm is at the firm-value maximizing value, policy pair V', B' then equity holders can costlessly move to any V'', B'' by paying (or receiving if negative) the difference in debt amounts $B' - B''$. Including this payment, equity holders will have achieved net value $V'' - B'$. But, by assumption, this is strictly less than the original

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value they had, $V' - B'$.

<div align="right">QED</div>

Proof of Corollary 2.1

Let P denote the value of an arbitrary debt contract and $p = P/B$ be its price per unit face value. Let τ denote the sooner of the firm's default time and the repayment time of the contract. (The debt contract considered in the paper has no formal maturity. However, the firm has the right to alter the amount outstanding costlessly at any time. We can consider a repayment of amount ΔB as applying *pro rata* randomly across bonds. So any individual bond can be considered to have a stochastic retirement time.) Then on $[0, \tau)$, p solves the valuation equation

$$\tfrac{1}{2}s^2(\sigma)p'' + [m(\sigma) + \eta_1(\sigma)s(\sigma)]p' + \left(\eta_0 + \tfrac{1}{2}\lambda[(u^{-\gamma}\mathrm{E}_t\left[\frac{p^+}{p}\right] - 1) + (d^{-\gamma}\mathrm{E}_t\left[\frac{p^-}{p}\right] - 1)]\right)p + \Gamma = 0.$$
$$(22)$$

where Γ is the coupon rate and $\frac{p^+}{p}$ and $\frac{p^-}{p}$ denote the fractional changes in p conditional on an up and down jump, respectively.

We require that Γ be set such that $p = 1$ solves this equation. And we are assuming $p = 0$ on default. In that case, the equation reads

$$\eta_0 + \tfrac{1}{2}\lambda[(u^{-\gamma} - 1) + (d^{-\gamma}\mathcal{F}^{\varphi^-}(\varphi^\star) - 1)] + \Gamma = 0.$$

or

$$\eta_0 + \tfrac{1}{2}\lambda[(u^{-\gamma} - 1) + (d^{-\gamma} - 1)] - \tfrac{1}{2}\lambda d^{-\gamma}(1 - \mathcal{F}^{\varphi^-}(\varphi^\star))] + \Gamma = 0.$$

We then recognize that the first two term are the drift rate of the pricing kernel, Λ, which is equal to minus the instantaneous riskless rate, r. Hence,

$$\Gamma = r + \tfrac{1}{2}\lambda d^{-\gamma}(1 - \mathcal{F}^{\varphi^-}(\varphi^\star)).$$

<div align="right">QED</div>

Proof of Corollary 2.2

The assumption now is that, creditors of a firm that has defaulted receive a payment ΘB_{t-} where B_{t-} is the face value of debt prior to default. The government does not have the ability to create the value lost due to default, however. Those losses create the same decline in aggregate consumption as in the base case. (So implicitly a tax on all households must fund the creditors' insurance payout.)

To derive the effect on optimal capital structure, we revisit the equation (21) for firm value. Previously, the contribution from the expected change in $\Lambda\, V^{(i)}$ from down jumps was $\frac{1}{2}\lambda$ times

$$d^{-\gamma} \int_{\varphi^\star}^0 e^\varphi \, d\mathcal{F}^{\varphi-} \;-\; 1.$$

Now there is an additional contribution to the left-hand term from the default insurance that creditors collect:

$$d^{-\gamma} \left[\int_{\varphi^\star}^0 e^\varphi \, d\mathcal{F}^{\varphi-} + \Theta \frac{B^{(i)}}{V^{(i)}} \int_{-\infty}^{\varphi^\star} d\mathcal{F}^{\varphi-} \right].$$

Differentiating the new term with respect to $\ell = B^{(i)}/V^{(i)}$ adds the two terms

$$\Theta \int_{-\infty}^{\varphi^\star} d\mathcal{F}^{\varphi-} + \Theta\, f^{\varphi-}(\varphi^\star).$$

So the full FOC becomes

$$\tfrac{1}{2}\lambda\, d^{-\gamma} \left[(1 - \Theta)\, f^{\varphi-}(\varphi^\star) - \Theta\, \mathcal{F}^{\varphi-}(\varphi^\star) \right] = \bar{r}\tau.$$

As in Proposition 1, this FOC can be solved jointly with the equation $d = \int_{\varphi^\star}^0 e^\varphi \, d\mathcal{F}^{\varphi-}$.

Besides altering the optimal leverage, the firm value equation must be solved with the extra term given above. In addition, the solution for the credit spread picks up a factor of $(1 - \Theta)$.   *QED*

Proposition 2 now simply finishes the characterizations of the quantities in Lemma 2 above. Now that d and $\varphi^\star$ have been determined explicitly, the coefficients in the differential equation for $j(\sigma)$ and the algebraic equation for $c(\sigma)$ are fully specified. The

proof just finishes the description of the pricing kernel.

Proof of Proposition 2

The Lemma determined that

$$\Lambda_t = e^{\int_0^t f_J(C_u, J_u) \ du} \ f_C(C_t, J_t),$$

and

$$f_C(C, J) = \beta \ c(\sigma)^{\rho-1} j(\sigma)^{1-\frac{1}{\theta}} \ Y^{-\gamma}.$$

Denote the product of c and j terms in this expression as $a(\sigma)$. Also, after some cancellations,

$$f_J(C, J) = \beta\theta \left[(1 - \frac{1}{\theta}) \ c(\sigma)^\rho j(\sigma)^{-\frac{1}{\theta}} - 1 \right].$$

The task is to evaluate $d\Lambda/\Lambda$. The integral term just contributes an f_J term to the drift. To this we add df_C/f_C, which is

$$\left[\frac{1}{2}\frac{a''}{a}s^2 + \frac{a'}{a}m + \mu_Y \right] \ dt + \frac{a'}{a}s \ dW + d \left[\sum_{j=1}^{J_t} \left((u^{-\gamma} - 1)1_{\{j,+\}} + (d^{-\gamma} - 1)1_{\{j,-\}} \right) \right].$$

The diffusion coefficient here is $sa'/a = s[(\rho - 1)c'/c + (1 - 1/\theta)j'/j]$, which is called η_1 in the Proposition. Likewise η_0 is the drift term plus f_J. The full expression for a''/a is omitted for brevity. The expression in the proposition for riskless rate is just minus the drift of $d\Lambda/\Lambda$. $\hspace{2cm}$ *QED*

Likewise, there is nothing formally to prove for Proposition 3, because the proof of Proposition 1 already deduced the ODE solved by $v(\sigma) = V^{(i)}/(1 - \tau)Y^{(i)}$. There it was only necessary to observe its form in order to take the first order condition for optimal debt. Now that the kernel and the debt policy have been explicitly obtained, the ODE is fully specified and (as observed above) a unique solution exists. We can redefine v to be that solution times $(1 - \tau)$ to obtain the solution in terms of pre-tax output $V^{(i)} = v(\sigma)Y^{(i)}$.

The following corollary computes the risk premia for the firm's claims.

Corollary A.1. *The expected excess return to the firm's assets is*

$$\pi_V = -\frac{v'}{v}s\eta_1 + \tfrac{1}{2}\lambda\left((u-1)+(d-1)+(u^{-\gamma}-1)+(d^{-\gamma}-1)-(u^{1-\gamma}-1)-(d^{1-\gamma}-1)\right).$$

The expected excess return to the firm's debt is

$$\pi_F = \tfrac{1}{2}\lambda\left(d(\sigma)^{-\gamma}-1\right)\left(1-\mathcal{F}^{-\varphi}(\varphi^\star)\right).$$

The expected excess return to the firm's equity, π_E is given by the solution to

$$\pi_V = \frac{1}{1-\ell}\pi_E + \frac{\ell}{1-\ell}\pi_F.$$

Proof of Corollary

The valuation ODE for V equates

$$\tfrac{1}{2}\frac{v''}{v}s^2 + \frac{v'}{v}m + \mu + \frac{1-\tau}{v} + \bar{r}\tau\frac{b}{v} + \eta_0$$

to

$$-\left(\eta_1 s\frac{v'}{v} + \tfrac{1}{2}\lambda((u^{1-\gamma}-1)+(d^{1-\gamma}-1))\right).$$

If we add to each side $\tfrac{1}{2}\lambda((u^{-\gamma}-1)+(d^{-\gamma}-1))$ we can then substitute out the sum of these terms and η_0 for $-r$ in the top expression. Then add to each side $\tfrac{1}{2}\lambda((u-1)+(d-1))$ and the top expression becomes the (true) expected excess returns to dV/V. We conclude

$$\pi_V = -\frac{v'}{v}s\eta_1 + \tfrac{1}{2}\lambda\left((u-1)+(d-1)+(u^{-\gamma}-1)+(d^{-\gamma}-1)-(u^{1-\gamma}-1)-(d^{1-\gamma}-1)\right).$$

which we can also write as

$$\pi_V = -\frac{v'}{v}s\eta_1 + \tfrac{1}{2}\lambda\left[(u-1)(u^{-\gamma}-1)+(d-1)(d^{-\gamma}-1)\right].$$

The debt contract has no expected change per unit time, outside of default. So its expected excess return is the coupon rate minus the riskless rate plus the instantaneous default intensity. But this is just the difference between the credit spread, determined above, (which is also the risk neutral default intensity) and the true default intensity, giving the expression in the corollary. Finally, by no arbitrage, the risk premium on the firm is the value weighted combination of debt and equity claims. This is the last assertion in the corollary.

<div align="right">*QED*</div>

B. Data and Estimation

This appendix describes the data and estimation procedures used in Section 3.

Aggregate Moments

The model has several simplifying assumptions about firms and the economy that make choice of empirical counterparts somewhat subjective. The list below discusses the proxies chosen and some possible alternatives.

Leverage:

The quantity b in the model is firm's debt face value divided by (pre-tax) cash-flow, or output. In the data, I thus need to choose pairs of (debt,output) measures that correspond to the same set of firms. The firm is supposed to be representative of the entire economy. The broadest measure, and the main proxy used, is from the Federal Reserves Z1 reports (The flow-of-funds accounts) for the U.S. nonfinancial corporate sector. Specifically debt is bank loans and bonds (long and short term), and output is net operating surplus plus consumption of fixed capital.[37] In one test, debt is scaled instead by firm assets (historical cost).[38]

The main tests in the paper use net debt, subtracting cash and cash equivalents.[39] To check robustness to the inclusion of off-balance sheet liabilities in total debt, another version adds retirement entitlements (pensions and healthcare liabilities, FL103152025.Q) to the numerator.

For further robustness, I also consider a broader measure of the private sector. Debt, also from the flow of funds accounts, includes the noncorporate sector, meaning

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[37]The respective Z1 data items are FL104122005.Q, FL104123005.Q and FU106402101.Q, FU106300005.Q.

[38]Series FL102000115.Q.

[39]Specifically, the definition follows the construction of Table L.103 in the Flow of Funds reports. Cash is checkable, time, and foreign deposits, holding of money market mutual funds, and Treasury securities and other bonds. These are Z1 series FL103020005.Q, FL103030003.Q, FL103034003.Q, FL103091003.Q, FL102051003.Q and FL103061103.Q.

primarily private firms, partnerships, and proprietorships. The debt measure is constructed from the same variables as for the corporate series (FL104104005.Q plus FL114123005.Q). The corresponding output measure is now taken to be the nonfarm business GDP number from NIPA Table 1.3.5. Fixed assets of nonfinancial corporate and noncorporate sectors are from NIPA Table 6.3.

To address concerns that aggregate data is dominated by large firms, a final alternative leverage measure is constructed from median firm values in quarterly Compustat data. Specifically, for all nonfinancial firms with a reporting quarter ending within each calendar quarter, I compute net debt as long term debt total plus debt in current liabilities minus cash and short term investment. Cash-flow is operating profits before interest, depreciation, and taxes. I then take the median value across firms of the ratio of net debt to cash-flow. Firms for which the denominator is non-positive are excluded, as are firms missing any of the numerator items. The quarterly Compustat series is available from 1976:Q1.

Credit spread:

In choosing a credit spread series, the main consideration is, again, that the model speaks to a firm representative of the entire corporate sector. The question then is how to define the average creditworthiness of firms overall.

Based on the average default rate of the entire private sector (see below) and long-term default frequencies from S&P by rating, the representative firm in the economy appears to be approximately of credit grade BB or BBB. The natural candidate to measure credit in this range is the time-series of seasoned Baa-rated yields-to-maturity from Moody's that goes back to 1916. (A Moody's Baa rating corresponds to an S&P BBB rating.) According to Moody's, this series is for debt with maturity of at least 20 years. So I subtract the constant maturity 20-year Treasury yield from the Federal Reserve Bank of St. Louis (FRED), interpolating between 10 and 30 year yields when the 20 year series in unavailable.[40] I measure

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[40] Choosing a series that fixes the rating level over time does impose a measurement bias because

both yields at the end of calendar quarters. The Treasury yields are available from 1953:Q1.

For comparison with other works in the asset pricing literature, I also consider the credit spread defined as the difference between Baa yields and Aaa yields. Some authors have viewed Treasury bonds as an inappropriate benchmark because of potential liquidity premia or tax effects embedded in their prices.

To address the concern that the firm's borrowing cost in the model is for floating-rate debt and should therefor correspond to a short-maturity interest rate, I also construct a credit spread based on commercial paper yields. The main drawback with this series is that commercial paper is only issued by large high-quality borrowers, making its spread unrepresentative. Commercial paper rates are obtained from FRED. I concatenate separate pre- and post-1998 series for 90-day maturity. (The post-1998 series is for A2/P2 rated nonfinancial issuers. The earlier series does not specify the issuer type.) The CP-TB spread is defined as the difference between this rate and the current 3-month Treasury bill rate, also from FRED.

Default rate:

To assess representative borrower quality in the U.S. corporate sector, I obtain a time-series of annual total bankruptcy filings by U.S. firms for 1981-2015 from the American Bankruptcy Institute. [41] I divide total bankruptcies by the total number of firms in the U.S. from the Statistics of U.S. Businesses [42] compiled from the U.S. Census Bureau's Survey of Business Ownership. The latter series are available from 1988-2012. I average the annual ratio of the two numbers to obtain the unconditional default frequency 0.0087 used in the estimation.

For comparison to rated bond issuers, average global default rates by rating and issuer type are available for 1981-2014 are obtained from S&P's Global Corporate

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it misses fluctuations in the population credit quality. Intuitively, the effect of this bias should be straightforward: it should mean that fluctuations in credit spreads are understated.

[41] http://www.abi.org/newsroom/bankruptcy-statistics

[42] https://www.sba.gov/advocacy/firm-size-data#susb

Default Study (Vazza and Kraemer (2015)). For such issuers, the average one-year default rate for nonfinancial firms worldwide over this period is 0.0181 (Table 16).

Investment rate:

In the model, investment is made directly by households through their savings decisions. Therefore I measure average investment as the personal savings rate (savings as a fraction of disposable household income) from NIPA Table 2.1. The value used in the estimation is the average of annual rates from 1980-2015.

Equity valuation:

The model's equity valuation as a fraction of output is again supposed to be representative of the entire economy. The flow of funds tables include market valuation of equity (less intercompany holdings) for the nonfinancial corporate public sector. This is value is divided by the cashflow series constructed as described above. The value used in the estimation is the average of quarterly ratios from 1980:Q1-2015:Q1.

Estimation

The two models in Section 2 are estimated by minimum-square-error criterion applied to the moments (or statistics) listed in the text's Table 1, with one exception. Instead of targeting the default rate itself, the estimation targets the ratio of the credit spread to the default rate in order to better identify the credit risk premium. In addition, because the trade-off model assumes zero recovery on debt, the estimation for that specification deflates the model's value by an average loss factor (one minus recovery rate) of 0.6 for comparison with the empirical counterpart.

Model moments at each point in the parameter space are computed by sampling from the stationary distribution of the σ process. Moment errors are scaled by the estimation error in each target statistic, and squared. Because the statistics are computed from

distinct samples over different dates and frequencies, I do not attempt to estimate cross-moment errors. Hence the scheme corresponds to a diagonal weighting matrix.[43]

The estimation fixes six of the parameters to be the same in both models. The jump intensity is held constant at $\lambda = 1$ for ease of interpretation, e.g., so that jump magnitudes can be viewed as expected annual rates. The jump shape parameter L is fixed at 4.0 and the scale of the production function ζ_1 is fixed at 0.975. These parameters are poorly identified by the data moments. For comparability across specifications, the upper and lower bounds of the state variable are held fixed at $\sigma_l = 0.05, \sigma_u = 0.60$. Finally, the tax-rate is fixed at 0.30, as it is not really a free parameter.

The resulting estimates for the remaining parameters are given in Table A1 for each specification.

Table A1: Parameter Estimates

Parameter		Model:T-O	Model:D-I
Risk aversion	γ	7.7074	8.4004
E.I.S.	ψ	0.3927	3.8295
Subjective discount rate	β	0.0595	0.1292
Interest deduction rate	$\bar{r}$	0.1246	0.0130
Debt recovery/insurance rate	θ	0.4000	0.4013
Production function curvature	ζ_0	0.1465	0.0808
Output growth constant	μ	0.0577	0.0814
Uncertainty mean	$\bar{\sigma}$	0.0925	0.0988
Uncertainty mean-reversion	κ	0.2271	0.1808
Uncertainty diffusion	s_0	0.2599	0.1805

Description: The table gives the point estimates of the parameters for the two versions of the model fitted by the method of simulated moments.

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[43]The delta method is used to approximate the sampling error of the credit spread-default rate ratio.

Panel Regressions

The panel regressions shown in Table 3 follow closely Covas and den Haan (2011) in the sample construction, definition of the variables, and controls used. Details on these may be found in the data appendix for that paper, available from `wouterdenhaan.com`. For reasons described there, the sample starts in 1980 and excludes financial firms, utilities, firms involved in major mergers, as well as Ford, Chrysler, GM, and GE. The sample runs through 2011, which is the extent of the uncertainty series from Jurado, Ludvigson, and Ng (2015). For parsimony, Covas and den Haan (2011) include lagged cash-flow and Tobin's Q as the sole controls. We do likewise.

Firm-quarter observations are required to have non-missing, strictly positive assets, sales, and shareholder equity. The debt numerator is long-term debt plus debt in current liabilities minus cash and short-term investments. This definition follows Strebulaev and Yang (2013). The three leverage measures – but not their changes – are winsorized at the 1% and 99% levels.

The definition of net external equity in Panel B of the table again follows Covas and den Haan (2011) and Fama and French (2005) in using the change in shareholder equity net of retained earnings. The Compustat data item SEQQ already nets out of balance sheet equity the cumulative total of extraordinary items as well as treasury stock repurchased. However it is still inclusive of retained earnings. So the the item REQ is subtracted out. Changes in the resulting quantity, known as paid-in capital, arise from stock issuance and repurchases. Dividend payments are not captured.

C. Robustness

This appendix provides additional evidence for the empirical relations documented in Section 3.

First, the empirical rejection of the trade-off model was primarily attributable to the positive correlation between credit spreads and leverage. That correlation was illustrated

visually in Figure 4 using the main measures of each quantity as described in Appendix B. Table A2 shows the correlations for pairwise combinations of the additional measures described there.

Next, Table A3 presents extended results on the empirical relation between aggregate debt and uncertainty that employ alternative uncertainty proxies. The table shows results for 12 regressions: using three proxies, two specifications, and two aggregate debt and output series. All 12 support the conclusion of a statistically and economically significant positive association.

The regressions in Section 3 utilize the JLN uncertainty series. Here the table also uses the well-known CBOE VIX index[44], the dispersion in economists' forecast of GDP, as tabulated from the Survey of Professional Forecasters (SPF)[45], and a third series (RED) that measures firm-level, rather than aggregate uncertainty, from Johnson and Lee (2014). It is constructed from the cross-firm dispersion of residual operating earnings, after orthogonalizing with respect to aggregate output changes. Note that each of the series is constructed from entirely distinct underlying data.

The table shows regressions of year-on-year changes in log debt on year-on-year log changes in the uncertainty series. In the top panel, debt is measured net of cash items using the Flow of Funds (Z.1) accounts for the nonfinancial corporate sector. The specification in the left panel scales debt by corporate cashflow. In the right panel debt changes themselves are the dependent variable, with contemporaneous cashflow changes included as a control variable. The tests are repeated in the bottom panel using a broader measure of debt that includes the noncorporate private sector, and scaling by Gross Domestic Product of nonfarm U.S. businesses, from NIPA Table 1.3.5. Standard errors correct for the serial correlation in overlapping residuals.

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[44]The VIX series is extend backward from 1990 to 1984 using implied volatility data on individual SPX options.

[45]The SPF series averages the dispersion in current-quarter forecasts of real and nominal GDP. Using other forecast horizons and extracting a principal component from the dispersion series produce similar results.

Table A2: Leverage Credit Spread Correlations

	Baa-20yr	Baa-Aaa	CP-TB
Nonfinancial corporate debt/cashflow	0.5229	0.2595	0.0278
minus cash	0.5641	0.3191	0.0603
plus retirement liabilities	0.4997	0.2568	0.0587
	1969:Q1-2015:Q1	1969:Q1-2015:Q1	1980:Q1-2015:Q1
Nonfinancial corporate+noncorporate net debt/ non-farm business GDP	0.3237	0.1025	0.2120
	1969:Q1-2015:Q1	1969:Q1-2015:Q1	1980:Q1-2015:Q1
Median Compustat nonfinancial net debt/cashflow	0.1187	0.1383	0.1248
	1976:Q1-2015:Q2	1976:Q1-2015:Q2	1980:Q1-2015:Q1

Description: The table shows the time-series correlation between credit spreads and leverage using three credit spread series and six leverage definitions.

Interpretation: The positive correlation is robust to the choice of measurement series.

Table A3: Aggregate Debt Dynamics

Panel A: Nonfinancial Corporate Debt ; Cash-flow

Dependent var	$(b_t - y_t) - (b_{t-4} - y_{t-4})$			$(b_t - b_{t-4})$		
Sample	1984Q1-2018:Q1	1969Q4-2014:Q4	1964Q2-2017:Q4	1984Q1-2018:Q1	1969Q4-2014:Q4	1964Q2-2017:Q4
Δ VIX	0.0827 (3.28)			0.0601 (3.01)		
Δ SPF		0.0762 (3.34)			0.0358 (2.13)	
Δ RED			0.2070 (3.79)			0.1297 (2.48)
$b_{t-4} - y_{t-4}$	-0.2341 (3.27)	-0.1785 (2.70)	-0.1286 (3.31)			
$y_t - y_{t-4}$				0.0161 (0.12)	-0.0351 (0.24)	0.0132 (0.10)
N	133	181	215	133	181	215

Panel B: Nonfin. Corp. & Noncorp. Debt; Bus. GDP

Dependent var	$(b_t - y_t) - (b_{t-4} - y_{t-4})$			$(b_t - b_{t-4})$		
Sample	1984Q1-2018:Q1	1969Q4-2014:Q4	1964Q2-2017:Q4	1984Q1-2018:Q1	1969Q4-2014:Q4	1964Q2-2017:Q4
Δ VIX	0.0591 (2.83)			0.0625 (3.37)		
Δ SPF		0.0454 (3.01)			0.0429 (2.86)	
Δ RED			0.1174 (2.92)			0.1225 (2.92)
$b_{t-4} - y_{t-4}$	-0.1521 (1.85)	-0.1364 (2.69)	-0.0970 (3.99)			
$y_t - y_{t-4}$				0.8120 (4.33)	0.7247 (4.10)	0.7446 (4.69)
N	133	181	215	133	181	215

Caption to Table A3 Description: The table reports time-series regressions of quarterly aggregate debt on measures of economic uncertainty, in changes. If the first panel, b is total debt securities and loans, minus cash items (including checkable, time and savings deposits, money-market funds, and foreign deposit) for the U.S. nonfinancial corporate sector and y is the total cashflow of this sector measured as net operating surplus plus consumption of fixed capital. All data are from the Federal Reserve's Flow of Funds accounts. In the second panel, b is the combined total debt (net of cash items) for corporate and noncorporate nonfinancial businesses and y is gross domestic product of nonfarm U.S. businesses from NIPA Table 1.3.5. Debt and output variables are in logarithms. VIX, SPF, and RED are respectively the CBOE VIX index (extend backward from 1990 by the author), the dispersion in current-quarter forecasts of real and nominal GDP as tabulated by the Survey of Professional Forecasters (SPF), and the residual earnings dispersion measure of Johnson and Lee (2014). The uncertainty series are year-on-year log differences contemporaneous with the dependent variable. Numbers in parentheses are Newey and West (1987) T-statistics (in absolute value) using 8 lags.
Interpretation: The positive leverage-uncertainty relation is robust to the choice of uncertainty proxy and the choice of aggregate debt. The same results hold using debt changes as the dependent variable.

Next, Table A4 reverts to the JLN uncertainty measure and shows additional regressions in which changes in (log) debt are the dependent variable.[46] Panel A shows the positive relation persists in these specifications, and is robust to controlling for additional predictors.

Because uncertainty changes are negatively autocorrelated, a natural question is whether the positive relationship in changes is actually proxying for a *negative* relationship between debt *changes* and uncertainty *levels*.[47] Panel B allows the data to consider both possibilities. The results unambiguously support a positive relation in changes, with no statistical support for a negative relation between debt changes and uncertainty levels.

The specifications so far do not distinguish between expected and unexpected changes in uncertainty. This is appropriate in the sense that the models in Section 2 imply that debt levels respond to levels of uncertainty, whether or not they were expected. Moreover,

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[46]The table utilizes the nonfinancial corporate series. Results using the broader private sector series are similar and are omitted for brevity. The variables and sample are defined in the caption to Table 2.

[47]The two relations are not econometrically inconsistent. In the context of the models studied here, however, a levels-on-changes regression is a misspecification. See also the discussion in footnote 26.

in the presence of real-world planning delays, it seems likely that debt issuance would be driven mostly by expected changes. Panel C of the Table quantifies both responses by splitting uncertainty changes into two components via an auxiliary regression of these changes on a set of lagged predictors (listed in the column labeled Projection). When both the fitted and the residual components of this projection are included in the debt regression, each is seen to have a statistically and economically significant positive impact. Summing the two coefficients, the total impact is substantially larger than in the baseline specifications in Panel A.

Table A4: Uncertainty and Debt Dynamics

	PANEL A: Baseline	
$\Delta v_{(t+4:t)}$		Controls:
0.3028 (3.69)		$\Delta y_{(t+4:t)}$
0.1989 (3.16)		$\Delta y_{(t+4:t)}, \Delta y_{(t:t-4)}, \Delta b_{(t:t-4)}$
0.1420 (2.56)		*all*

	PANEL B: Uncertainty changes vs levels	
$\Delta v_{(t+4:t)}$	v_t	Controls:
0.2777 (3.79)	-0.0486 (0.57)	$\Delta y_{(t+4:t)}$
0.1457 (2.34)	0.0155 (0.23)	*all*

PANEL C: Uncertainty innovations vs expected changes

$U\ \Delta v_{(t+4:t)}$	$E\ \Delta v_{(t+4:t)}$	Controls:	Projection:
0.2450 (3.71)	0.4691 (2.28)	$\Delta y_{(t+4:t)}$	$v_t, \Delta v_{(t:t-4)}, R_t^{mkt}, |R_t^{mkt}|$
0.1247 (2.15)	0.2197 (2.81)	*all*	$v_t, \Delta v_{(t:t-4)}, R_t^{mkt}, |R_t^{mkt}|$

Description: The table shows regressions of aggregate changes in log debt, $\Delta b_{(t+4:t)}$, on uncertainty changes. The variables are described in the caption to Table 2. In the column labeled Controls, *all* refers to the variables $\Delta y_{t+4:t}, \Delta y_{t:t-4}, \Delta b_{t:t-4}, R_t^{mkt}, SP\_PE_t, Y_t^{10yr}, CR\_SP_t, \Delta CPI_{t:t-4}$. In Panel C, $U\ \Delta v_{(t+4:t)}$ and $E\ \Delta v_{(t+4:t)}$ denote the residual and fitted values, respectively, from a first-stage regression of uncertainty changes on the variables listed in the Projection column. Numbers in parentheses are Newey and West (1987) T-statistics (absolute value) using 8 quarterly lags.

Interpretation: The positive debt-uncertainty relation is using the JLN measure is explored further. The positive relation with uncertainty changes is not masking a negative relation with uncertainty levels. The relation is found in both expected and unexpected components of uncertainty changes.

Another way of quantifying the response of debt to orthogonalized uncertainty shocks is via impulse response functions in vector autoregressions. Figure 9 shows two response

functions, each computed in non-overlapping quarterly specifications, using four lags. The left-hand panel is a bivariate specification in levels where debt is scaled by output (both in logs) to achieve stationarity. (This specification also includes linear trends.) The right-hand panel is from a trivariate system – uncertainty, output, and debt – in changes. In each system, the variables are ordered with the debt series last and the shocks are orthogonalized via Cholesky decomposition.[48]

Consistent with the change regressions above, the right panel affirms a significant positive debt change response at one and two quarters. Uncertainty shocks predict changes in debt. These responses then mean-revert as uncertainty itself mean-reverts. The left panel shows that the cumulative positive effect on leverage levels (which includes the negative response of the denominator to an uncertainty shock) remains positive for around two years.

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[48]The data series are the same ones used in Table A4.

Figure 9: Impulse response functions
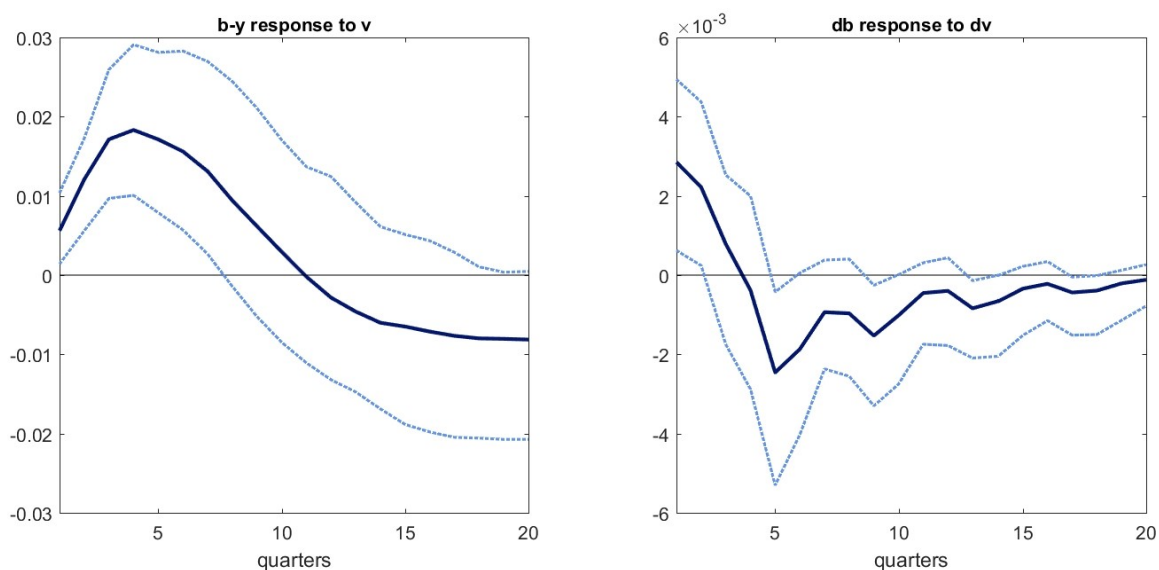
Description: The figure shows impulse responses to a one standard deviation shock to uncertainty in quarterly vector autoregressions. The left panel uses a bivariate specification of logs of debt-to-income $(b - y)$ and uncertainty (v) and includes linear trends. The right panel uses a trivariate system of log changes in uncertainty, output, and debt. Both specifications include four lags. The horizontal axis is in quarters. The dashed lines are bootstrapped 95% confidence intervals.

Interpretation: Uncertainty shocks produce statistically significant positive impulse responses of leverage levels and debt changes in quarterly vector autoregressions.