Internet Appendix

The Value of a Cure: An Asset Pricing Perspective*

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September 2021

Appendix A includes news articles from the Introduction and Section 3 of the paper.

Appendix B includes additional details on the vaccine progress indicator as described in Section 3 of the paper.

Appendix C contains proofs to Section 4 of the paper.

Appendix D derives the solution to the regime-switching model with just two states. The solution technique is then applied to solving the regime-switching model with S states in Section 4 of the paper.

Appendix E derives Proposition 4 in Section 5 of the paper.

Appendix F derives (39)-(40) in Section 5 of the paper.

*First draft: November 15, 2020.

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References

Mariano M Croce, Thien T Nguyen, and Lukas Schmid. The market price of fiscal uncertainty. *Journal of Monetary Economics*, 59(5):401–416, 2012.

Chi Heem Wong, Kien Wei Siah, and Andrew W Lo. Estimation of clinical trial success rates and related parameters. *Biostatistics*, 20(2):273–286, 01 2018.

A News Articles

This section includes news articles from the Introduction and Section 3 of the paper.

A.1 News Articles from the Introduction

On May 18, 2020 *Moderna* released positive interim clinical data from their Phase I trials and announced a Phase III trial.

> Federal Reserve chair Jay Powell has warned that a full US economic recovery may take until the end of next year and require the development of a COVID-19 vaccine: "For the economy to fully recover, people will have to be fully confident. And that may have to await the arrival of a vaccine", Mr. Powell told CBS News on Sunday.
>
> Lauren Fedor and James Politi, Financial Times, May 18, 2020

> U.S. stocks gained about $1 trillion of market capitalization yesterday, and while there are lots of reasons why any particular stock may have gone up or down, good news about a vaccine that might allow reopening of the economy seems like a common factor for a lot of stocks.
>
> "U.S. Stocks Surge as Hopes for Coronavirus Vaccine Build," was the Wall Street Journal's headline, citing the Moderna results... It is almost fair to say that Moderna added $1 trillion of value to all the other stocks yesterday.
>
> Matt Levine, Money Stuff, May 19, 2020

On July 14, 2020 *Moderna* publishes positive Phase I data in the New England Journal of Medicine, highlighted by its vaccine candidate producing antibodies in all patients.

> The most interesting correlation in the stock market right now is the one between (1) the prices of airline stocks and (2) the amount of antibodies produced by coronavirus vaccine candidates in clinical trials. So far the vaccines are experimental and uncertain. If you knew that they'd work really well—protect everyone perfectly, no side effects, easy to produce, etc.—then you'd know

> with a pretty high degree of certainty that airline stocks (and cruise ships, hotels, casinos, retailers, etc.) would go up. If you knew that they'd be a disaster then you'd probably be short airlines.
>
> So on Tuesday Moderna announced good news, and yesterday:... Royal Caribbean Cruises Ltd. was up 21.2%. Norwegian Cruise Line Holdings Ltd. was up 20.7%. Carnival Corp. was up 16.2%. American Airlines Group Inc. was also up 16.2%. United Airlines Holdings was up 14.6%. The biggest gainers were the vaccine sensitive industries, not Moderna itself.
>
> Matt Levine, Money Stuff, July 16, 2020

On November 9, 2020 *Pfizer* and *BioNTech* announced positive news regarding interim analysis from their Phase III Study.

> Markets received a shot in the arm Monday from Pfizer Inc. and its encouraging Stage III tests on a COVID-19 vaccine. As a result, the S&P 500, the MSCI World and the MSCI All-World indexes all rose to records. But that misses the point of the impact. The news triggered the biggest single-day market rotation I've witnessed in the 30 years since I started covering markets...
>
> In technical terms, the clearest expression of the violence of the turnaround comes from tracking the performance of stocks that have had the greatest positive momentum, relative to the market. Bloomberg's measure of the pure momentum factor in the U.S. stock market shows that momentum dropped 4% Monday. Since Bloomberg started tracking daily moves in 2008, it had never before fallen as much as 2%.
>
> John Authers, Bloomberg Opinion, November 10, 2020

> Monday's news that a COVID-19 vaccine being developed by Pfizer and Germany's BioNTech was more than 90 per cent effective sent markets soaring. But it also prompted an abrupt switch out of sectors that have prospered during the pandemic, such as technology, and into beaten-down stocks such as real estate and airlines — and triggered an earthquake in some popular investment "factors" such as value and momentum...

The value factor, which is centered on lowly-priced, unfashionable stocks, enjoyed a 6.4 per cent uplift, its strongest one-day gain since the 1980s, while the momentum factor — essentially stocks on a hot streak — tumbled 13.7 per cent, its worst ever loss, according to JPMorgan.

Laurence Fletcher and Robin Wigglesworth, Financial Times, November 14, 2020

A.2 News Articles from Section 3

Our duration estimates are based on projections from the pharmaceutical and financial press during 2020. For example, see (1) Damian Garde, STAT News, January 24, 2020, (2) Chelsea Weidman Burke, BioSpace, February 17, 2020, (3) Hannah Kuchler, Clive Cookson and Sarah Neville, Financial Times, March 5, 2020, (4) Bill Bostock, Business Insider, April 1, 2020, (5) Derek Lowe, Science Translational Medicine, April 15, 2020, (6) The Economist, April 16, 2020, (7) Nicoletta Lanese, Live Science, April 16, 2020, and (8) James Paton, Bloomberg, April 27, 2020.

B Vaccine Progress Indicator

This section includes additional details on the vaccine progress indicator as described in Section 3 of the paper.

The simulation takes as input a timeline of COVID-19 vaccine candidates' stage-by-stage progress from the London School of Hygiene & Tropical Medicine.[1] We observe the start dates of each pre-clinical and clinical trial, along with their vaccine strategy. Table A.1 breaks down the number of candidates at each state at the end of our sample. We also observe each candidate's strategy. Table A.2 summaries the main strategies along with the number of candidates following each.

We then augment π_s^{base} with 233 news articles from FactSet StreetAccount, split into positive and negative news types. Table A.3 shows the number of articles by news type, while Table A.4 shows the top ten candidates by news count.

[1] This version of the paper uses the timeline available on November 2, 2020.

B.1 Data and Parameters

The simulation takes as input a timeline of COVID-19 vaccine candidates' stage-by-stage progress from the London School of Hygiene & Tropical Medicine.[2] We observe the start dates of each pre-clinical and clinical trial, along with their vaccine strategy. Vaccines typically take years to develop, and institutes have combined phases in an effort to accelerate the timeline. Following Wong et al. (2018), we adopt each candidate's most advanced state. We also observe each candidate's strategy.

Since candidates share a common virus target, and potentially common institutes or strategies, we define pairwise correlations in an additive manner. For two candidates $n \neq n'$:

$$\rho(n, n') = \begin{cases} 0.2 & \text{baseline} \\ \text{add } 0.2 & \text{if shared institute} \\ \text{add } 0.1 & \text{if shared strategy.} \end{cases}$$

Table A.5 lists our parameter choices of durations and baseline probabilities of success.[3] Table A.6 summarizes the distribution of time spent in each state in our simulation. Following Wong et al. (2018), we adopt each candidate's most advanced state. We track days spent in each state until the next state starts, only among candidates that have successfully transitioned to the next state. The realized outcomes for durations are reasonably consistent with our choices of parameters, in particular for Phase I and Phase II. And the standard deviations of durations are less than the mean is consistent with the Gaussian copula assumption of positively correlated outcomes.

We then augment π_s^{base} with 233 news articles from FactSet StreetAccount, split into positive and negative news types. Table A.7 lists the news types along with their changes in probabilities.

C Proofs to Section 4

This section contains the proofs to the propositions in Section 4 of the paper.

[2]This version of the paper uses the timeline available on November 2, 2020.

[3]Our success probabilities are taken from pharmaceutical research firm BioMedTracker and are based upon historical outcomes of infectious disease drug trials. Our duration estimates are based on projections from the pharmaceutical and financial press during 2020.

C.1 Proof of Proposition 1

Proposition 1. *Denote*

$$g(s) \equiv \frac{(1-\gamma)\rho}{(1-\psi^{-1})} - (1-\gamma)\left(\mu(s) - \frac{1}{2}\gamma\sigma(s)^2\right) - \left([1-\chi(s)]^{1-\gamma} - 1\right) \tag{A.1}$$

Let $H(s)$'s denote the solution to the following system of S recursive equations:

$$g_0 \equiv g(0) = \frac{(1-\gamma)}{(\psi-1)}\rho^{\psi}\left(H(0)\right)^{-\psi\theta^{-1}} + \eta\left[\frac{H(1)}{H(0)} - 1\right] \tag{A.2}$$

$$g_1 \equiv g(1) = \frac{(1-\gamma)}{(\psi-1)}\rho^{\psi}\left(H(s)\right)^{-\psi\theta^{-1}} + \lambda_d\left[\frac{H(s-1)}{H(s)} - 1\right] + \lambda_u\left[\frac{H(s+1)}{H(s)} - 1\right], \tag{A.3}$$

for $s \in \{1,\ldots,S-1\}$.

Assuming the solutions are positive, optimal consumption in state s is

$$C(s) = \frac{\left(H(s)\right)^{-\psi\theta^{-1}} q}{\rho^{-\psi}}, \tag{A.4}$$

and the value function of the representative agent is

$$\mathbb{J}(s) \equiv \frac{H(s)q^{1-\gamma}}{1-\gamma}. \tag{A.5}$$

Proof. From the evolution of capital stock for the representative agent (25), we obtain the Hamilton-Jacobi-Bellman (HJB) equation as follows for each state s:

$$\begin{aligned} 0 = \max_C \Big[& f(C,\mathbb{J}(s)) - \rho\mathbb{J}(s) + \mathbb{J}_q(s)(q\mu(s) - C) \\ & + \frac{1}{2}\mathbb{J}_{qq}(s)q^2\sigma(s)^2 + \zeta(s)\left[\mathbb{J}(s)\left(q(1-\chi(s))\right) - \mathbb{J}(s)(q)\right] \\ & + \lambda_u(s)\left[\mathbb{J}(s+1)(q) - \mathbb{J}(s)(q)\right] + \lambda_d(s)\left[\mathbb{J}(s-1)(q) - \mathbb{J}(s)(q)\right] \Big] \end{aligned} \tag{A.6}$$

Taking the first-order condition with respect to $C(s)$ in HJB equation (A.6), we obtain

$$f_c(C,\mathbb{J}(s)) - \mathbb{J}_q(s) = 0. \tag{A.7}$$

Using $f(C,\mathbb{J})$ from (11) and taking the derivative with respect to C, we obtain

$$f_c = \frac{\rho C^{-\psi^{-1}}}{[(1-\gamma)\mathbb{J}(s)]^{\frac{1}{\theta}-1}}. \tag{A.8}$$

Substituting the conjecture $\mathbb{J}(s)$ in equation (A.5) yields

$$f_c = \frac{\rho C^{-\psi^{-1}}}{H(s)^{\frac{\gamma-\psi^{-1}}{1-\gamma}} q^{\gamma-\psi^{-1}}}. \tag{A.9}$$

Then, for state $s \in \{0, \ldots, S\}$, we obtain by substituting $\mathbb{J}_q(s) = H(s)q^{-\gamma}$ in (A.7), and simplifying:

$$C(s) = \frac{H(s)^{-\theta\psi^{-1}} q}{\rho^{-\psi}}. \tag{A.10}$$

To verify the conjectured form of the value function, we plug it in to the HJB equation (A.6) and reduce it to the recursive system in the proposition via the following steps:

1. substitute the optimal policy $C(s)$ into the HJB equation (A.6);
2. cancel the terms in q which have the same exponent; and
3. group terms not involving $H(s)$ constants into $g(0)$ for state $s = 0$ and $g(s)$ for state $s \in \{1, \ldots, S-1\}$

to reach equations (A.1) – (A.3). This system of recursive equations can be solved numerically with the final condition in Proposition 2: $H(s) = H(0)$, that states 0 and S are non-pandemic states. □

A detailed analysis of the system for the two-state case ($S = 2$) with endogenous labor is provided for illustration in the internet appendix where we refer to the non-pandemic state as state 0 and the pandemic state as state 1.

□

C.2 Proof of Proposition 2

Proposition 2. *The value of a cure in the pandemic state s is determined by the ratio of marginal propensity to consume ($c \equiv dC/dq$) in the pandemic state s relative to that in the non-pandemic state, adjusted by the agent's elasticity of intertemporal substitution (EIS):*

$$V(s) = 1 - \left(\frac{c(s)}{c(0)}\right)^{-\frac{1}{\psi-1}} = 1 - \left(\frac{C(s)}{C(0)}\right)^{-\frac{1}{\psi-1}} \tag{A.11}$$

Proof. The value of a cure (vaccine) $V(s)$ satisfies:

$$\mathbb{J}(0)(q) = \mathbb{J}(0)\left[(1 - V(s))\, q\right] \tag{A.12}$$

where $\mathbb{J}(0)$ is evaluated at $(1 - V(s))\, q$. Substituting for $\mathbb{J}(s)$ from (A.5), we obtain

$$\frac{H(0)q^{1-\gamma}}{(1-\gamma)} = \frac{H(0)\left[(1 - V(s))\, q\right]^{1-\gamma}}{(1-\gamma)} \tag{A.13}$$

which yields

$$V(s) = 1 - \left(\frac{H(s)}{H(0)}\right)^{\frac{1}{1-\gamma}}. \tag{A.14}$$

Then, substituting for $C(s)$ from (A.4) and recognizing marginal propensity to consume, $c(s)$, equals $\frac{dC}{dq} = \frac{C(s)}{q}$, yields Proposition 2. □

C.3 Proof of Proposition 3

This appendix first describes and discusses a decentralization of the economy under which a claim to the flow of output (not consumption) is the net profit of the corporate sector. It then provides the proof to Proposition 3.

The decentralization we have in mind is as follows.

1. Households own the capital stock and rent it to consumption goods-producing firms.

2. These firms produce output $\mu(s)\, q\, dt + \sigma(s)\, q\, dW_t$ per unit time.

3. This sector purchases insurance against pandemic shocks $-\chi q\, dJ$ from an insurance sector.

4. The market portfolio consists of a claim to the profits of both sectors plus the rental contract for the capital stock.

The assumption that firms rent productive capital from households is common in macroeconomic models. Households retain ownership of the capital stock and their savings in not retained by firms as investment. Any model in which household savings is not equal to the corporate capital stock[4] will likewise not have net corporate cash flow equal to aggregate consumption. Notice that in step 1, the rental is effectively a riskless bond in that the "face value" of q is insured. Thus in this economy households separate risky and safe claims. Both are in positive net supply.

In this setting, the risky component of cash flow can be negative. We assume the parameter values are such that the market portfolio has positive value in all states, so that limited liability obtains, and we verify this for each case in our numerical work. It is worth clarifying that it is not necessary for our results to assume that holders of the portfolio bear the losses of the pandemic shock as a negative dividend. Our results are mathematically the same using an alternative equity claim that instead pays the (risk-neutral) expected output rate per unit time in each state.

We now turn to the proposition.

Proposition 3. *The price of the output claim is $P = p(s)q$ where the constants $p(s)$ solve a matrix system $Y = Xp$ where X is an $S+1$-by-$S+1$ matrix and Y is an $S+1$ vector both of whose elements are given in the appendix.*

Proof. To begin, we derive the pricing kernel and the risk-free rate. Under stochastic differential utility, the kernel can be represented as

$$\Lambda_t = e^{\int_0^t f_{\mathbb{J}} du} f_C \tag{A.15}$$

where

$$f(C,J) = \rho \frac{C^{\varrho}}{\varrho} \left((1-\gamma)\mathbb{J}\right)^{1-\frac{1}{\theta}} - \rho\theta\mathbb{J} \tag{A.16}$$

[4]Examples include Croce et al. (2012) where the savings technology is government investment.

where $\varrho = 1 - \frac{1}{\psi}, \theta = \frac{1-\gamma}{\varrho}$. As shown in Section 4, the value function and the consumption flow rates are:

$$\mathbb{J} = q^{1-\gamma} H(s)/(1-\gamma) \quad \text{and} \quad C = \rho^{\psi} H(s)^{e}\, q(s) q \tag{A.17}$$

where $e = \frac{1-\psi}{1-\gamma}$. Together these imply

$$f_C = \rho C^{\varrho - 1} \left((1-\gamma)\mathbb{J}\right)^{1-\frac{1}{\theta}} \tag{A.18}$$

or

$$f_C = \rho \left(\rho^{\psi} H(s)^{e} q\right)^{\varrho - 1} \left((1-\gamma)\left(q^{1-\gamma} H(s)/(1-\gamma)\right)\right)^{1-\frac{1}{\theta}}. \tag{A.19}$$

Simplifying, we get:

$$f_C = \rho^{1+\psi(\varrho-1)} H(s)^{e(\varrho-1)+\frac{\theta-1}{\theta}} q^{(\varrho-1)+\frac{(1-\gamma)(\theta-1)}{\theta}}. \tag{A.20}$$

The exponent of ρ is: $1+\psi(\varrho-1) = 1+\psi(-\frac{1}{\psi}) = 0$. The exponent of q is: $(\varrho-1)+\frac{(1-\gamma)(\theta-1)}{\theta}$. Substitute $\theta = \frac{1-\gamma}{\varrho}$ to get: $(\varrho-1)+\varrho(\frac{1-\gamma}{\varrho}-1) = -\gamma$. The exponent of $H(s)$ is

$$e(\varrho-1)+\frac{\theta-1}{\theta} \Rightarrow \frac{1-\psi}{1-\gamma}\left(-\frac{1}{\psi}\right)+\frac{1-\gamma\psi}{\psi(1-\gamma)} = 1 \tag{A.21}$$

Hence, $f_C = H(s)q^{-\gamma}$. Next, to evaluate $f_{\mathbb{J}}$, note that

$$f_{\mathbb{J}} = \rho \frac{C^{\varrho}}{\varrho}\left(1-\frac{1}{\theta}\right)\left[(1-\gamma)\mathbb{J}\right]^{-\frac{1}{\theta}}(1-\gamma) - \rho\theta \tag{A.22}$$

Plugging in for C and $\mathbb{J}$ we get:

$$f_{\mathbb{J}} = \rho \frac{\left(\rho^{\psi} H(s)^{e} q\right)^{\varrho}}{\varrho}\left(1-\frac{1}{\theta}\right)\left[(1-\gamma)\left(q^{1-\gamma} H(s)/(1-\gamma)\right)\right]^{-\frac{1}{\theta}}(1-\gamma) - \rho\theta \tag{A.23}$$

or

$$f_{\mathbb{J}} = \rho \frac{\left(\rho^{\psi} H(s)^{e} q\right)^{\varrho}}{\varrho}\left(\frac{\theta-1}{\theta}\right)\left[\left(q^{1-\gamma} H(s)\right)\right]^{-\frac{1}{\theta}}(1-\gamma) - \rho\theta. \tag{A.24}$$

This can be expressed as:

$$f_{\mathbb{J}} = \frac{1}{\varrho}\rho^{1+\psi\varrho}H(s)^{e\varrho}q^{\varrho}\left(\frac{\theta-1}{\theta}\right)(1-\gamma)q^{\frac{\gamma-1}{\theta}}H(s)^{-\frac{1}{\theta}} - \rho\theta. \tag{A.25}$$

Collecting terms:

$$f_{\mathbb{J}} = \frac{1}{\varrho}\rho^{1+\psi\varrho}H(s)^{e\varrho-\frac{1}{\theta}}q^{\varrho+\frac{\gamma-1}{\theta}}\left(\frac{\theta-1}{\theta}\right)(1-\gamma) - \rho\theta. \tag{A.26}$$

Here the exponent of ρ is : $1+\psi\varrho = \psi$, and the exponent of $H(s)$ is: $e\varrho - \frac{1}{\theta} = e\varrho - \frac{\varrho}{1-\gamma} = e$, and the exponent of q is: $\varrho + \frac{\gamma-1}{\theta} = 0$. Hence,

$$f_{\mathbb{J}} = \frac{1}{\varrho}\rho^{\psi}H(s)^{e}\left(\frac{\theta-1}{\theta}\right)(1-\gamma) - \rho\theta = \rho^{\psi}H(s)^{e}(\theta-1) - \rho\theta = c(s)(\theta-1) - \rho\theta. \tag{A.27}$$

So, we conclude that

$$\Lambda_t = e^{\int_0^t f_{\mathbb{J}} du} f_C = q^{-\gamma}H(s)e^{\int_0^t [c(s)(\theta-1)-\rho\theta]du}. \tag{A.28}$$

The riskless interest rate, $r(s)$ is minus the expected change of $d\Lambda/\Lambda$ per unit time. Applying Itô's lemma to the above expression yields drift (or dt terms)

$$c\ (\theta-1) - \rho\theta - \gamma(\ell^{\alpha}\mu - c) + \gamma(\gamma+1)\ell^{\alpha}\sigma^2 \tag{A.29}$$

where $\ell(0) = \bar{\ell} = 1$ and $\ell(s) = \ell^{\star}$ for $s > 0$. Note that the term $(\ell^{\alpha}\mu - c)$ is the drift of dq/q. To these terms we add the expected change from the jumps in the state s for $s = 0$:

$$\eta\ \left(\frac{H(1)}{H(0)} - 1\right) \equiv \tilde{\eta} - \eta \tag{A.30}$$

which serves to define the risk-neutral jump intenstity $\tilde{\eta}$. For $s > 0$ the expected jumps include both up and down changes in s as well as jumps in $q^{-\gamma}$:

$$\lambda_u \left(\frac{H(s+1)}{H(s)} - 1\right) + \lambda_d \left(\frac{H(s-1)}{H(s)} - 1\right) + \zeta((1-\chi)^{-\gamma} - 1) \equiv (\tilde{\lambda_u} - \lambda_u) + (\tilde{\lambda_d} - \lambda_d) + (\tilde{\zeta} - \zeta) \tag{A.31}$$

where the risk neutral intensities are defined as for η. The full expression for $r(0)$ is then

$$-\left\{c(0)\ (\theta-1) - \rho\theta - \gamma(\mu - c(0)) + \gamma(\gamma+1)\sigma^2 + (\tilde{\eta} - \eta)\right\}. \tag{A.32}$$

For $s > 0$ we have $r(s)$ as

$$-\left\{c(s)(\theta-1) - \rho\theta - \gamma((\ell^\star)^\alpha\mu - c(s)) + \frac{1}{2}\gamma(\gamma+1)(\ell^\star)^\alpha\sigma^2 + (\tilde{\lambda_u} - \lambda_u) + (\tilde{\lambda_d} - \lambda_d) + (\tilde{\zeta} - \zeta))\right\}. \tag{A.33}$$

We return to these expressions after deriving the pricing equation for the output claim.

By the fundamental theorem of asset pricing, the instantaneous expected excess return to the claim $P(q,s)$ must equal minus covariance of the returns to P with the pricing kernel. Deriving these two quantities and setting them equal yields the pricing system, to which the proof will construct the solution.

The expected excess return to the claim $P(q,s)$ is the sum of its expected capital gain and its expected payout, minus rP. In the nonpandemic state, this is

$$\frac{1}{2}\sigma^2 q^2 P_{qq}(q,0) + (\mu - c(0))qP_q(q,0) + \eta(P(q,1) - P(q,0)) + \mu q - r(0)P(q,0) \tag{A.34}$$

whereas in the pandemic states it is

$$\begin{aligned} &\frac{1}{2}(\ell^\star)^\alpha\sigma^2 q^2 P_{qq}(q,s) + ((\ell^\star)^\alpha\mu - c(s))qP_q(q,s) \\ &+\lambda_u(P(q,s+1) - P(q,s)) + \lambda_d(P(q,s-1) - P(q,s)) + \zeta(P((1-\chi)q,s) - P(q,s)) \\ &+\mu(\ell^\star)^\alpha q - \zeta\chi q - r(s)P(q,s). \end{aligned} \tag{A.35}$$

Next, we need to derive the covariance of the returns to P with $d\Lambda/\Lambda$. As mentioned in the text, in addition to the usual contribution of covariance from the capital gains dP/P, the covariance also includes the contribution from the dividends themselves, which are risky in this model. There are also contributions from both Brownian comovement and co-jumps in q and s. The Brownian terms are

$$-\gamma(\ell^\star)^\alpha\sigma^2[qP(q,s)-q] \tag{A.36}$$

for $s>0$, or just $-\gamma\sigma^2[qP-q]$ for $s=0$. The co-jump terms for $s>0$ are

$$\begin{aligned}&\zeta[P((1-\chi)q,s)-P(q,s)-\chi q]\ [(1-\chi)^{-\gamma}-1]\\ &+\lambda_u[P(q,s+1)-P(q,s)]\left[\frac{H(s+1)}{H(s)}-1\right]+\lambda_d[P(q,s-1)-P(q,s)]\left[\frac{H(s-1)}{H(s)}-1\right]\end{aligned} \tag{A.37}$$

or

$$\begin{aligned}&[P((1-\chi)q,s)-P(q,s)-\chi q]\ [\tilde{\zeta}-\zeta]\\ &+[P(q,s+1)-P(q,s)][\tilde{\lambda_u}-\lambda_u]+[P(q,s-1)-P(q,s)][\tilde{\lambda_d}-\lambda_d].\end{aligned} \tag{A.38}$$

For $s=0$ the corresponding expression is just

$$[P(q,1)-P(q,0)][\tilde{\eta}-\eta]. \tag{A.39}$$

We now equate the expected excess return to minus the above covariance to obtain the difference/differential equation system that P must solve. Rather than repeating the general expressions, we instead conjecture that the the solutions are linear in q and deduce the resulting system. Under linearity $P_{qq}=0$ and $P_q=p$, a constant that depends on s.

Plugging in the conjectured form, and cancelling a q, in states $s>0$ the pricing equation says

$$((\ell^\star)^\alpha\mu-c(s))p(s)+\lambda_u(p(s+1)-p(s))+\lambda_d(p(s-1)-p(s)) \tag{A.40}$$

$$-\chi\zeta p(s)+\mu(\ell^\star)^\alpha-\zeta\chi-r(s)p(s)-\gamma(\ell^\star)^\alpha\sigma^2[p(s)+1] \tag{A.41}$$

$$
\begin{aligned}
&-\chi[p(s)+1]\,[\tilde{\zeta}-\zeta]+[p(s+1)-p(s)][\tilde{\lambda_u}-\lambda_u]+[p(s-1)-p(s)][\tilde{\lambda_d}-\lambda_d]\\
&=0.
\end{aligned} \tag{A.42}
$$

Leaving the constant terms on the left, the right side consists of

$$
p(s+1) \quad \text{terms:} \quad -\lambda_u-[\tilde{\lambda_u}-\lambda_u]=-\tilde{\lambda_u}, \tag{A.43}
$$

$$
p(s-1) \quad \text{terms:} \quad -\lambda_d-[\tilde{\lambda_d}-\lambda_d]=-\tilde{\lambda_d}, \tag{A.44}
$$

and $p(s)$ terms:

$$
-((\ell^\star)^\alpha\mu-c(s))+\lambda_u+\lambda_d+\chi\zeta+r(s)+\gamma(\ell^\star)^\alpha\sigma^2+\chi[\tilde{\zeta}-\zeta]+[\tilde{\lambda_u}-\lambda_u]+[\tilde{\lambda_d}-\lambda_d] \tag{A.45}
$$

or

$$
r(s)+c(s)-(\ell^\star)^\alpha(\mu-\gamma\sigma^2)+\tilde{\lambda_u}+\tilde{\lambda_d}+\chi\tilde{\zeta}. \tag{A.46}
$$

The remaining constants on the left are

$$
\mu(\ell^\star)^\alpha-\zeta\chi-\gamma(\ell^\star)^\alpha\sigma^2-\chi[\tilde{\zeta}-\zeta]. \tag{A.47}
$$

or

$$
(\ell^\star)^\alpha(\mu-\gamma\sigma^2)-\chi\tilde{\zeta}. \tag{A.48}
$$

The above equations define a linear system for $p(1)$ to $p(S-1)$. The pricing equation for $s=0$ says

$$
(\mu-c(s))p(0)+\eta(p(1)-p(0))+\mu-r(0)p(0)-\gamma\sigma^2[p(0)+1]+[p(1)-p(0)][\tilde{\eta}-\eta]=0, \tag{A.49}
$$

or

$$
\mu-\gamma\sigma^2=p(0)[r(0)+c(0)-(\mu-\gamma\sigma^2)+\tilde{\eta}]-p(1)\,\tilde{\eta}. \tag{A.50}
$$

This equation closes the system on the low end. At the high end, the system is closed via $p(S)=p(0)$.

Altogether the system may be written in matrix form,

$$\begin{bmatrix} r(0) + c(0) - (\mu - \gamma\sigma^2) + \tilde{\eta} & -\tilde{\eta} & 0 & \cdots \\ -\tilde{\lambda_d} & r(s) + c(s) - (\ell^*)^\alpha(\mu - \gamma\sigma^2) + \chi\tilde{\zeta} + \tilde{\lambda_d} + \tilde{\lambda_u} & -\tilde{\lambda_u} & 0 \\ 0 & \ddots & \ddots & \ddots \\ \vdots & \ddots & \ddots & \ddots \\ -\tilde{\lambda_u} & 0 & \cdots & \end{bmatrix} p = \begin{bmatrix} (\mu - \gamma\sigma^2) \\ (\ell^\star)^\alpha(\mu - \gamma\sigma^2) - \chi\tilde{\zeta} \\ \vdots \\ \vdots \\ \vdots \end{bmatrix}.$$

Assuming the parameters are such that the right-hand matrix is of full rank, the system has a unique, finite solution. Since the output flow being priced is not guaranteed to be positive, it need not be the case that the price of the claim is positive either.

Finally, the proposition also identifies a minimal set of parameters that characterize the system. It has been shown that the value function solution functions $H(s)$ and consumption propensities $c(s)$ depend only on the pandemic parameters $\alpha, k, K, \epsilon, \zeta, , \chi$, and $\ell^\star$ (the latter two of which are endogenous) via the important variable we have called g_1. The pricing system explicitly references $\alpha, \zeta, , \chi$, and $\ell^\star$. We now show that the equations can be written in terms of g_1 and one additional combination of these variables.

In fact, the second combination of parameters is the constant term on the right hand side, $(\ell^\star)^\alpha(\mu - \gamma\sigma^2) - \chi\tilde{\zeta}$, which may be seen to be the risk-neutral expected output per unit time in the pandemic. So it suffices to show that the diagonal term can be written solely in terms of g_1.

To do this, it is necessary to unpack the dependence of the riskless rate on the parameters. From above, the diagonal coefficient for $s > 0$ is

$$r(s) + c(s) - (\ell^\star)^\alpha(\mu - \gamma\sigma^2) + \chi\tilde{\zeta} + (\tilde{\lambda_u} - \lambda_u) + (\tilde{\lambda_d} - \lambda_d) + (\lambda_u + \lambda_d) \tag{A.51}$$

And $r(s)$ is

$$-[c(s)\ (\theta - 1) - \rho\theta - \gamma((\ell^\star)^\alpha\mu - c(s)) + \frac{1}{2}\gamma(1+\gamma)(\ell^\star)^\alpha\sigma^2 + (\tilde{\lambda_u} - \lambda_u) + (\tilde{\lambda_d} - \lambda_d) + (\tilde{\zeta} - \zeta)]. \tag{A.52}$$

Collecting terms, we have

$$\rho\theta + [(1-\theta) + (1-\gamma)]c(s) + [\lambda_u + \lambda_d] - (\tilde{\zeta} - \zeta) + \chi\tilde{\zeta} - (\ell^\star)^\alpha(1-\gamma)\mu + (\ell^\star)^\alpha(\gamma - \frac{1}{2}\gamma(1+\gamma)). \tag{A.53}$$

Recall that we defined

$$g_1 = \rho\theta - (\ell^\star)^\alpha(1-\gamma)(\mu - \frac{1}{2}\gamma\sigma^2) - \zeta((1-\chi)^{1-\gamma} - 1). \tag{A.54}$$

Then note that $\gamma - \frac{1}{2}\gamma(1+\gamma) = -\frac{1}{2}\gamma(1-\gamma)$, and that

$$\begin{aligned} \zeta((1-\chi)^{1-\gamma} - 1) &= \zeta((1-\chi)(1-\chi)^{-\gamma} - 1) && \text{(A.55)} \\ &= \zeta((1-\chi)^{-\gamma} - 1) + \zeta\chi(1-\chi)^{-\gamma} && \text{(A.56)} \\ &= \tilde{\zeta} - \zeta - \chi\tilde{\zeta}. && \text{(A.57)} \end{aligned}$$

Using these, the expression for the coefficient becomes

$$g_1 + [(1-\theta) + (1-\gamma)]c(s) + [\lambda_u + \lambda_d]. \tag{A.58}$$

This establishes the claim.

□

D Solution to the Regime-Switching Model with 2 States

We derive below the solution to the regime-switching model with just two states. The solution technique is then applied to solving the regime-switching model with S states.

D.1 Description of States & Regime Switch

State $s \in \{0, \underbrace{1}_{pandemic}\}$ The transition probabilities are $P(s_{t+dt} = 1 | s_t = 0) = \eta dt$ and $P(s_{t+dt} = 0 | s_t = 1) = \lambda dt$. Let $\mathbb{J}(0)$ and $\mathbb{J}(1)$ be the two value functions. The HJBs for the two states are:

For the non-Pandemic state

$$0 = \max_{c} \left[f(C, \mathbb{J}) + \mathbb{J}_q(0) \left(\bar{\ell}^{\alpha} \mu q - C \right) + \frac{1}{2} \mathbb{J}_{qq}(0) q^2 \bar{\ell}^{\alpha} \sigma^2 + \eta \left[\mathbb{J}(1) - \mathbb{J}(0) \right] \right] \tag{A.59}$$

and for the Pandemic state

$$\begin{aligned} 0 = \max_{c,l} \Bigg[& f(C, \mathbb{J}) + \mathbb{J}_q(1) \left(l^{\alpha} \mu q - C \right) + \frac{1}{2} \mathbb{J}_{qq}(1) q^2 l^{\alpha} \sigma^2 + \\ & \zeta [\mathbb{J}(1)(q[1-\chi]) - \mathbb{J}(1)(q)] + \lambda \left[\mathbb{J}(0) - \mathbb{J}(1) \right] \Bigg] \end{aligned} \tag{A.60}$$

Assume $\mathbb{J}(1) = H(1) \frac{q^{1-\gamma}}{1-\gamma}$ and $\mathbb{J}(0) = H(0) \frac{q^{1-\gamma}}{1-\gamma}$. In the Pandemic state, optimal labor supply is

$$\frac{\alpha[\mu - \frac{1}{2}\gamma\sigma^2]}{\zeta\varepsilon} = l^{1-\alpha} [1 - (\varepsilon l + k + Kl)]^{-\gamma} \tag{A.61}$$

Define

$$\nu \equiv \left(\frac{\alpha[\mu - \frac{1}{2}\gamma\sigma^2]}{\zeta\varepsilon} \right)^{-\frac{1}{\gamma}} \tag{A.62}$$

Combine (A.61) and (A.62) to get:

$$\nu l^{\frac{1-\alpha}{\gamma}} = [1 - (\varepsilon l + k + Kl)] \tag{A.63}$$

Or,

$$\chi(l, l) = \varepsilon l + k + Kl = \left[1 - \nu l^{\frac{1-\alpha}{\gamma}} \right] \tag{A.64}$$

Then,

$$C(s) = \frac{(H(s))^{-\theta^{-1}\psi} q}{\rho^{-\psi}} \tag{A.65}$$

D.2 Non-Pandemic state

$$0 = \max_{c}\left[f(C^*,\mathbb{J}(0)) + \mathbb{J}_q(0)\left(\overline{\ell}^{\alpha}\mu q - C^*\right) + \frac{1}{2}\mathbb{J}_{qq}(0)q^2\overline{\ell}^{\alpha}\sigma^2 + \eta[\mathbb{J}(1) - \mathbb{J}(0)]\right] \quad \text{(A.66)}$$

1. $f(C,\mathbb{J}):$

$$\begin{aligned} f(C^*,\mathbb{J}(0)) =& \frac{\rho}{1-\psi^{-1}}\frac{(C^*)^{1-\psi^{-1}} - ((1-\gamma)\mathbb{J}(0))^{\theta^{-1}}}{((1-\gamma)\mathbb{J}(0))^{\theta^{-1}-1}} \\ =& \frac{\rho}{1-\psi^{-1}}\frac{(C^*)^{1-\psi^{-1}} - ((1-\gamma)\mathbb{J}(0))^{\theta^{-1}}}{((1-\gamma)\mathbb{J}(0))^{\theta^{-1}-1}} \\ =& \frac{\rho^{\psi}}{1-\psi^{-1}}(H(0))^{1-\theta^{-1}\psi}q^{1-\gamma} - \frac{\rho}{1-\psi^{-1}}H(0)q^{1-\gamma} \end{aligned} \quad \text{(A.67)}$$

2. $\mathbb{J}_q(0)\left(\overline{\ell}^{\alpha}\mu q - C^*\right):$

$$H(0)\left(\overline{\ell}^{\alpha}\mu - \frac{(H(0))^{-\theta^{-1}\psi}}{\rho^{-\psi}}\right)q^{1-\gamma} \quad \text{(A.68)}$$

3. $\frac{1}{2}\mathbb{J}_{qq}(0)q^2\overline{\ell}^{\alpha}\sigma^2:$

$$-\frac{1}{2}\gamma H(0)\overline{\ell}^{\alpha}\sigma^2 q^{1-\gamma} \quad \text{(A.69)}$$

4. $\eta\left[\mathbb{J}(1) - \mathbb{J}(0)\right]:$

$$\eta[H(1) - H(0)]\frac{q^{1-\gamma}}{1-\gamma} \quad \text{(A.70)}$$

HJB simplifies to

$$0 = \frac{\rho^{\psi}}{1-\psi^{-1}}(H(0))^{1-\theta^{-1}\psi}q^{1-\gamma} - \frac{\rho}{1-\psi^{-1}}H(0)q^{1-\gamma} + H(0)\left(\overline{\ell}^{\alpha}\mu - \frac{(H(0))^{-\theta^{-1}\psi}}{\rho^{-\psi}}\right)q^{1-\gamma}$$

$$-\frac{1}{2}\gamma H(0)\overline{\ell}^{\alpha}\sigma^2 q^{1-\gamma}+\eta\left[H(1)-H(0)\right]\frac{q^{1-\gamma}}{1-\gamma} \tag{A.71}$$

Cancelling out $q^{1-\gamma}$

$$\begin{aligned}0=&\frac{\rho^{\psi}}{1-\psi^{-1}}(H(0))^{1-\theta^{-1}\psi}-\frac{\rho}{1-\psi^{-1}}H(0)+H(0)\left(\overline{\ell}^{\alpha}\mu-\frac{(H(0))^{-\theta^{-1}\psi}}{\rho^{-\psi}}\right)\\&-\frac{1}{2}\gamma H(0)\overline{\ell}^{\alpha}\sigma^2+\eta\left[H(1)-H(0)\right]\frac{1}{1-\gamma}\end{aligned} \tag{A.72}$$

Dividing by $H(0)$, we get:

$$0=\frac{\rho^{\psi}}{1-\psi^{-1}}(H(0))^{-\theta^{-1}\psi}-\frac{\rho}{1-\psi^{-1}}+\left(\overline{\ell}^{\alpha}\mu-\frac{(H(0))^{-\theta^{-1}\psi}}{\rho^{-\psi}}\right)-\frac{1}{2}\gamma\overline{\ell}^{\alpha}\sigma^2+\eta\left[\frac{H(1)}{H(0)}-1\right]\frac{1}{1-\gamma} \tag{A.73}$$

$$=\frac{\rho^{\psi}\psi^{-1}}{1-\psi^{-1}}(H(0))^{-\theta^{-1}\psi}-\frac{\rho}{1-\psi^{-1}}+\overline{\ell}^{\alpha}\mu-\frac{1}{2}\gamma\overline{\ell}^{\alpha}\sigma^2+\eta\left[\frac{H(1)}{H(0)}-1\right]\frac{1}{1-\gamma} \tag{A.74}$$

$$=\frac{\rho^{\psi}\psi^{-1}}{1-\psi^{-1}}(H(0))^{-\theta^{-1}\psi}-\frac{\rho}{1-\psi^{-1}}+\overline{\ell}^{\alpha}\mu-\frac{1}{2}\gamma\overline{\ell}^{\alpha}\sigma^2+\eta\left[\frac{H(1)}{H(0)}-1\right]\frac{1}{1-\gamma} \tag{A.75}$$

$$=\frac{\rho^{\psi}}{\psi-1}(H(0))^{-\theta^{-1}\psi}(1-\gamma)-\frac{\rho(1-\gamma)}{1-\psi^{-1}}+(1-\gamma)\overline{\ell}^{\alpha}\left(\mu-\frac{1}{2}\gamma\sigma^2\right)+\eta\left[\frac{H(1)}{H(0)}-1\right] \tag{A.76}$$

Define

$$g(\overline{\ell},0)\equiv\frac{\rho(1-\gamma)}{(1-\psi^{-1})}-\overline{\ell}^{\alpha}(1-\gamma)(\mu-\frac{1}{2}\gamma\sigma^2) \tag{A.77}$$

Then, (A.76) can be written as:

$$0=\frac{\rho^{\psi}}{\psi-1}(H(0)^{-\theta^{-1}\psi}(1-\gamma)-g(\overline{\ell},0)+\eta\left[\frac{H(1)}{H(0)}-1\right] \tag{A.78}$$

Rearranging

$$H(0)=\left(\frac{g(\overline{\ell},0)-\eta\left[\frac{H(1)}{H(0)}-1\right]}{(1-\gamma)\frac{\rho^{\psi}}{\psi-1}}\right)^{-\frac{1}{\theta^{-1}\psi}} \tag{A.79}$$

Define

$$1+\delta \equiv \frac{H(1)}{H(0)} \tag{A.80}$$

Then, we get:

$$H(0) = \left(\frac{g(\bar{\ell},0) - \eta\delta}{(1-\gamma)\frac{\rho^{\psi}}{\psi-1}} \right)^{-\frac{1}{\theta^{-1}\psi}} \tag{A.81}$$

D.3 Pandemic state

$$\begin{aligned} 0 = \max_{C,l} \Bigg[& f(C^*, \mathbb{J}(1)) + \mathbb{J}_q(1)\left(l^{\alpha}\mu q - C^*\right) + \frac{1}{2}\mathbb{J}_{qq}(1) q^2 l^{\alpha}\sigma^2 + \\ & \zeta\left[\mathbb{J}(1)(q[1-\chi]) - \mathbb{J}(1)(q)\right] + \lambda\left[\mathbb{J}(0) - \mathbb{J}(1)\right] \Bigg] \end{aligned} \tag{A.82}$$

1. $f(C, \mathbb{J}):$

$$\begin{aligned} f(C^*, \mathbb{J}(1)) = & \frac{\rho}{1-\psi^{-1}} \frac{(C^*)^{1-\psi^{-1}} - ((1-\gamma)\mathbb{J}(1))^{\theta^{-1}}}{((1-\gamma)\mathbb{J}(1))^{\theta^{-1}-1}} \\ = & \frac{\rho}{1-\psi^{-1}} \frac{(C^*)^{1-\psi^{-1}} - ((1-\gamma)\mathbb{J}(1))^{\theta^{-1}}}{((1-\gamma)\mathbb{J}(1))^{\theta^{-1}-1}} \\ = & \frac{\rho^{\psi}}{1-\psi^{-1}} (H(1))^{1-\theta^{-1}\psi} q^{1-\gamma} - \frac{\rho}{1-\psi^{-1}} H(1) q^{1-\gamma} \end{aligned} \tag{A.83}$$

2. $\mathbb{J}_q(1)\left(l^{\alpha}\mu q - C^*\right):$

$$H(1)\left(l^{\alpha}\mu - \frac{(H(1))^{-\theta^{-1}\psi}}{\rho^{-\psi}}\right) q^{1-\gamma} \tag{A.84}$$

3. $\frac{1}{2}\mathbb{J}_{qq}(1)q^2\overline{\ell}^{\alpha}\sigma^2$:

$$-\frac{1}{2}\gamma H(1)l^{\alpha}\sigma^2 q^{1-\gamma} \tag{A.85}$$

4. $\zeta[\mathbb{J}(1)(q[1-\chi]) - \mathbb{J}(1)(q)]$

$$\zeta\left[H(1)(q[1-\chi]) - H(1)(q)\right]q^{1-\gamma}\frac{1}{1-\gamma} \tag{A.86}$$

5. $\lambda\left[\mathbb{J}(0) - \mathbb{J}(1)\right]$:

$$\lambda\left[H(0) - H(1)\right]\frac{q^{1-\gamma}}{1-\gamma} \tag{A.87}$$

HJB simplifies to

$$\begin{aligned}0 =&\frac{\rho^{\psi}}{1-\psi^{-1}}(H(1))^{1-\theta^{-1}\psi}q^{1-\gamma} - \frac{\rho}{1-\psi^{-1}}H(1)q^{1-\gamma} + H(1)\left(\overline{\ell}^{\alpha}\mu - \frac{(H(1))^{-\theta^{-1}\psi}}{\rho^{-\psi}}\right)q^{1-\gamma}\\ &-\frac{1}{2}\gamma H(1)l^{\alpha}\sigma^2 q^{1-\gamma} + \zeta[H(1)([1-\chi])^{1-\gamma} - H(1)]q^{1-\gamma}\frac{1}{1-\gamma} + \lambda\left[H(0) - H(1)\right]\frac{q^{1-\gamma}}{1-\gamma}\end{aligned} \tag{A.88}$$

Cancelling out $q^{1-\gamma}$

$$\begin{aligned}0 =&\frac{\rho^{\psi}}{1-\psi^{-1}}(H(1))^{1-\theta^{-1}\psi} - \frac{\rho}{1-\psi^{-1}}H(1) + H(1)\left(\overline{\ell}^{\alpha}\mu - \frac{(H(1))^{-\theta^{-1}\psi}}{\rho^{-\psi}}\right)\\ &-\frac{1}{2}\gamma H(1)l^{\alpha}\sigma^2 + \zeta[H(1)([1-\chi]^{1-\gamma} - 1)]\frac{1}{1-\gamma} + \lambda\left[H(0) - H(1)\right]\frac{1}{1-\gamma}\end{aligned} \tag{A.89}$$

Dividing by $H(1)$, we get:

$$\begin{aligned}0 =&\frac{\rho^{\psi}}{1-\psi^{-1}}(H(1))^{-\theta^{-1}\psi} - \frac{\rho}{1-\psi^{-1}} + \left(\overline{\ell}^{\alpha}\mu - \frac{(H(1))^{-\theta^{-1}\psi}}{\rho^{-\psi}}\right)\\ &-\frac{1}{2}\gamma l^{\alpha}\sigma^2 + \zeta[([1-\chi]^{1-\gamma}) - 1]\frac{1}{1-\gamma} + \lambda\left[\frac{H(0)}{H(1)} - 1\right]\frac{1}{1-\gamma}\end{aligned} \tag{A.90}$$

$$
\begin{aligned}
=&\frac{\rho^{\psi}}{1-\psi^{-1}}(H(1))^{-\theta^{-1}\psi}-\frac{\rho}{1-\psi^{-1}}+\left(\overline{\ell}^{\alpha}\mu-\frac{H(1)^{-\theta^{-1}\psi}}{\rho^{-\psi}}\right)\\
&-\frac{1}{2}\gamma l^{\alpha}\sigma^{2}+\zeta[([1-\chi]^{1-\gamma})-1]\frac{1}{1-\gamma}+\lambda\left[\frac{H(0)}{H(1)}-1\right]\frac{1}{1-\gamma} \qquad (A.91)\\
=&\frac{\rho^{\psi}\psi^{-1}}{1-\psi^{-1}}(H(1))^{-\theta^{-1}\psi}-\frac{\rho}{1-\psi^{-1}}+\overline{\ell}^{\alpha}\mu\\
&-\frac{1}{2}\gamma l^{\alpha}\sigma^{2}+\zeta[([1-\chi]^{1-\gamma})-1]\frac{1}{1-\gamma}+\lambda\left[\frac{H(0)}{H(1)}-1\right]\frac{1}{1-\gamma} \qquad (A.92)\\
=&\frac{\rho^{\psi}\psi^{-1}}{1-\psi^{-1}}(H(1))^{-\theta^{-1}\psi}(1-\gamma)-\frac{\rho(1-\gamma)}{1-\psi^{-1}}\\
&+\left[\overline{\ell}^{\alpha}\mu-\frac{1}{2}\gamma l^{\alpha}\sigma^{2}+\zeta[([1-\chi]^{1-\gamma})-1]\frac{1}{1-\gamma}\right](1-\gamma)+\lambda\left[\frac{H(0)}{H(1)}-1\right] \qquad (A.93)\\
=&\frac{\rho^{\psi}}{\psi-1}(H(1))^{-\theta^{-1}\psi}(1-\gamma)-\frac{\rho(1-\gamma)}{1-\psi^{-1}}+(1-\gamma)\overline{\ell}^{\alpha}\left(\mu-\frac{1}{2}\gamma\sigma^{2}\right)\\
&+\zeta[([1-\chi]^{1-\gamma})-1]+\lambda\left[\frac{H(0)}{H(1)}-1\right] \qquad (A.94)
\end{aligned}
$$

Define

$$
g(l,\zeta)\equiv\frac{\rho(1-\gamma)}{(1-\psi^{-1})}-l^{\alpha}(1-\gamma)(\mu-\frac{1}{2}\gamma\sigma^{2})-\zeta[([1-\chi(l)]^{1-\gamma})-1] \qquad (A.95)
$$

Then, (A.82) can be written as:

$$
0=\frac{\rho^{\psi}}{\psi-1}(H(1))^{-\theta^{-1}\psi}(1-\gamma)-g(l,\zeta)+\lambda\left[\frac{H(0)}{H(1)}-1\right] \qquad (A.96)
$$

Rearranging

$$
H(1)=\left(\frac{g(l,\zeta)-\lambda\left[\frac{H(0)}{H(1)}-1\right]}{(1-\gamma)\frac{\rho^{\psi}}{\psi-1}}\right)^{-\frac{1}{\theta^{-1}\psi}} \qquad (A.97)
$$

Using the definition of δ

$$
H(1)=\left(\frac{g(l,\zeta)+\lambda\frac{\delta}{1+\delta}}{(1-\gamma)\frac{\rho^{\psi}}{\psi-1}}\right)^{-\frac{1}{\theta^{-1}\psi}} \qquad (A.98)
$$

We can solve for δ from (A.81) and (A.98).

E HJB System with Endogenous Pandemic Severity and Labor Externalities

This appendix derives Proposition 4 in Section 5 of the paper.

Proposition 4. *Equilibrium labor in the non-pandemic state is given by*

$$L(0) = L(S) = \overline{\ell} \tag{A.99}$$

Equilibrium labor in pandemic states $L^*(s)$ $\forall s \in \{1, \ldots, S-1\}$ solves[5]

$$\chi\left(L(s), L(s)\right) = k + (\varepsilon + K)L(s) = \left[1 - \left(L(s)\right)^{\frac{1-\alpha}{\gamma}} \nu\right] \tag{A.101}$$

where

$$\nu \equiv \left[\frac{\alpha\left(\mu - \frac{1}{2}\gamma\sigma^2\right)}{\zeta\varepsilon}\right]^{-\frac{1}{\gamma}} . \tag{A.102}$$

Proof. The HJB equation for each state $s \in \{1, \ldots, S-1\}$ is now

$$0 = \max_{C,\ell}\Big[f(C, \mathbb{J}(s)) - \rho\mathbb{J}(s) + \mathbb{J}_q(s)(\ell^\alpha q\mu - C) + \frac{1}{2}\mathbb{J}_{qq}(s)\ell^\alpha q^2\sigma^2 + \zeta\left[\mathbb{J}(s)\left(q(1-\chi)\right) - \mathbb{J}(s)(q)\right] + \lambda_u(s)\left[\mathbb{J}(s+1)(q) - \mathbb{J}(s)(q)\right] + \lambda_d(s)\left[\mathbb{J}(s-1)(q) - \mathbb{J}(s)(q)\right]\Big] \tag{A.103}$$

Using the conjecture for the objective function (A.5) for $\mathbb{J}(s)$, calculating the derivatives with respect to q, $\mathbb{J}_q(s) = H(s)q^{-\gamma}$ and $\mathbb{J}_{qq}(s) = -\gamma H(s)q^{-\gamma-1}$, and differentiating with

[5] It can be shown that given $\alpha \in (0,1)$, the second order condition for a maximum is satisfied whenever

$$\mu - \frac{1}{2}\gamma\sigma^2 > 0 \tag{A.100}$$

which also implies $\nu > 0$.

respect to labor ℓ, we obtain the first-order condition as

$$\mathbb{J}_q(q)\alpha\ell^{\alpha-1}\mu q + \frac{1}{2}\mathbb{J}_{qq}(q)\alpha\ell^{\alpha-1}\sigma^2 q^2 - \mathbb{J}_q\left(q(1-\chi)\right)\zeta\varepsilon q = 0 \tag{A.104}$$

where we have suppressed state s in the notation. This in turn simplifies to

$$\left[\frac{\alpha\left(\mu - \frac{1}{2}\gamma\sigma^2\right)}{\zeta\varepsilon}\right]\ell^{\alpha-1} - [1-\chi]^{-\gamma} = 0 \tag{A.105}$$

where $\chi(\ell, L) = k + \varepsilon\ell + KL$. In rational expectations equilibrium $L(s) = \ell(s)$, which gives us that optimal labor in pandemic state $L^{\star}(s)\ \forall s \in \{1, \ldots, S-1\}$ satisfies (A.101):

$$\chi\left(L(s), L(s)\right) = k + (\varepsilon + K)L(s) = \left[1 - (L(s))^{\frac{1-\alpha}{\gamma}}\nu\right] \tag{A.106}$$

where

$$\nu \equiv \left[\frac{\alpha\left(\mu - \frac{1}{2}\gamma\sigma^2\right)}{\zeta\varepsilon}\right]^{-1/\gamma}. \tag{A.107}$$

The second-order condition with respect to ℓ is satisfied (footnote 7, equation A.100) whenever $\left(\mu - \frac{1}{2}\gamma\sigma^2\right) > 0$. For the non-pandemic state $s = 0$ or $s = S$, the third term in first-order condition (A.104) is absent; therefore, we obtain that labor is at the highest possible level $L(0) = L(S) = \overline{\ell}$, whenever $\alpha\left(\mu - \frac{1}{2}\gamma\sigma^2\right) > 0$. □

F HJB System with Parameter Uncertainty

This appendix derives equations (39)-(40) in Section 5 of the paper.

As noted in the text, the model can be parameterized in terms of the state variables $M, \hat{\eta}, \hat{\lambda}$, and q, where $M = M_t$ is an integer counter that increases on a state switch such that $M_0 = 0$ and even numbered states are the non-pandemic epochs and odd numbered states are the pandemics. Also, in the non-pandemic states, $\hat{\lambda}$ is constant, while $\hat{\eta}$ is constant in pandemics. As a consequence, compared with the derivation above for the full-information case, there is now only one additional source of variability in each regime.

The dynamics of $\hat{\eta}$ and $\hat{\lambda}$ are given in (35)-(37). And note that, under the agents' information set, the dynamics of the wealth variable q are identical to the full information dynamics.

As a result, the HBJ equations under partial information are the same as (A.59) and (A.60) a above (with state 0 and state 1 being replaced by M and $M+1$ in (A.59), and by $M+1$ and M in (A.60)) with the addition of a single term on the right side of each:

$$-\frac{(\hat{\eta})^2}{a^{\eta}}\frac{\partial \mathbb{J}(0)}{\partial \hat{\eta}} \tag{A.108}$$

in (A.59), and

$$-\frac{(\hat{\lambda})^2}{a^{\lambda}}\frac{\partial \mathbb{J}(1)}{\partial \hat{\lambda}} \tag{A.109}$$

in (A.60). Since, under the agent's information set, the state switches are a point-process with instantaneous intensities $\hat{\eta}$ and $\hat{\lambda}$, these quantities also replace their full information counterparts, η and λ, in multiplying the jump terms in the respective equations.

As discussed in Section 5, the next steps in the derivation involving the first order conditions for optimal consumption and labor are unchanged from the full-information case. The derivation proceeds to replace $\mathbb{J}$ by the conjecture $\frac{q^{1-\gamma}}{1-\gamma}H(\hat{\eta},\hat{\lambda},M)$, then a common power of q term is cancelled, and the whole equation is divided by H. These manipulations lead to the above two terms becoming the right-most terms in (39) and (40), which are otherwise identical to the full-information system (14), (15) and (32).

Table A.1: Vaccine States

State	# Candidates	Example Candidates
Preclinical	210	Amyris Inc Baylor College of Medicine Mount Sinai
Phase I Safety Trials	20	Clover/GSK/Dynavax CSL/University of Queensland Imperial College London
Phase II Expanded Trials	18	Arcturus/Duke Osaka/AnGes/Takara Bio Sanofi Pasteur/GSK
Phase III Efficacy Trials	11	AstraZeneca/Oxford BioNTech/Fosun/Pfizer Moderna

Note: Table describes the number of vaccine candidates in each state, along with example institutes. Data are from the London School of Hygiene & Tropical Medicine's COVID-19 Tracker. Data are as of November 2, 2020.

Table A.2: Vaccine Strategies

Type	Description	# Candidates
RNA (genetic)	Consist of messenger RNA molecules which code for parts of the target pathogen that are recognised by our immune system ('antigens'). Inside our body's cells, the RNA molecules are converted into antigens, which are then detected by our immune cells.	33
DNA (genetic)	Consist of DNA molecules which are converted into antigens by our body's cells (via RNA as an intermediate step). As with RNA vaccines, the antigens are subsequently detected by our immune cells.	21
Viral Vector	Consist of harmless viruses that have been modified to contain antigens from the target pathogen. The modified viruses act as delivery systems that display antigens to our immune cells. Replicating make extra copies of themselves in our body's cells. Non-replicating do not.	56
Protein	Consist of key antigens from the target pathogen that are recognised by our immune system.	78
Inactivated	Consist of inactivated versions of the target pathogen. These are detected by our immune cells but cannot cause illness.	16
Attenuated	Consist of living but non-virulent versions of the target pathogen. These are still capable of infecting our body's cells and inducing an immune response, but have been modified to reduce the risk of severe illness.	4

Note: Table describes the number of vaccine candidates in each strategy. 51 candidates have other, virus-like particle or unknown strategies. Data from the London School of Hygiene & Tropical Medicine's COVID-19 Tracker. Data as of November 2, 2020.

Table A.3: Number of Articles by News Type

News Type	Number of Articles
Release positive data	79
Announce next state	45
Positive regulatory action	30
Positive preclinical progress	22
Announce dosage start	21
Positive enrollment	17
State ahead of schedule	7
State resumed	5
State paused	4
State behind schedule	1
Negative regulatory action	1
Negative enrollment	1
Total	233

Note: Table shows the count of news articles by news type.

Table A.4: Number of Articles by Top 10 Candidates

Candidate	Number of Articles
Moderna	37
BioNTech / Fosun Pharma / Pfizer	25
Oxford / AstraZeneca	23
Johnson & Johnson / Beth Israel Deaconess Medical Center	21
Inovio Pharmaceuticals	18
Novavax	14
Arcturus / Duke	10
Vaxart	9
Medicago / GSK / Dynavax	8
Takis / Applied DNA / Evvivax	8

Note: Table the number of news articles for the top ten candidates by article count.

Table A.5: State Durations and Probabilities of Success

State	τ_s (years)	π_s^{base} (%)
Preclinical	0.6	5
Phase I	0.2	70
Phase II	0.2	44
Phase III	0.4	69
Application	0.1	88
Approval	0.5	95

Table A.6: Vaccine States

	Days in State				
	Min	Max	Mean	Median	SD
Preclinical	1.0	233.0	94.6	90.5	59.2
Phase I	17.0	103.0	51.9	27.0	39.8
Phase II	6.0	152.0	86.8	89.0	54.5

Note: Table shows statistics on days spent in each state before transitioning, among candidates that have successfully transitioned to the next state. Following Wong et al. (2018), we adopt each candidate's most advanced state. Data are from the LSHTM and are as of November 2, 2020.

Table A.7: News and Changes in Probabilities

Positive		Negative	
News type	$\Delta\pi$ (%)	News type	$\Delta\pi$ (%)
Announce next state	+5	Pause in state	-25
State ahead of schedule	+2	State behind schedule	-15
Release positive data	+5	Release negative data	-60
Positive regulatory action	+3	Negative regulatory action	-50
Positive preclinical progress	+1	Negative preclinical progress	-2
Positive enrollment	+1	Negative enrollment	-5
Dose starts	+1		
State resumes after pause	+5		

Note: Table shows the positive and negative news types, along with their changes in probabilities.