We analyze a model that integrates demand shaping via dynamic pricing and risk mitigation via supply diversification. The firm under consideration replenishes a certain product from a set of capacitated suppliers for a price-dependent demand in each period. Under deterministic capacities, we derive a multilevel base stock list price policy and establish the optimality of cost-based supplier selection, that is, ordering from a cheaper source before more expensive ones. With general random capacities, however, neither result holds. While it is optimal to price low for a high inventory level, the optimal order quantities are not monotone with respect to the inventory level. In general, a near reorder-point policy should be followed. Specifically, there is a reorder point for each supplier such that no order is issued to him when the inventory level is above this point and a positive order is placed almost everywhere when the inventory level is below this point. Under this policy, it may be profitable to order exclusively from the most expensive source. We characterize conditions under which a strict reorder-point policy and a cost-based supplier-selection criterion become optimal. Moreover, we quantify the benefit from dynamic pricing, as opposed to static pricing, and the benefit from multiple sourcing, as opposed to single sourcing. We show that these two strategies exhibit a substitutable relationship. Dynamic pricing is less effective under multiple sourcing than under single sourcing, and supplier diversification is less valuable with price adjustments than without. Under limited supply, dynamic pricing yields a robust, long-term profit improvement. The value of supply diversification, in contrast, mainly comes from added capacities and is most significant in the short run.

Key words: procurement policies; dynamic pricing; supplier diversification

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1. Introduction

Matching supply with demand in the face of uncertainties has always been a challenge of supply-chain management. The major difficulties of accurate demand prediction emanate from changing consumer tastes, competition dynamics, and economic conditions. Firms have realized the inextricable connections between price and inventory, as well as the importance of demand shaping to achieve profitability. Dynamic pricing has become a major strategy of many firms including Dell, Amazon, FairMarket, Lands End, J. C. Penney, MSN Auction, and Grainger (Chan et al. 2005, Elmaghraby and Keskinocak 2003). Recent developments in this area have demonstrated major benefits derived from coordinating inventory and pricing decisions (e.g., Chen and Simchi-Levi 2004a, 2004b; Federgruen and Heching 1999).

Meanwhile, supply uncertainty has also become an increasing concern in procurement planning due to the growing trend of outsourcing and globalization. Unpredictable supply streams often induce fluctuations in stock availability, which in turn dampen firms’ ability to capture market demand. Mitigating supply risks via a diversified supplier portfolio has become a common procurement strategy in practice (Anupindi and Akella 1993, Babich et al. 2007, Kouvelis and Li 2010, Lester 2002). Multiple sourcing allows firms to spread procurement orders among different suppliers to achieve the best cost–risk mix.

While the benefits of dynamic pricing and supply diversification have been well documented, it is commonly agreed that implementation of these strategies may not always justify the increased operating costs and planning complexities. Previous studies tend to address this issue by analyzing either dynamic pricing under single sourcing or supply diversification under an exogenous price. The joint effects of dynamic pricing and supply diversification on firms’ policies and profits are yet to be explored. There is also a lack of understanding on when firms should deploy either or both strategies.

To address these issues, we present a single-item, multiperiod model. There is a set of potential suppliers, each characterized by a random capacity and a unit procurement cost. In every period, the firm needs
to determine the order quantities from the suppliers and the selling price for the product before supply and demand uncertainties are resolved. The delivery quantities obtained at the end of the period may turn out to be less than the corresponding order quantities due to random supply capacities. Upon demand realization, leftover inventory is carried over to the next period and unmet demand is fully backlogged. The objective of our analysis is twofold. First, we characterize the optimal policy for the integrated decisions of product pricing and multiple sourcing. Second, we quantify and compare the benefits of dynamic pricing and supply diversification.

The consideration of supply uncertainties introduces a major technical challenge—the resulting objective function is not concave in the procurement quantities (section 3). Consequently, the traditional concave or convex analysis does not work for our dynamic programming model. A critical development from our analysis is to establish the concavity of the optimal profit function. More importantly, this result is derived under fairly general conditions.

The concavity of the value function allows for analytical derivation of the optimal policy. In the special case of deterministic capacities (section 4.1), it is optimal to follow a multilevel base stock list price policy. This policy specifies a base-stock level and a list price for each supplier. A low-cost supplier is associated with a high base-stock level and a low list price. Under this policy, the order quantities increase and the price decreases in the inventory level. We further show that this multidimensional optimization problem reduces to one of finding the optimal price for any available stock level.

In the general case of random supplies (section 4.2), the optimal price decreases in the inventory level as it does in the case of deterministic supplies. The optimal order quantities, however, may reveal nonmonotone responses to changes in the inventory level. Nevertheless, we prove that a near reorder-point procurement policy is optimal. In particular, there is a unique threshold, called the reorder point, for each supplier. No order is issued to that supplier if the inventory level is above the threshold. For an inventory level below the threshold, a positive order is issued with exceptions over a set of measure zero. Furthermore, contrary to previous studies that propose a cost-based supplier-selection criterion (e.g., Anupindi and Akella 1993, Dada et al. 2007, Federgruen and Yang 2009), our analysis suggests the possibility of ordering from a more expensive source without first ordering from a cheaper source. We derive conditions for the cost-based selection criterion to be optimal. Under the same conditions, we prove the optimality of a strict reorder-point policy, which calls for a positive order if and only if the inventory level is below the reorder point.

Because it integrates dynamic price adjustment and multiple sourcing options, our model allows for examining the individual and combined effects of supply diversification and dynamic pricing in a unified framework (section 5). Such a quantification is valuable to justify the adoption of these strategies against their associated implementation costs in practice. Via an intensive numerical study, we demonstrate that multiple sourcing, compared to single sourcing, is valuable when the additional suppliers bring in extra capacity. In this case, the benefit of supply diversification is subject to rapidly diminishing returns. Under a limited total average capacity, changing the supply-risk profile via diversification does not have a significant impact on the optimal profit. In contrast, dynamic pricing, as opposed to static pricing, yields a significant profit improvement under limited supply. Moreover, such an improvement is robust to supply-risk profiles.

We uncover that, on the one hand, supply diversification and dynamic pricing reveal several similarities. For example, when the firm’s ability to buy and hold inventory increases (via reductions in procurement and holding costs), both strategies become more effective at raising profit. They can also effectively mitigate supply uncertainties. On the other hand, the two strategies exhibit several differences. For example, dynamic pricing induces a long-term profit improvement, while the benefit from supply diversification decreases over time.

In what follows, we first review the related literature and discuss our contributions in section 2. We lay out the model in section 3 and present the analysis in section 4. We examine the benefits obtained from deploying supply diversification and dynamic pricing strategies in section 5. Section 6 concludes the study and provides future research directions. The proofs of key results are relegated to an Appendix, which is supplemented by Appendix S1.

2. Literature Review

Our work is related to two streams of studies. The first stream deals with sourcing from multiple suppliers and the second with joint inventory–pricing decisions. In the domain of multiple sourcing, there has been an extensive discussion on the trade-off between cost and lead time of reliable suppliers. This line of research can be traced back to Barankin (1961), with further developments by Daniel (1963), Fukuda (1964), Whittmore and Saunders (1977), Lawson and Porteus (2000), and Sheopuri et al. (2010), among others. The optimal policy for this problem can be very complex unless there are only two suppliers with a special lead-time structure (e.g., Feng et al. 2006). Recently, Kouvelis and Li (2010) studied a model
involving an unreliable source with a long lead time and a reliable source with a much shorter lead time. The reliable source is only used as a backup when observing a low output from the unreliable source. The lead-time structure in their model allows for at most one outstanding order, which is critical for analytical tractability. There has not been any study on procurement from unreliable suppliers with general lead-time structures. The problem is challenging even for single-supplier settings. This suggests that we restrict our analysis to one period lead time.

When supplies are random, there is a trade-off between procurement cost and reliability. Supply unreliability is often modeled using random yield (Yano and Lee 1995). For example, Gerchak and Parlar (1990) consider two suppliers with random yields and identical cost in an EOQ setting. They show that it is optimal to allocate procurement orders proportional to the mean–variance ratio of each supplier’s yield rate. For the general cost structure, there is no simple measure to determine order allocation, as suggested by Parlar and Wang (1993) and Ilan and Yadin (1985). Ciarallo et al. (1994) capture supply uncertainty by assuming a random capacity in their single-supplier setting. They prove the optimality of a base-stock policy.

In the context of multiple sourcing, several studies (e.g., Anupindi and Akella 1993, Chen et al. 2001, Dada et al. 2007, Federgruen and Yang 2009) have established the optimality of cost-based supplier selection (i.e., selecting a cheaper source before more expensive ones). These discussions are limited to the case of an exogenous product price. It is worth mentioning that the cost-based selection principle derived from the above studies relies critically on their assumption of a continuous demand density. This result breaks down in the case of discrete demands, as pointed out by Swaminathan and Shanthikumar (1999). We consider general demand distributions in our model. Our analysis enhances that of Swaminathan and Shanthikumar by characterizing conditions for cost-based selection to be optimal.

The second stream of related literature underscores the importance of coordinating pricing and inventory decisions. Whitin (1955) was the first to consider pricing in an inventory-control model. Subsequently, many authors have investigated the problem in various contexts (e.g., Amihud and Mendelson 1983, Chen and Simchi-Levi 2004a, 2004b, 2006, Chen et al. 2010, Federgruen and Heching 1999; Li 1998, Thomas 1974, Thowsen 1975, Zabel 1972). Comprehensive surveys on this literature are provided by Elmaghraby and Keskinocak (2003), Yano and Gilbert (2003), and Chen and Simchi-Levi (2012). These studies are restricted to a single reliable supplier. In particular, Federgruen and Heching (1999), assuming an unconstrained supplier, prove the optimality of a base-stock list price policy. A similar policy structure is derived by Chao et al. (2008) who consider a linear price-dependent demand with an additive noise and a Markov-modulated supply capacity. They assume that at the beginning of each period, the firm observes the supply capacity for that period before determining the order quantity.

Li and Zheng (2006) and Feng (2010) analyze the impact of supply uncertainty on inventory–pricing decisions. Li and Zheng analyze a random-yield model. They prove that the optimal order quantity and price are nonincreasing in the inventory level when the demand shock is additive. With a multiplicative demand noise, they provide an example to demonstrate the complex policy structure. In this case, it may be optimal to order for a certain inventory level but not for lower inventory levels within a nondegenerate interval. Feng studies a special case of our model with a single supplier and an additive demand noise. She demonstrates the suboptimality of the base stock list price policy. In particular, the optimal price and order decisions reveal nonlinear responses to the inventory level. The target safety stock, however, does not change with respect to the inventory level whenever an order is issued. Our analysis of multiple suppliers with random capacities suggests that none of the policies derived in the above mentioned studies is optimal for our model. Moreover, we extend Feng’s study along several key dimensions. First, her proof for the concavity of the value function cannot be extended to multisupplier settings. The latter requires a very different approach as illustrated in this study. Second, our results imply that the optimal policy derived in her model is generally suboptimal when either a multiplicative random noise or a second unreliable supplier is involved. Third, we extend her analysis of the benefit from dynamic pricing to multisupplier settings. The consideration of multiple sourcing also allows us to compare the effects of dynamic price and supply diversification.

In quantifying the value of dynamic pricing, many studies report that there is no need to adjust price under unconstrained supply. Specifically, a fixed or static pricing policy performs efficiently in a stationary system with a long enough planning horizon (e.g., Chan et al. 2005, Dong et al. 2009, Federgruen and Heching 1999, Gimpl-Heersin et al. 2008). Amihud and Mendelson (1983) offer an explanation that adjusting inventories helps to absorb demand shocks, leading to price rigidity. Ashley and Orr (1985) also find that production smoothing can bring about price smoothing. Feng (2010), however, suggests that firms exposed to significant supply uncertainty can derive a long-term benefit from dynamic pricing. In an essential contrast to these studies, we model multiple
unreliable suppliers. Our analysis enhances the results obtained by Feng and shows their robustness with respect to suppliers’ risk profile. Moreover, we analyze the joint effect of dynamic pricing and supply diversification to characterize conditions under which one strategy may outperform the other.

3. Model Development and Preliminary Analysis

In this section, we formulate a dynamic programming model for our problem and derive several properties of the optimal profit function.

3.1. Problem Formulation

The firm faces a $T$-period planning problem. There is a set of potential suppliers, denoted by $\mathcal{N} = \{1, 2, \ldots, n\}$. Supplier $j \in \mathcal{N}$ offers a capacity $K_j$ in period $t \in \{1, 2, \ldots, T\}$, which is non-negative with a finite mean. The processes $\{K_j\}_{t=1}^{T}$, $j \in \mathcal{N}$, each form an independent sequence of independent and identically distributed random variables. Let $G_t(\cdot)$ denote the distribution of $K_j$, and $K_j$ a generic random variable following this distribution. We shall impose that $\{K_1, K_2, \ldots, K_n\}$ are independent of one another. When an order of $q_j$ is issued to supplier $j$, the actual amount delivered is

$$q_j \wedge K_j = \min\{q_j, K_j\}.$$  

The firm pays $c_j$ dollars for each unit received from supplier $j$. Without loss of generality, we assume $c_1 \leq c_2 \leq \cdots \leq c_n$.

The demand materialized at the end of period $t$ is

$$D_t(p_t) = A_t \mu_D(p_t) + B_t,$$

where $A_t$ and $B_t$ are, respectively, the multiplicative and additive demand noises, and $\mu_D(p_t)$ is the mean demand for a given price $p_t$. We assume that $\{A_t\}_{t=1}^{T}$ and $\{B_t\}_{t=1}^{T}$ are independent sequences of independent and identically distributed random variables. They are also independent of the capacity processes. We use $A$ and $B$ to denote the generic components $A_t$ and $B_t$, respectively. Thus, $A$ and $B$ are mutually independent, and they are independent of $\{K_{1}, K_2, \ldots, K_n\}$.

We also assume that $A$ has mean 1 and support $[a, \bar{a}]$, and $B$ has mean zero and support $[b, \bar{b}]$. For any feasible choice of $p_t \in [\underline{p}, \overline{p}]$ and any possible realization of $\{A, B\}$, the demand is non-negative. We impose the condition $c_1 < \overline{p}$ so that it is profitable to order from the cheapest supplier for some feasible price. The average demand, $\mu(p_t)$, has a decreasing inverse $\mu^{-1}(d)$, where $\mu_D(p_t)$ and $\overline{d} = \mu_D(\overline{p})$. Then, choosing an average demand $d \in [\underline{d}, \overline{d}]$ is equivalent to charging a price $\mu^{-1}_D(d) \in [\underline{p}, \overline{p}]$. The expected revenue, $R(d) = d \times \mu^{-1}_D(d)$, is assumed to be concave in $d$.

Upon demand realization, leftover inventory is carried to the next period and unmet demand is fully backordered. Denote $H(x)$ to be the surplus and shortage cost. We assume that $H(\cdot)$ is continuous and convex, $H(0) = 0$, $\lim_{x \to -\infty} H(x) = \infty$, and $|H(x_1) - H(x_2)| \leq \epsilon H|_\lim(x_1 - x_2)|$ for some $\epsilon H|_\lim > 0$. Also, $\lim_{x \to -\infty} [H(x - \delta) - H(x)]/\delta > c_n$ for some $\delta > 0$, so that the firm should replenish inventory when the backlog becomes extremely large.

In our model, the customer pays the current price $p_t$ upon demand realization. This price may not be the same as the price quoted at the time when he actually obtains the product if the demand is backordered. Thus, the delay in filling the order does not affect the price charged directly. Instead, the firm incurs a penalty cost via $H(\cdot)$, which accounts for the loss of goodwill and the compensation to the customer. Such a pricing scheme, allowing for analytical tractability, is commonly assumed in the literature (e.g., Chen and Simchi-Levi 2004a; Federgruen and Heching 1999; Li and Zheng 2006).

At the beginning of period $t$, the firm reviews the inventory level $x_t$ and determines ordering quantities $q_t = \{q_{t,1}, q_{t,2}, \ldots, q_{t,n}\}$ and average demand $d_t$ for this period. These decisions are made before knowing $\{K_t, K_{t+1}, \ldots, K_n\}$ and $\{A, B\}$. The firm observes suppliers’ capacities when receiving the shipments from them at the end of period $t$. Following delivery, the demand uncertainty is resolved and the firm realizes its profit for this period. The beginning inventory in the next period, that is period $t + 1$, is updated to

$$x_{t+1} = x_t + \sum_{j=1}^{n} (q_{t,j} \wedge K_j) - Ad_t - B.$$  

To develop a dynamic programming formulation, we consider the problem over the horizon $(t,T)$. The goal is to find a sequence of history-dependent admissible decisions

$$(q_t, d_t) = \{(q_{t,1}, d_t), (q_{t+1,1}, d_{t+1}), \ldots, (q_{T,1}, d_T)\},$$

that maximize the discounted total profit from period $t$ to period $T$ expressed as

$$J_t(x_t, q_t, d_t) = \sum_{i=1}^{T} \gamma^{T-i} \left\{ R(d_i) - \sum_{j=1}^{n} c_j \mathbb{E}[\{(q_{i,j} \wedge K_j)\}] - \mathbb{E}\left[ H\left(x_i + \sum_{j=1}^{n} (q_{i,j} \wedge K_j) - Ad_i - B\right)\right] \right\},$$

where $\gamma \in (0,1)$ is the discount factor. Let $V_t(x)$ denote the optimal profit function in period $t$ when the inventory level is $x$. We obtain
In general, the objective function

\[ V_t(x) = \max_{q \geq 0, d \leq d} J_t(x; q, d), \tag{2} \]

where

\[ J_t(x; q, d) = R(d) - \sum_{j \in \mathcal{F}} c_j \mathbb{E}[(q_j \wedge K_j)] + \mathbb{E} \left[ L_t \left( x + \sum_{j \in \mathcal{F}} (q_j \wedge K_j) - Ad \right) \right], \tag{3} \]

and

\[ L_t(y) = -\mathbb{E}[H(y - B)] + 2\mathbb{E}[V_{t+1}(y - B)]. \tag{4} \]

Equations (2)–(4) constitute the dynamic program of the problem.

For ease of exposition, we have assumed that \( c_j, K_j, A, B, \mu_D(\cdot), H(\cdot), \) and \( R(\cdot) \) are stationary over time. It should be noted that our analysis can be easily extended to nonstationary systems. Furthermore, the results derived in this study can be generalized to the case when \( \{K_{ij}\}_{i=1}^{\mathcal{I}}, \{K_{ij}\}_{i=1}^{\mathcal{I}}, \ldots, \{K_{ij}\}_{i=1}^{\mathcal{I}}, \{A_i\}_{i=1}^{\mathcal{I}}, \{B_i\}_{i=1}^{\mathcal{I}} \) each form a Markov process, or when \( A \) and \( B \) are correlated (see Remark 1 in Appendix).

However, our results may not hold when supply capacities are correlated among one another or when the supply capacities are correlated with the demand noises.

Standard dynamic programming arguments (see Lemma 3 in Appendix S1) show that the objective function \( J_t(x; q, d) \) is continuous and thus \( V_t(x) \) is bounded. Therefore, an optimal feedback policy always exists. Denote \( q_j^* (x), j \in \mathcal{F}, \) to be the optimal order issued to supplier \( j \) and \( d(x) \) to be the optimal average demand. In general, the optimal solution may not be uniquely determined due to the high dimensionality of the problem. When this happens, we choose the one with the smallest \( d \). If there are multiple solutions containing the smallest \( d \), we pick the one with a larger quantity from the supplier with a lower index. Take an example with \( c_1 = c_2, K_1 \in [2,10], \) and \( K_2 \in [5,15]. \) A solution with \( q_1 = 2 \) and \( q_2 = 3 \) is equivalent to one with \( q_1 = 2 - z \) and \( q_2 = 3 + z \) for \( z \in [0,2]. \) Our criterion suggests a choice of \( z = 0. \)

\[ \text{Lemma 1. Let } \phi(q), \ q \geq 0, \text{ be a concave function with } q^* = \arg\max_{q \geq 0} \phi(q), \text{ and } K \text{ is a non-negative random variable.} \]

(i) For any sufficiently small \( \delta > 0, \) \( \mathbb{E}[\phi(q \wedge K)] - \mathbb{E}[(\phi(q^* - \delta) \wedge K)] \geq 0. \)

(ii) For any \( \delta \in [0,q], \) \( \mathbb{E}[\phi(q^* \wedge K)] \leq \mathbb{E}[(\phi(q^* - \delta) \wedge K + \delta)]. \)

(iii) For any \( a \geq b \) and \( q > 0, \) \( \phi(a + q) - \phi(b + q) \leq \mathbb{E}[\phi(a + q \wedge K)] - \mathbb{E}[\phi(b + q \wedge K)] \leq \phi(a) - \phi(b). \)

To interpret Lemma 1, consider ordering \( q \) units from a supplier with capacity \( K \) to earn a profit of \( \mathbb{E}[\phi(q \wedge K)]. \) Part (i) implies \( \mathbb{E}[\phi(q \wedge K)] \) is increasing (decreasing) whenever \( \phi(q) \) is increasing (decreasing), and thus both are maximized at \( q^* . \) In other words, the optimal order quantity for a given supplier does not directly depend on the capacity of that supplier (but may depend on other suppliers’ capacities). A version of this result has been proven by Ciarallo et al. (1994). Our proof presented in the Appendix generalizes theirs by allowing for possibly nondifferentiable profit functions. Furthermore, because they consider a single supplier and exogenous price, this result directly leads to the optimality of a base-stock policy in their model. As we show later, base-stock policies are generally suboptimal in our model.

Part (ii) of Lemma 1 suggests that the profit of purchasing all \( q^* \) units from a random source with capacity \( K \) is less than the profit of ordering \( \delta \) units from a reliable source and \( (q^* - \delta) \) units from that random source. The relation in part (iii) states that the marginal value of inventory is lower when ordering than when not ordering. Furthermore, the marginal value of inventory when ordering from an uncapacitated reliable source is lower than that from an unreliable source.

Lemma 1 establishes some connections between the ordering decisions and the profit function. These connections lead to a proof for the concavity of the value function stated in the next theorem.

\[ \text{Theorem 1. The optimal profit function, } V_t(x), \text{ is concave in } x \text{ for any } t. \]

The key to establishing Theorem 1 is the following observation. For fixed order quantities \( q_i, i \in \mathcal{I} \setminus \{j\}, \) and a fixed average demand \( d(x), \) we compare two alternatives to determine \( q_j. \) In the first alternative, we order an optimal \( q_j \) from a single supplier with capacity \( K_j. \) In the second alternative, we have two suppliers with capacities equal to \( K_j \) that is, their capacities are identical in each realization. If orders \( q^a \) and \( q^b \) are issued, then \( \theta \) and \( (1 - \theta) \) proportions of the outputs, respectively, are delivered. Thus, the delivery quantity is \( \theta(K_j \wedge q^a) + (1 - \theta)(K_j \wedge q^b). \) It turns out that
the resulting profit from the second alternative is weakly lower than that from the first regardless of the choice of \( (q^a, q^b) \). In other words, ordering from a single source is better than splitting the order between two identical sources with a perfect, positive correlation. The concavity of \( V_t(x) \) is derived by inductively repeating this argument for all suppliers.

Theorem 1 extends its counterpart in Ciarallo et al. (1994). We shall note that the approach developed by Ciarallo et al. (1994) cannot be generalized to treat our problem. In analyzing their model, they first identify an optimal base-stock policy and then obtain the concavity of the value function by applying this simple policy. In contrast, we establish Theorem 1 without much knowledge of the optimal policy.

Before ending this section, we state two additional properties of the optimal profit function implied by demand and supply uncertainties.

**Proposition 1.** \( V'_t(x) \leq R'(d^*(x)) \) whenever the derivatives exit. The equality holds when \( A = 1 \).

It is well known that a concave function can have at most a finite number of nondifferentiable points and thus the condition imposed in Proposition 1 is not restrictive. With only an additive demand noise (i.e., \( A = 1 \)) the marginal profit of having one extra unit of inventory is simply the corresponding marginal revenue. The optimal pricing decision works effectively in the sense that any incremental revenue earned from inventory is directly translated into an incremental profit. A random \( A \), however, introduces a gap between the marginal profit and the marginal revenue of the inventory.

The next result indicates that an increased capacity or a decreased capacity uncertainty can lead to an increased profit. These properties, though intuitive, are not trivial in view of the nonconcave objective function.

**Proposition 2.** Consider two otherwise identical systems with the capacities \( \mathbf{K}^a = \{K^n_1, \ldots, K^n_n\} \) and \( \mathbf{K}^b = \{K^n_1, \ldots, K^n_n\} \), respectively.

(i) If \( \mathbf{K}^a \) is larger than \( \mathbf{K}^b \) in the stochastic order, then \( V^a_t(x) \geq V^b_t(x) \) for each \( x \) and \( t \).

(ii) If \( \mathbf{K}^a \) is larger than \( \mathbf{K}^b \) in the convex order, then \( V^a_t(x) \leq V^b_t(x) \) for each \( x \) and \( t \).

### 4. The Optimal Policy

In this section, we derive the optimal procurement and pricing decisions based on the properties obtained in the previous section. To highlight the effect of supply uncertainty, we first treat the problem with deterministic capacities in section 4.1, where we generalize the notion of the base stock list price policy for single-supplier settings to multiple-supplier settings. In section 4.2, we show how the consideration of random capacities leads to a different policy.

#### 4.1. Reliable Supplies

This section considers a special case in which the suppliers’ capacities are known upon ordering. This model applies to two situations. First, each supplier guarantees a delivery amount up to a certain limit via some long-term capacity agreement with the firm under consideration. In this case, supply capacities are specified at the beginning of the planning horizon. Second, the firm can quote from each supplier a delivery limit when placing procurement orders. In this case, the supply capacities may fluctuate over time and may not be predictable in advance (see Chao et al. [2008] for a single-supplier version of this problem). In particular, one can model such supply streams as independent Markov-modulated processes with the state of the system being the current capacity levels. In this model, the capacities for each period is observed at the beginning of that period when placing the corresponding orders and the capacities for future periods are random. The structural properties derived in section 4.1 apply to this model since all of our formal results can be easily generalized under Markovian supply processes. In both situations, the optimal policy is characterized as follows.

**Theorem 2.** When \( K_j = k_j \), \( j \in \mathcal{N} \), is observed at the beginning of period \( t \), there exist a base-stock level, \( \bar{y}_j \), and a list price, \( \bar{p}_j = \mu^\text{mc}_j(d_j) \), for supplier \( j \) such that the optimal order quantity for supplier \( j \) in period \( t \) is

\[
q^*_j(x) = \begin{cases} 
  k_j & \text{if } x \leq \bar{y}_j - s_j \\
  \bar{y}_j - x - s_{j-1} & \text{if } \bar{y}_j - s_j < x \leq \bar{y}_j - s_{j-1} \\
  0 & \text{if } x > \bar{y}_j - s_{j-1},
\end{cases}
\]

and the optimal average demand in period \( t \) is

\[
d^*(x) = \begin{cases} 
  \tilde{d}(x + s_n) & \text{if } x \leq \bar{y}_n - s_n \\
  \tilde{d}_j & \text{if } \bar{y}_j - s_j < x \leq \bar{y}_j - s_{j-1}, j \in \mathcal{N} \\
  \tilde{d}(x) & \text{if } x > \bar{y}_1,
\end{cases}
\]

where \( s_n = 0, s_j = \sum_{i<j} k_i \) and

\[
\tilde{d}(y) = \arg\max_{d \in \mathcal{D}} \{R(d) + E[L_s(y - Ad)]\}. 
\]

Furthermore, \( \bar{y}_1 \geq \cdots \geq \bar{y}_n \) and \( \tilde{d}_1 \geq \cdots \geq \tilde{d}_n \).

To establish Theorem 2, we first recognize that it is suboptimal to order from an expensive supplier before the capacities of cheap suppliers are used up.
This observation allows us to examine one supplier at a time in the sequence of their cost rankings. Furthermore, the prior-order inventory position of supplier \(j\) can be defined as \(x + s_j-1\) and the associated post-order inventory position becomes \(x + s_j-1 + q_j\). We then observe that the profit function is separable in these two inventory positions and thus an optimal base-stock level can be obtained. Finally, we note that the optimal price depends on the inventory level and the orders only via their sum, that is, the available stock to meet the demand. This gives rise to Equation (5) and the optimal list price corresponding to the base-stock level.

The policy described in Theorem 2 represents a generalization of the base stock list price policy (Chao et al. 2008; Federgruen and Heching 1998) for single-supplier problems. When multiple reliable suppliers are involved, it is optimal to follow the multilevel base stock list price policy demonstrated in Figure 1. In particular, there is an optimal base-stock level, \(y_j\), and an optimal list price, \(p_{opt}^j(d_j)\), associated with each supplier \(j\). When the capacities of cheaper sources (e.g., suppliers \(\{1,2,\ldots,j-1\}\)) are exhausted, we may order from supplier \(j\), depending on his base-stock level and prior-order inventory position. If the former is below the latter, that is, \(y_j \leq x + s_j-1\), then no order is placed and the price \(p_{opt}^j(d(x + s_j-1))\) is charged. If, however, \(y_j > x + s_j-1\), then an order is issued to bring the post-order inventory level up to \(y_j\) or \(x + s_j\) whichever is smaller. In other words, \(x_j^* \equiv y_j - s_j-1\) constitutes the reorder point for supplier \(j\) so that an order is issued to him if and only if the inventory level is below this point. Whenever the available stock to meet demand equals supplier \(j\)'s base-stock level \(y_j\), the corresponding list price, \(p_{opt}^j(d_j)\), is charged.

\[ M_t(y) - M_t(y - \delta) \geq c_j \geq M_t(y + \delta) - M_t(y), \quad (6) \]

for arbitrarily small \(\delta\) and \(d_j = \hat{d}(y_j)\).

Theorem 3 also suggests a relation between the base-stock level and the marginal value of inventory. Since there is a one-to-one correspondence between the base-stock level, \(y_j\), and the reorder point, \(x_j^* \equiv y_j - s_j-1\), relation (6) implies that \(x_j^*\) satisfies

\[ V_t(x_j^*) - V_t(x_j^* - \delta) \geq c_j \geq V_t(x_j^* + \delta) - V_t(x_j^*), \quad (7) \]

for any \(\delta > 0\). This relation implies that an order is issued to supplier \(j\) if and only if the marginal value of inventory is above \(c_j\) (recall that \(V_t\) is concave by Theorem 1). Similar observations have been obtained in previous studies of procurement policies in various inventory problems (e.g., Anupindi and Akella 1993, Federgruen and Heching 1999, Federgruen and Yang 2009, Feng 2010, Li and Zheng 2006). As we will show in our analysis in Section 4.2, this is no longer true under random supply capacities.

### 4.2. Unreliable Suppliers

Next we consider the general model with random supply capacities that are only known upon order deliveries. Unfortunately, the multilevel base stock list price policy derived in Theorem 2 fails to be...
optimal in this case even for $n = 1$ (Feng 2010). Moreover, the optimal policy may reveal unintuitive behaviors and thus Corollary 1 also fails to hold. To obtain some insights, we first examine the relation between the order quantities and the marginal value of the inventory.

**Lemma 2.** Let $q_i(x)$ denote the smallest unconstrained maximizer of $f_i(x; q, d)$ when $q_i = q_i^*(x)$ for $i \in N \setminus \{j\}$ and $d = d^*(x)$. For a small enough $\delta > 0$,

$$
\begin{aligned}
V_i(x) - V_i(x - \delta) &\leq c_j &\text{if } q_j(x) < 0, \\
\frac{V_i(x + \delta) - V_i(x)}{\delta} &\geq c_j &\text{if } q_j(x) > 0.
\end{aligned}
$$

The inequality in the second relation is strict if $G_j(\delta) > 0$ for arbitrarily small $\delta > 0$.

In deriving Lemma 2, we allow for a negative order quantity from supplier $j$ (i.e., relax the constraint $q_j(x) \geq 0$). If the resulting unconstrained order $q_j(x)$ is nonpositive, then $q_j^*(x) = 0$. If $q_j(x)$ is positive, then $q_j^*(x) = q_j(x)$. According to Equation (8), a negative $q_j(x)$ implies a marginal value of inventory lower than $c_j$, and a positive $q_j(x)$ indicates a marginal value of inventory higher than $c_j$. These observations are consistent with our earlier discussion of Theorem 3 and Equation (7) for reliable supplies. The difference is that Equation (8) does not specify the case when $q_j^*(x) = q_j(x) = 0$. Our analysis of reliable supplies suggests that $q_j(x)$ corresponds to a marginal value of increasing inventory strictly lower than $c_j$ and thus a unique threshold (i.e., the reorder point) can be determined. Under random supplies, however, these results no longer hold as illustrated by the example in Figure 2. Specifically, it is not always optimal to order from a supplier when the marginal value of inventory exceeds the procurement cost of that supplier. Consequently, a reorder-point policy is generally suboptimal under unreliable supplies.

Nevertheless, the trajectory of the optimal ordering quantity depicted in Figure 2 is close to a reorder-point policy. In particular, we can treat $x^*$ as the reorder point. When the inventory level is above $x^*$, no order is issued to the supplier. When the inventory level is below $x^*$, a positive order is placed except for a set of discrete points. In other words, it is impossible to have $q_j^*(x) > 0$ over an interval $[x_1, x_2]$ with $x_1 < x_2 < x^*$. This policy, termed the near reorder point policy, is formally established in the next theorem.

**Theorem 4.** There exists a reorder point $x_j^*$ for supplier $j$, $j \in N$, such that

$$
q_j^*(x) \begin{cases} 
0 & \text{if } x \in (x_j^*, \infty) \cup S_j, \\
> 0 & \text{if } x \in (-\infty, x_j^*) \setminus S_j,
\end{cases}
$$

where $S_j$ has Lebesgue measure of zero and $x > x_j^*$ for any $x \in S_j$. Furthermore, if either

(i) $R(d)$ is differentiable and $Ad + B$ has a continuous distribution for each $d$ or

(ii) $\lim_{\delta \to 0} G_j(\delta) = 0$,

then $S_j$ is empty and $x_j^*$ satisfies Equation (7). When $S_j$ is empty for each $j \in N$, $x_1^* \geq x_2^* \geq \cdots \geq x_n^*$.

In general, the reorder point $x_j^*$ derived in Theorem 4 is lower than the level satisfying Equation (7); see also Figure 2, in which $x^*$ is lower than the dashed line. They coincide when either the optimal profit function is differentiable (condition (i)) or supplier $j$ offers a minimum delivery guarantee (condition (ii)). Under these conditions, the optimal ordering policy becomes a strict reorder-point policy, which calls for a

![Figure 2 An Example of the Near Reorder-Point Policy](image-url)

**Notes:** $n = 1, T = 1, c = 8, H(x) = \max\{x, 0\} + 30 \max\{-x, 0\}, \mu_0^\infty(d) = 20 - 0.18d$, and $B = 0$. The demand component $A$ satisfies $Pr(A = 0.5l) = 0.2$ for $l = 0, 1, 2, 3, 4$. The capacity $K$ satisfies $Pr(K = 0) = 0.5, Pr(K = 1) = 0.1, Pr(K = 10) = 0.3$, and $Pr(K = 40) = 0.1$. 


positive order whenever the inventory level is below the reorder point.

Theorem 4 also states that when a strict reorder-point policy is followed for each supplier, a higher reorder point is set for a supplier with a lower cost. In other words, the cost-based supplier selection criterion (i.e., always ordering from the cheapest supplier first) should be deployed. This leads to our next result.

**Corollary 2.** For \( j < i \), \( q_j^*(x) = 0 \) implies \( q_i^*(x) = 0 \) when either

(i) \( R(d) \) is differentiable and \( Ad + B \) has a continuous distribution for each \( d \) or

(ii) \( \lim_{\delta \to 0} G_j(\delta) = 0 \).

Under condition (i), when the demand has a fine granularity (e.g., under retailing), or under condition (ii), when the chance of no delivery is tiny, it is always more profitable to order from supplier \( j \) than to order exclusively from a more expensive supplier. This is because a small order from the cheaper source allows for cost saving without adding much delivery risk. To see the intuition, we note that the delivery quantity \( q_j \wedge K_j \) is increasing in \( q_j \) in the dispersive order (see Shaked and Shanthikumar 2006), which, in particular, suggests an increasing variance of \( q_j \wedge K_j \). In the extreme case, when \( q_j \) approaches zero, the variability of \( q_j \wedge K_j \) is negligible. Thus, if a more expensive source (i.e., supplier \( i \)) is used, it is always profitable to order a sufficiently small amount from a cheaper source (i.e., supplier \( j \)) regardless how unreliable the latter is.

When the demand becomes highly discrete (e.g., under wholesaling) and the no-delivery probability is significant, an exclusive order from the more expensive supplier may be desired. This is demonstrated by an example in Figure 3. If the demand distribution were continuous, then Equation (7) implies that the reorder point of supplier 1 is strictly higher than that of supplier 2. Because, however, the demand is always a multiple of five, the optimal ordering decisions must match the granularity of the demand. This is reflected by the fact that the reorder points and \( x + q_1^*(x) + q_2^*(x) \) are always multiples of five. We further note that \( x + q_1^*(x) + q_2^*(x) = 15 \) whenever an order is issued and the maximum capacities of both suppliers are not exhausted (i.e., \( x \in [-85,15] \)).

In other words, the optimal policy should allocate orders between the suppliers to achieve a target stock level of 15, the maximum demand. When the inventory level is close to 15 (i.e., \( x \in [10,15] \)), it is optimal to order only from supplier 2. This is because supplier 1, though having a cost advantage, is much more unreliable than supplier 2. Ordering from supplier 1 induces a high risk of no-delivery and a high risk of shortage. Instead, if ordering \( q_2^* = 15 - x \) from supplier 2, there is a good chance to obtain 15 units to meet the maximum demand. This also eliminates the need to order an arbitrarily small amount from supplier 1 because the chance of selling it is small.

It is worth mentioning that most of previous studies on multiple unreliable supplies (e.g., Anupindi and Akella 1993, Dada et al. 2007, Federgruen and Yang 2009) assume condition (i) and obtain the cost-based supplier-selection criterion. Swaminathan and Shanthikumar (1999) provide a counterexample with discrete demand distribution when allowing for only integral order quantities. Our example in Figure 3 suggests that the cost-based selection criterion can be suboptimal even with continuous order quantities.

We shall remark that the nonmonotone response of the order quantity to changes in the inventory level is not due to the failure of conditions (i) and (ii). In general, when \( A \) is random (see Feng 2010) or when \( A = 1 \) and \( n \geq 3 \), the optimal order quantities are not monotone with respect to the inventory level. Even though the optimal ordering quantities reveal a complex behavior, the optimal price, to our surprise, is monotone in the inventory level.

**Theorem 5.** \( d'(x) \) is increasing in \( x \).

5. Evaluating the Benefits of Dynamic Pricing and Supply Diversification

We have analyzed the optimal policy for our model that integrates dynamic price adjustments and
diversified procurement options. Both dynamic pricing and supply diversifications have shown to be effective strategies to mitigate demand and supply risks in many applications. In practice, implementing either strategy can involve substantial costs due to changes in business process, increased administrative efforts, and added coordination complexities. It is therefore important to understand the economic implications of these strategies before implementation. The purpose of this section is to understand how the benefits derived from dynamic pricing and supply diversification depend on system characteristics and when a firm should adopt none, either, or both of these strategies.

For that, we measure the *value of dynamic pricing* as the percentage profit improvement,

$$\lambda^p = \frac{V^d - V^s}{V^s} \times 100\%,$$

by comparing the optimal profits under static ($V^s$) and dynamic ($V^d$) pricing. Also, we measure the *value of supply diversification* as the percentage profit improvement,

$$\lambda_n = \frac{V^n - V^1}{V^1} \times 100\%,$$

by comparing the optimal profits with one ($V^1$) and $n$ ($V^n$) suppliers. In our subsequent discussions, we first examine the individual effects of supply diversification (section 5.1) and dynamic pricing (section 5.2), and then make comparisons (section 5.3).

In the base setting, we solve a 15-period problem with a discount factor of $\alpha = 0.95$. There is a single supplier with a unit procurement cost of $c = 2$. We take $H(x) = h \max(x|0) + s \max(-x|0)$ with $h = 1$ and $s$ determined via the relation $\pi = (s - (1 - x)c)/\left(h + s\right) = 0.95$. Note that $\pi$ represents the optimal service level (i.e., the probability of meeting demand) when the supply is uncapacitated. The component $A$ is normal with mean 1 and standard deviation $\sigma_A = 0.3$, and the component $B$ is normal with mean 0 and standard deviation $\sigma_B = 6$. We consider a linear demand $\mu_D(p) = a - bp$ with $a = 70$ and $b = 2$. The set of feasible prices is given by $\{p: A\mu_D(p) + B \geq 0 \text{ a.s.}\}$. The supply capacity, $K$, is a normal random variable with mean $\mu_K = 20$ and standard deviation $\sigma_K = 6$. We focus on the case of limited supply capacity. Previous studies suggest that when there is an ample supply capacity, the system can be managed efficiently by following a base stock list price policy. This is because it is always possible to bring the stocks to the desired

### Table 1: The Value of Supply Diversification ($\lambda_n$)

<table>
<thead>
<tr>
<th>Profile</th>
<th>Pricing strategy</th>
<th>Suppliers</th>
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<th>Add 2</th>
<th>Add 1</th>
<th>Add 2</th>
<th>Add 1</th>
<th>Add 2</th>
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<td></td>
<td>Static</td>
<td>Dynamic</td>
<td>Static</td>
<td>Dynamic</td>
<td>Static</td>
<td>Dynamic</td>
<td>Static</td>
<td>Dynamic</td>
<td>Static</td>
<td>Dynamic</td>
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<td>$\pi = 0.95$</td>
<td>$\alpha = 2$</td>
<td>$\lambda_2$</td>
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<td>39.35%</td>
<td>23.39%</td>
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<td>0.04%</td>
<td>0.00%</td>
<td>0.00%</td>
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<td>0.16%</td>
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</tr>
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<td>$\pi = 0.95$</td>
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<td>25.57%</td>
<td>28.23%</td>
<td>1.13%</td>
<td>3.28%</td>
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</tr>
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<td>50.38%</td>
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</tr>
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<td>24.08%</td>
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<td>1.13%</td>
<td>3.27%</td>
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<td>3.28%</td>
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</tr>
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<td>3.28%</td>
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<td>2.42%</td>
</tr>
<tr>
<td>$\pi = 0.95$</td>
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<td>3.28%</td>
<td>0.85%</td>
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</tr>
<tr>
<td>$\pi = 0.95$</td>
<td>$\alpha = 20$</td>
<td>$\lambda_{10}$</td>
<td>46.88%</td>
<td>51.38%</td>
<td>27.35%</td>
<td>29.63%</td>
<td>1.38%</td>
<td>3.64%</td>
<td>1.30%</td>
<td>2.44%</td>
</tr>
<tr>
<td>$\pi = 0.95$</td>
<td>$\alpha = 25$</td>
<td>$\lambda_{11}$</td>
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<td>41.92%</td>
<td>24.08%</td>
<td>26.14%</td>
<td>1.13%</td>
<td>3.27%</td>
<td>0.85%</td>
<td>2.43%</td>
</tr>
<tr>
<td>$\pi = 0.95$</td>
<td>$\alpha = 30$</td>
<td>$\lambda_{12}$</td>
<td>40.32%</td>
<td>44.46%</td>
<td>24.84%</td>
<td>27.14%</td>
<td>1.14%</td>
<td>3.28%</td>
<td>0.85%</td>
<td>2.43%</td>
</tr>
<tr>
<td>$\pi = 0.95$</td>
<td>$\alpha = 35$</td>
<td>$\lambda_{13}$</td>
<td>42.46%</td>
<td>47.29%</td>
<td>25.57%</td>
<td>28.23%</td>
<td>1.13%</td>
<td>3.28%</td>
<td>0.85%</td>
<td>2.42%</td>
</tr>
<tr>
<td>$\pi = 0.95$</td>
<td>$\alpha = 40$</td>
<td>$\lambda_{14}$</td>
<td>43.81%</td>
<td>47.39%</td>
<td>26.26%</td>
<td>28.50%</td>
<td>1.13%</td>
<td>3.28%</td>
<td>0.85%</td>
<td>2.42%</td>
</tr>
</tbody>
</table>

* The capacity of each supplier is $K_j \sim N(\mu_K, \sigma^2_K), j \in \mathcal{N}$. The total supply capacity is $\sum_{j \in \mathcal{N}} K_j \sim N(\mu_K, \sigma^2_K)$.

1. The capacity of each supplier is $K_j \sim N(\mu_K/n, \sigma^2_K/n), j \in \mathcal{N}$. The total supply capacity is $\sum_{j \in \mathcal{N}} K_j \sim N(\mu_K, \sigma^2_K/n)$.

2. The capacity of each supplier is $K_j \sim N(\mu_K, \sigma^2_K/n^2), j \in \mathcal{N}$. The total supply capacity is $\sum_{j \in \mathcal{N}} K_j \sim N(\mu_K, \sigma^2_K/n^2)$.
level and charge a fixed price. When this happens, there is not much need to adjust price dynamically or to add additional suppliers with the same cost. Hence, we set the average capacity, \( \mu_K = 20 \), to be lower than the riskless demand, \( d = \arg \max_R \{ |R(d) - cd| \} = 33 \), the optimal average demand when no uncertainty is involved.

### 5.1. The Value of Supply Diversification

Table 1 presents the value of supply diversification, for which we compute the profit improvement from adding one and two additional suppliers. We consider three different ways to diversify the supplier portfolio.

Under Profile 1 (columns 2–5), each added supplier has the same procurement cost (\( c_i = c \)) and capacity (\( K_i = \beta d \)) as the existing supplier. When the number of suppliers increases, the average total supply capacity increases and the coefficient of variation of the total supply capacity decreases. We observe that adding a second supplier leads to a significant profit improvement. In particular, \( \lambda_2 > 37\% \) under static pricing (column 2) and \( \lambda_2 > 23\% \) under dynamic pricing (column 4). When a third supplier is included, however, the additional gain reduces to 2–5% under both static pricing (the difference between columns 3 and 2) and dynamic pricing (the difference between columns 5 and 4). In other words, the benefit from increasing supply capacity is subject to rapidly diminishing returns.

Under Profile 2 (columns 6–9), we keep the mean and the variance of the total supply capacity fixed. When the number of supplier increases, each individual supplier’s capacity has a decreased mean (\( \mu_K/n \)), a decreased variance (\( \sigma_K^2/n \)), and an increased coefficient of variation (\( \sqrt{\sigma_K^2/\mu_K} \)). We observe that the number of suppliers does not have a significant effect on the profit. Moreover, when \( \sigma_K \) is low, adding an additional supplier may even lead to a slight profit reduction.

Under Profile 3 (columns 10–13), we keep the mean of the total supply capacity and the coefficient of variation of individual supply capacity fixed. When additional suppliers are added to the system, each supplier’s capacity has a decreased mean (\( \mu_K/n \)) and a decreased variance (\( \sigma_K^2/n \)). Also, the variance of the total supply capacity (\( \sigma_K/n \)) decreases. We observe that the profit improvement is slightly larger than that under Profile 2 and significantly smaller than that under Profile 1.

In summary, Table 1 suggests that multiple sourcing, compared to single sourcing, may lead to dramatic profit improvement when the additional suppliers bring extra capacity into the system. Under a fixed average supply capacity, however, changing suppliers’ risk profile via diversification does not have as significant an impact on the profit.

### 5.2. The Value of Dynamic Pricing

In Table 2, we compute the value of dynamic pricing, \( \lambda^\prime \), under different supplier profiles. Under single sourcing (column 2), a significant gain

<table>
<thead>
<tr>
<th>Profile 2(^\dag)</th>
<th>Profile 3(^\dag)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma_K = 2 )</td>
<td>( \sigma_K = 4 )</td>
</tr>
<tr>
<td>( \sigma_K = 6 )</td>
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<tr>
<td>( \sigma_K = 4 )</td>
<td>( \sigma_K = 6 )</td>
</tr>
</tbody>
</table>

\(^{\ast}\) The capacity of each supplier is \( K_j \sim N(\mu_K, \sigma_K^2) \), \( j \in N \). The total supply capacity is \( \sum_{j \in N} K_j \sim N(n \mu_K, n \sigma_K^2) \).

\(^{\dag}\) The capacity of each supplier is \( K_j \sim N(\mu_K/n, \sigma_K^2/n^2) \), \( j \in N \). The total supply capacity is \( \sum_{j \in N} K_j \sim N(\mu_K, \sigma_K^2) \).

\(^{\ddag}\) The capacity of each supplier is \( K_j \sim N(\mu_K/n, \sigma_K^2/n^2) \), \( j \in N \). The total supply capacity is \( \sum_{j \in N} K_j \sim N(\mu_K, \sigma_K^2) \).
Comparing Tables 1 and 2, we find that the values of dynamic pricing, as opposed to static pricing. When one or two additional suppliers are included under Profile 1 (columns 3–4), the advantage of dynamic pricing compared to static pricing is significantly reduced. This is because multiple sourcing relaxes capacity restrictions and increases procurement flexibility. Consequently, inventory adjustment via procurement orders can effectively absorb the fluctuations in demand and supply processes, reducing the need to change price.

In contrast, under Profile 2 (columns 5–6) and Profile 3 (columns 7–8), a significant profit increase is obtained in each instance. While the gain decreases when adding additional suppliers, the differences are very small. In other words, with a fixed average total capacity, the value of dynamic pricing is robust with respect to the supply-risk profile.

In general, dynamic pricing and supply diversification reveal a substitutable relationship. The profit improvement from adding suppliers is greater under static pricing than under dynamic pricing in Table 1, and the value of dynamic pricing decreases as the number of suppliers increases in Table 2. Furthermore, the substitutable effect is much stronger (i.e., a smaller incremental value of using two strategies compared to using just one) under Profile 1 than under Profiles 2 and 3.

5.3. Comparisons

Comparing Tables 1 and 2, we find that the values of supply diversification and dynamic pricing reveal several similar patterns with respect to model inputs. When supply becomes more random (i.e., \( \sigma_k \) increases), both \( \lambda_k \) and \( \lambda^p \) are larger. This suggests that both strategies can effectively mitigate supply uncertainty.

We also observe that with an enhanced ability to adjust (i.e., a decreased \( \epsilon \)) or hold (i.e., an increased \( \pi \)), both strategies lead to increased profit improvements. When it is cheap to buy and carry inventory, the supply capacity seems even more restricted. It is thus impossible to maintain a desired stock level and balance inventory fluctuations over time with a static price and a single supplier. Moreover, when \( \pi \) approaches one, \( \lambda_k \) and \( \lambda^p \) increase dramatically. This suggests that firms offering a high service level (recall \( \pi \) represents the optimal probability of meeting demand in uncapacitated systems) can profit from implementing either strategy. It is worth mentioning that Amihud and Mendelson (1983) report a contrary observation from their model of a single, uncapacitated supplier. They show that the value of dynamic pricing increases in the procurement and holding costs. Our result suggests that capacity restriction is an important factor to consider when evaluating dynamic pricing strategies.

The values of supply diversification and dynamic pricing also exhibit several important differences. While dynamic pricing can effectively mitigate both additive and multiplicative demand uncertainties (i.e., \( \lambda^p \) increases in \( \sigma_B \) and \( \sigma_A \)), supply diversification is only useful to cope with additive demand uncertainty (i.e., \( \lambda_k \) increases in \( \sigma_B \)) and not multiplicative demand uncertainty (i.e., \( \lambda_k \) decreases in \( \sigma_A \)). This is because diversifying the supply profile leads to an increased total supply capacity (Profile 1), a reduced variance for individual supply capacity (Profile 2), or a reduced variance for total supply capacity (Profile 3). In any case, the risk of not obtaining ordered units is reduced, allowing the firm to sell more. A larger sales volume and thus a larger average demand amplify the uncertainty associated with the multiplicative demand component via \( Ad \), which, in turn, induces a lower profit. Consequently, the benefit from supply diversification decreases in \( \sigma_A \).

We observe that \( \lambda^p \) increases as the demand is more price sensitive (i.e., \( b \) is larger). This is because price adjustments become more effective for a larger \( b \). We also note that \( \lambda_k \) increases in \( b \) under static pricing due to the reduced influence of the multiplicative demand noise. In particular, the average demand, \( d = \mu_\pi(p) = a - bp \), decreases in \( b \) for a fixed price \( p \). This indicates a reduced uncertainty associated with the multiplicative demand component via \( Ad \). Under dynamic pricing, however, because the optimal price \( p \) depends on inventory level and may increase or decrease, the response of \( \lambda_k \) is not clear.

We further observe different impacts of the planning horizon on the values of dynamic pricing and supply diversification. Inventory-based price revisions can effectively balance the stock level over time when the supply capacity restricts the procurement order, bringing the inventory to the desired level. Consequently, the profit improvement obtained from price adjustments accumulates over time, leading to a long-term benefit. The value of supply diversification, in contrast, is most significant in the short run. This is because the major advantage of multiple sourcing comes from the ability to mitigate supply risk by splitting a large bet into smaller ones. This can also be achieved by spreading the order quantities across multiple periods. As a result, the need to introduce additional suppliers reduces as the planning horizon increases.

6. Conclusions

We have analyzed a model that integrates multiple sourcing and dynamic pricing under supply and demand uncertainties. Our study provides both theoretical and managerial contributions to the literature.

From the theoretical perspective, we analyze a nonconcave dynamic programming model and provide
analytical characterizations of the optimal policy. For deterministic supply, a multilevel base stock list price policy is derived, which represents a generalization of the base stock list price policy in single-supplier settings (Chao et al. 2008, Federgruen and Heching 1999). For random supply, our results indicate that the optimal policy is neither a simple threshold policy (Anupindi and Akella 1993, Dada et al. 2007, Federgruen and Yang 2009) nor a complex one without any structure (Li and Zheng 2006). Instead, the optimal ordering decisions follow a near reorder-point policy and the optimal price is decreasing in the inventory level. Moreover, in our model, the comparison between the marginal value of inventory and the procurement cost alone is not sufficient to determine whether or not to place an order, as it often is in previous studies. Consequently, it is generally suboptimal to select suppliers based solely on their procurement costs. We also derive conditions under which a reorder-point policy and a cost-based supplier selection are optimal.

From the managerial perspective, our study provides useful insights to practice. While both supply diversification and dynamic pricing can improve profit, they exhibit substitutive relations. Supplier diversification is less valuable with price revisions than without, and dynamic pricing is less useful under multiple sourcing than under single sourcing. Dynamic pricing yields a robust benefit under limited supply, while supplier diversification is only valuable if it can relax the capacity limit. Both strategies can become more useful when supply is more uncertain, buying or holding inventory is cheaper, or the price sensitivity of demand is stronger. There are, however, fundamental differences between these strategies. While dynamic pricing can effectively mitigate demand risk, supply diversification may yield less value with a higher demand uncertainty. More interestingly, dynamic pricing can generate a long-term benefit, but supply diversification is more effective in a shorter planning horizon.

The problem analyzed in this study can be further explored in a number of directions. We have focused on deriving the optimal policy, and evaluating the benefits from dynamic pricing and supply diversification under the optimal policy. Design of simple and efficient heuristic policies would facilitate implementations of both strategies in practice, as well as quantifying their business impact. The challenge though is that the profit function under a suboptimal policy may not be concave and thus approximations of the profit function need to be explored. Also, this article, as well as the existing literature, is restricted to the case of one-period procurement lead time. Impacts of lead-time differential on supplier selection, order allocation, and demand shaping are important practical consider-
To see that, we have two cases to consider, depending on the realization $k_j$ of $K_j$:

Case 1: $\tilde{q}_j \leq k_j$. By the definition of $\tilde{q}_j$, we obtain

$$
\pi'[\theta(\tilde{q}_j(x) \land k_j) + \bar{\theta}(\tilde{q}_j(x) \land k_j)] \leq \pi'(\tilde{q}_j) = \pi'(\tilde{q}_j \land k_j).
$$

Case 2: $\tilde{q}_j > k_j$. We have $\theta(\tilde{q}_j(x_1) \land k_j) + \bar{\theta}(\tilde{q}_j(x_2) \land k_j) \leq k_j < \tilde{q}_j$, Since $\pi'(\cdot)$ is concave and $\tilde{q}_j$ is a maximizer, we deduce that

$$
\pi'[\theta(\tilde{q}_j(x_1) \land k_j) + \bar{\theta}(\tilde{q}_j(x_2) \land k_j)] \leq \pi'(k_j) = \pi'(\tilde{q}_j \land k_j).
$$

Combining the two cases gives rise to Equation (A3). Together with Equation (A2), we conclude

$$
E[\pi'(\tilde{q}_j \land K_j)] \leq E[\pi'(\tilde{q}_{j+1} \land K_{j+1})],
$$

for $j = 1, \cdots, n - 1$.

Now we note that the right-hand side of Equation (A1) can be written in terms of $\pi'$ as follows

$$
\partial W_i(x_1, d(x_1)) + \bar{\partial} W_i(x_2, d(x_2)) \leq R(d(x)) + II(\theta(\tilde{q}_1(x_1) \land K_1) + \bar{\theta}(\tilde{q}_2(x_2) \land K_2))
$$

$$
= R(d(x)) + E[\pi'(\theta(\tilde{q}_1(x_1) \land K_1) + \bar{\theta}(\tilde{q}_2(x_2) \land K_2))]
$$

$$
\leq R(d(x)) + E[\pi'(\tilde{q}_1 \land K_1)]
$$

$$
\leq R(d(x)) + E[\pi'(\tilde{q}_n \land K_n)]
$$

$$
= R(d(x)) + E[II(\tilde{q}_1 \land K_1, \ldots, \tilde{q}_n \land K_n)].
$$

Furthermore, by the definition of $\tilde{q}(x)$, we have

$$
W_i(x, d(x)) = R(d(x)) + E[II(\tilde{q}_1(x) \land K_1, \ldots, \tilde{q}_n(x) \land K_n)]
$$

$$
\geq R(d(x)) + E[II(\tilde{q}_1 \land K_1, \ldots, \tilde{q}_n \land K_n)].
$$

We deduce

$$
\partial W_i(x_1, d(x_1)) + \bar{\partial} W_i(x_2, d(x_2)) \leq W_i(x, d(x)).
$$

Thus, $W_i(x, x + \beta)$ is concave in $x$. By Lemma (4-i) stated in Appendix S1, we conclude that $W_i(x, d)$ is jointly concave in $(x, d)$ and thus $V_i(x)$ is concave in $x$. \(\square\)

**Proof of Lemma 2.** Define

$$
f_i(q) = R(d^*(x)) - \sum_{i \in I \setminus \{i\}} c_i E[\bar{q}_i^*(x) \land K_i] - c_j q
$$

$$
+ E \left[ L_i \left( x + \sum_{i \in I \setminus \{i\}} q_i^*(x) \land K_i + q - Ad^*(x) \right) \right].
$$

(A4)

From Theorem 1, $f_2(q)$ is concave in $q$. If $\tilde{q}_j(x) < 0$, then $q_j^*(x) = 0$. Because $f_2(q)$ is concave and by definition, $\tilde{q}_j(x)$ is the smallest maximizer, we have $f_2(0) \leq f_2(-\delta)$ for $0 < \delta \leq -\tilde{q}_j(x)$. It follows

$$
c_j\delta \geq E \left[ L_i \left( x + \sum_{i \in I \setminus \{i\}} q_i^*(x) \land K_i - Ad^*(x) \right) \right]
$$

$$
- E \left[ L_i \left( x - \delta + \sum_{i \in I \setminus \{i\}} q_i^*(x) \land K_i - Ad^*(x) \right) \right]
$$

$$
= f_i(x, q^*(x), d^*(x)) - f_i(x - \delta, q^*(x), d^*(x))
$$

$$
\geq V_i(x) - V_i(x - \delta).
$$

The last inequality follows from the fact that $V_i(x - \delta)$ is the optimal profit. Thus, the first relation in Equation (8) is obtained.

Next we prove the second relation in Equation (8) by contradiction. Suppose it is not true for some $x$ satisfying $q_i^*(x) = \tilde{q}_j(x) > 0$. By the concavity of $V_i$, we must have $[V_i(x + \delta) - V_i(x)]/\delta < c_j$ for any $\delta > 0$. Then, for a given realization $k_j$ of $K_j$, we have

$$
c_j(q_i^*(x) \land k_j) > V_i(x + q_i^*(x) \land k_j) - V_i(x)
$$

$$
\geq R(d^*(x)) - \sum_{i \in I \setminus \{i\}} c_i E[q_i^*(x) \land K_i]
$$

$$
+ E \left[ L_i \left( x + q_i^*(x) \land k_j \right) \right]
$$

$$
+ \sum_{i \in I \setminus \{i\}} c_i E[q_i^*(x) \land K_i]
$$

$$
= c_j(\tilde{q}_j(x) \land k_j) + E \left[ L_i \left( x + q_i^*(x) \land k_j \right) \right]
$$

$$
+ \sum_{i \in I \setminus \{i\}} q_i^*(x) \land K_i - Ad^*(x)
$$

$$
\geq V_i(x) - V_i(x - \delta).
$$

The first inequality follows from our hypothesis and the second from the optimality of $V_i(x + q_i^*(x) \land k_j)$. Taking expectation with respect to $K_j$ on both sides of the above relation, we deduce that the left-hand side must equal the right-hand side. This leads to a contradiction and thus the second relation in Equation (8) must hold.

Finally, we check the case when $G(\epsilon) > 0$ for an arbitrarily small $\epsilon$. Define $\bar{k}_i = \inf\{k_j\}$ and $\bar{k}_i =$
inf\{k_j\}. Then, in this case, \( k_j = 0 \). Suppose that the result is not true and \( |V_i(x + \delta) - V_i(x)|/\delta \leq c_j \) for any \( \delta \). In other words, Equation (A5) holds with “>” replaced by “\( \geq \)”. Since the equality must hold in Equation (A5), the equality must hold for each realization of \( K_j = k_j \). In particular, \( |V_i(x + \delta) - V_i(x)|/\delta = c_j \) at \( \delta = q_j^*(x) \wedge k_j \) and \( \delta = q_j^*(x) \wedge k_j \). By the concavity of \( V_i \), and our hypotheses that \( |V_i(x + \delta) - V_i(x)|/\delta \leq c_j \) for any \( \delta > 0 \), we deduce that

\[
\frac{V_i(x + \delta) - V_i(x)}{\delta} = c_j, \quad \delta \in [0, \delta].
\]

(A6)

This implies that \( V_i'(\hat{x}) = c_i \) is well defined for \( \hat{x} \in (x, x + \delta) \). Applying the Envelope Theorem in Equation (2), we obtain

\[
c_j = V_i'(\hat{x})
\]

\[
\frac{\partial E}{\partial \hat{x}} \left( L_i \left( \hat{x} + q_j^*(\hat{x}) \wedge K_j + \sum_{i \in V \setminus \{j\}} q_i^*(\hat{x}) \wedge K_i - Ad^*(\hat{x}) \right) \right) = 0.
\]

(A7)

Furthermore, the first-order condition of \( \tilde{q}_j(\hat{x}) \) yields,

\[
- c_j \frac{\partial E}{\partial \hat{x}} \left( \hat{x} + q_j^*(\hat{x}) + \sum_{i \in V \setminus \{j\}} q_i^*(\hat{x}) \wedge K_i - Ad^*(\hat{x}) \right) = 0.
\]

Comparing the above two equations, we must have \( q_j^*(\hat{x}) = \hat{x} = k_j \). Since \( \hat{x} \) is arbitrarily chosen, we deduce that \( q_j^*(\hat{x}) = \hat{x} = 0 \) for any \( \hat{x} \in (x, x + \delta) \).

Because \( q_j^*(x) > 0 \) is the smallest maximizer of \( f_s(q) \) and \( f_s(q) \) is concave in \( q \), we must have

\[
f_s(q_j^*(x) \wedge k_j) \leq f_s(\hat{x}),
\]

(A8)

for each realization of \( K_j = k_j \). Also,

\[
c_j \frac{\partial E}{\partial \hat{x}} \left( x + q_j^*(\hat{x}) \wedge K_j - Ad^*(\hat{x}) \right)
\]

\[
\frac{\partial}{\partial \hat{x}} \left( L_i \left( \hat{x} + q_j^*(\hat{x}) \wedge K_j + \sum_{i \in V \setminus \{j\}} q_i^*(\hat{x}) \wedge K_i - Ad^*(\hat{x}) \right) \right) = 0.
\]

(A7)

To show that \( S \) has a Lebesgue measure of zero, we only need to prove that \( q_j^*(x) = 0 \) cannot be sustained over a nondegenerate interval containing \( \hat{x} \). That is, for an arbitrarily small \( \delta > 0 \), \( q_j^*(\hat{x} - \delta) = q_j^*(\hat{x} - \delta) > 0 \). Suppose that this is not true. Then, there exists an \( \epsilon > 0 \) such that \( q_j^*(x) \leq 0 \) and \( q_j^*(x) = 0 \) for any \( x \in [\hat{x} - \epsilon, \hat{x}] \). Since \( V_j(x) \) is concave in \( x \), it is differentiable with respect to \( x \) except for a finite number of points (Folland 1999). Therefore, there must be an \( x^0 \in (\hat{x} - \epsilon, \hat{x}) \) such that \( V_j(x) \) is differentiable at \( x = x^0 \). Thus, by the Envelope Theorem, we have

\[
V_j'(x^0) = \left. \frac{\partial}{\partial x} \left[ L_i \left( x + \sum_{i \in V \setminus \{j\}} q_i^*(x^0) \wedge K_i - Ad^*(x^0) \right) \right] \right|_{x=x^0} \leq c_j.
\]

(A10)

The last equation comes from the first-order condition of \( \tilde{q}_j(x^0) \leq 0 \). This contradicts (A9). Therefore, we cannot have \( q_j^*(x + \epsilon) > 0 \) for any \( \epsilon \in (0, \delta) \) and \( S \) has a Lebesgue measure of zero.

Next we check the conditions for a strict reorder-point policy. Under condition (i), the optimal profit function is differentiable. If \( \tilde{q}_j(x) < 0 \), the first relation in Equation (8) suggests \( V_j'(x) \leq c_j \). Also, when \( q_j^*(x) = \tilde{q}_j(x) = 0 \), we have
Thus, \( q_j(x) \leq 0 \) implies \( V_t(x) \leq c_j \). In other words, \( V_t(x) > c_j \) implies \( q_j(x) > 0 \). Likewise, from the second relation in Equation (8), \( V_t(x) < c_j \) implies \( q_j(x) \leq 0 \).

We need to check the case when \( V_t(x) = c_j \). We have

\[
\begin{align*}
    c_j &= V_t(x) = E \left[ I_t \left( x + \sum_{i \in I} q_i^*(x) \wedge K_i - Ad^*(x) \right) \right] \\
    &\geq E \left[ I_t \left( x + q_j^*(x) + \sum_{i \in I \setminus \{j\}} q_i^*(x) \wedge K_i - Ad^*(x) \right) \right] = c_j.
\end{align*}
\]

The above relation suggests that \( V_t(x) = c_j \) if and only if \( q_j^*(x) \leq k = \inf \{ K_i \} \). When this happens, it is optimal to order up to \( x^*_j \), which solves Equation (A11). Thus, \( x^*_j \) defines the reorder point.

Under condition (ii), we show that \( q_j(x) = 0 \) implies the first relation in Equation (7). When \( q_j^*(x) = q_j(x) = 0 \), we have

\[
\begin{align*}
    V_t(x) - V_t(x - \delta) &\leq R(d^*(x)) - \sum_{i \in I} c_i E[q_i^*(x) \wedge K_i] \\
    &+ E \left[ I_t \left( x + \sum_{i \in I} q_i^*(x) \wedge K_i - Ad^*(x) \right) \right] \\
    &- R(d^*(x)) + \sum_{i \in I} c_i E[q_i^*(x) \wedge K_i] + c_j E[\delta \wedge K_j] \\
    &- E \left[ I_t \left( x - \delta + \sum_{i \in I \setminus \{j\}} q_i^*(x) \wedge K_i + \delta \wedge K_j - Ad^*(x) \right) \right] \\
    &= c_j \delta + \int_0^\delta \left\{ c_i(k_i - \delta) \right\} dG_i(k_i).
\end{align*}
\]

The inequality follows from the optimality of \( V_t(x - \delta) \). Note that the second term in the last equation is of the order \( o(\delta) \). Hence, there exists a small enough \( \delta \) such that

\[
    V_t(x) - V_t(x - \delta) \leq c_j \delta.
\]

Together with Lemma 2, we deduce that \( q_j(x) \leq 0 \) implies \( V_t(x) - V_t(x - \delta) \leq c_j \delta \) for a small enough \( \delta > 0 \). If \( k = 0 \), the second inequality in Equation (7) is strict and thus the reorder point can be identified as the smallest \( x \) satisfying \( c_j > [V_t(x + \delta) - V_t(x)] / \delta \) for arbitrarily small \( \delta \). If \( k > 0 \), then we can analyze the interval of \( \delta \) over which \( V_t(x + \delta) - V_t(x) = c_j \delta \). Repeating our early argument under condition (i) for this interval leads to the result.

\[ \blacksquare \]

Remark 1. The analysis of our model can be extended to the case when \( A \) and \( B \) are correlated. In this case, we define

\[
    \hat{L}_t(y, a) = E[-H(y - B) + \alpha V_{t+1}(y - B) | A = a].
\]

The objective in Equation (3) can be rewritten as

\[
\begin{align*}
    J_t(x; q, d) &= R(d) - \sum_{j \in I} c_j E[q_j \wedge K_j] \\
    &+ \int \left[ \left( x + \sum_{j \in I} (q_j \wedge K_j) - Ad, A \right) \right] \frac{dG}{k_j}.
\end{align*}
\]

We can replace \( L_t \) by \( \hat{L}_t \) in our proofs and show that all the formal results derived continue to hold.

References


**Supporting Information**

Additional supporting information may be found in the online version of this article:

**Appendix S1.** Sourcing from Multiple Suppliers for Price-Dependent Demands.

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