



5 - 1:

Shares		Price	Market Value	β		
100,000	Alliance Gaming	4.000	\$400,000.	2.00	\$400,000 * 2.00 =	800,000.00
25,000	Burlington Northern	35.000	\$875,000.	0.92	\$875,000 * 0.92 =	805,000.00
25,000	Ameren Corp	25.000	\$625,000.	0.71	\$625,000 * 0.71 =	443,750.00
	Cash		\$100,000.	0.00	\$100,000 * 0.00 =	0.00
			<u>\$2,000,000.</u>			<u>2,048,750.00</u>
	TOTAL PORTFOLIO				\$2,048,750. / \$2,000,000. =	1.024375

- A. Cash has a $\beta = 0.00$. The return on cash, if any, is independent of conditions in the equity market.
- B. $\beta = 1.024$
- C. The portfolio β characterizes this portfolio as fairly close to the market. On average we would expect this portfolio to move with the market. We expect that when the market declines by 1% this portfolio would decline by 1.024%, but if the market advances 1% this portfolio would advance by 1.024%.



5 - 2:

The first thing to do is to write down the information you are given: The weighted average of the 20 stocks in the portfolio is 0.60. One of the twenty stocks has a $\beta = 0.35$. We do not know any of the other β s but we do know that each of the twenty stocks has a market value of \$100,000. Thus

$$\beta = 0.60 = \frac{\$100,000 (0.35) + \left(\sum_{i=2}^{20} \$100,000 \beta_i \right)}{\$2,000,000}$$

Simple algebra gives us

$$\left(\sum_{i=2}^{20} \$100,000 \beta_i \right) = 1,165,000$$

and we can put Caterpillar's β of 1.12 into the portfolio β calculation.

$$\beta = \frac{\$100,000 (1.12) + \left(\sum_{i=2}^{20} \$100,000 \beta_i \right)}{\$2,000,000}$$

$$\beta = 0.6385$$

We could accomplish the same thing by calculating:

$$\beta = 0.60 - \left(\frac{0.35}{20} \right) + \left(\frac{1.12}{20} \right) = 0.6385$$



5 - 3:

The SML shows us how much investors require in compensation for the systematic risk they bear. Investors require some return for postponing consumption. This return is the intercept of the SML and represents investments with no systematic risk. In other words, all investments must earn at least this minimum return, otherwise investors will consume their income rather than investing it.

Stocks with an "average" amount of systematic risk ($\beta=1$; the market risk) should provide a return equal to the average of all investments. Thus the SML must pass through both the risk free rate of return ($\beta=0$) and the market rate of return ($\beta=1$).

The SML has a positive slope. This reflects the premise that investors are risk averse: the more risky an investment is, the greater the return an investor will demand for investing in it. The slope of the line is known as the risk premium; this is the premium that investors demand by taking on an additional unit of systematic risk as measured by β .

Note that the SML and CAPM deal solely in systematic risk. Unsystematic risk is assumed eliminated through diversification.

@ Finance 254 [G. McIntire]



5 - 4:

- A. The T-bill return does not depend on the state of the economy because the Treasury must and will redeem the bills at par regardless. The T-Bills are risk free in the nominal sense because the 8% return will be realized in all possible states. However, this 8% return is composed of the real risk free rate (say 3%) plus an inflation premium (5%). If inflation averaged 9% over the year rather than the expected 5% then the real returns realized on the T-bills would be -1%. Thus, in real terms, T-bills are not riskless.
- B. Compulectric's returns are positively correlated with the economy because the firm's sales, and hence profits, will generally experience the same ups and downs as the economy as a whole. If the economy grows, so will Compulectrics. On the other hand Gold Mines is considered a hedge against both slow growth and high inflation. When the market turns sour investors will retreat into gold and gold production. This stock is thus counter-cyclical. (This is a totally fictional stock. It is almost impossible to find real stocks that run counter to the market on a systematic basis. Even Homestake Mining had a low but positive β (.35)).

	E [R]	σ	$CV = \frac{\sigma}{E[R]}$
T-Bills	8.0%	0.0	0.00
Index Fund	15.0%	15.3	1.02
Compulectrics	17.4%	20.0	1.15
American Rubber	13.8%	18.8	1.36
Gold Mines	1.7%	13.4	7.88

To demonstrate, calculate for the Index Fund

$$E[R] = 0.1 (0.430) + 0.2 (0.290) + 0.4 (0.150) + 0.2 (0.010) + 0.1 (-0.130) = 0.150$$

$$E[R^2] = 0.1 (0.430)^2 + 0.2 (0.290)^2 + 0.4 (0.150)^2 + 0.2 (0.010)^2 + 0.1 (-0.130)^2 = 0.04602$$

$$\sigma^2 = 0.04602 - (0.150)^2 = 0.02352$$

$$\sigma = 0.15336232 = 15.3\%$$

- C. If the trust is risk neutral we would invest in Compuelectrics; it has the highest expected return.
- D. American Rubber has a higher σ than the market fund yet is expected to earn a lower return. When we measure risk per unit of expected return, Gold Mines, with its low expected rate of return, becomes the most risky stock.
- E. We calculate the covariance as follows:

Scenario	Compuelectrics: R - E[R _C]	Gold Mines: R - E[R _{GM}]	(R - E[R _C]) (R - E[R _{GM}])
Rapid Growth	50.0 - 17.4 = 32.6%	-20.0 - 1.74 = -21.74%	-708.724
Above Average	35.0 - 17.4 = 17.6%	-10.0 - 1.74 = -11.74%	-206.624
Average	20.0 - 17.4 = 2.6%	0.0 - 1.74 = -1.74%	-4.524
Below Average	-2.0 - 17.4 = -19.4%	14.7 - 1.74 = 12.96%	-251.424
Recession	-22.0 - 17.4 = -39.4%	28.0 - 1.74 = 26.26%	-1034.644
Covariance:			-267.756

The covariance is -267.756% and the correlation coefficient is $\rho = \frac{COV_{C,GM}}{\sigma_C \sigma_G} = \frac{-267.756}{20.0 * 13.4} = -1.00$

There are two ways to calculate the standard deviation of the portfolio.

One, you can use the correlation coefficient you just calculated:

$$\sigma^2 = (x_C^2 \sigma_C^2) + 2(x_C \sigma_C (-1) x_{GM} \sigma_{GM}) + (x_{GM}^2 \sigma_{GM}^2)$$

$$= (.5 * 0.2004)^2 + 2 (.5 * 0.2004 * -1 * .5 * 0.1336) + (.5 * 0.1336)^2$$

$$= (0.01004004) + 2 (-0.00669336) + (0.00446224) = 0.00111556$$

$$\sigma = 0.03340000$$

Two, you can calculate the portfolio's return in each state of the economy and then calculate

$$E[R] = 9.57\%$$

$$E[R^2] = 102.7145\%$$

$$\sigma = 3.3, \text{ and}$$

$$CV = 0.34.$$

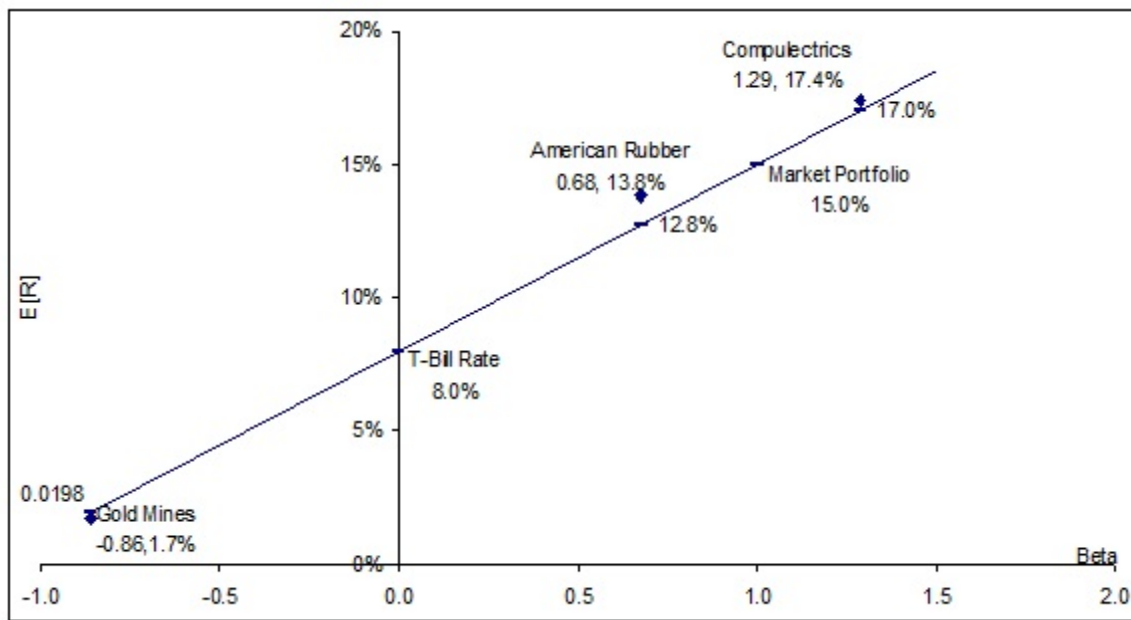
State of Economy	Prob	E[R]
Rapid Growth	0.1	15.00%
Above Average	0.2	12.50%
Average	0.4	10.00%
Below Average	0.2	6.35%
Recession	0.1	3.00%

The portfolio risk is much lower than the risk associated with either of the individual stocks. This is because the two stocks are negatively correlated: when Compuelectrics goes down Gold Mines goes up, and vice versa.

- F. Use the T-bill rate to proxy the risk free rate of return. The expected market rate of return is estimated at 15.0% (remember the index fund), thus the risk premium is 7.0% (15.0 - 8.0). The rates of return required to justify the risk are:

	CAPM: Required Return	E[R]	
Compulectrics	$8.0 + 1.29 (7.0) = 17.0\%$	17.4%	undervalued (buy)
American Rubber	$8.0 + 0.68 (7.0) = 12.8\%$	13.8%	undervalued (buy)
Gold Mines	$8.0 - 0.86 (7.0) = 2.0\%$	1.7%	overvalued (sell)
Portfolio $\beta = 0.215 [\beta=.5(1.29) + .5(-0.86)]$	$8.0 + .215 (7.0) = 9.505\%$	9.6%	slightly undervalued

- G. The undervalued securities are above the line; their rate of return is higher than the return required to compensate for the risk. The overvalued securities are below the line; their rate of return is less than the return required to compensate for the risk.



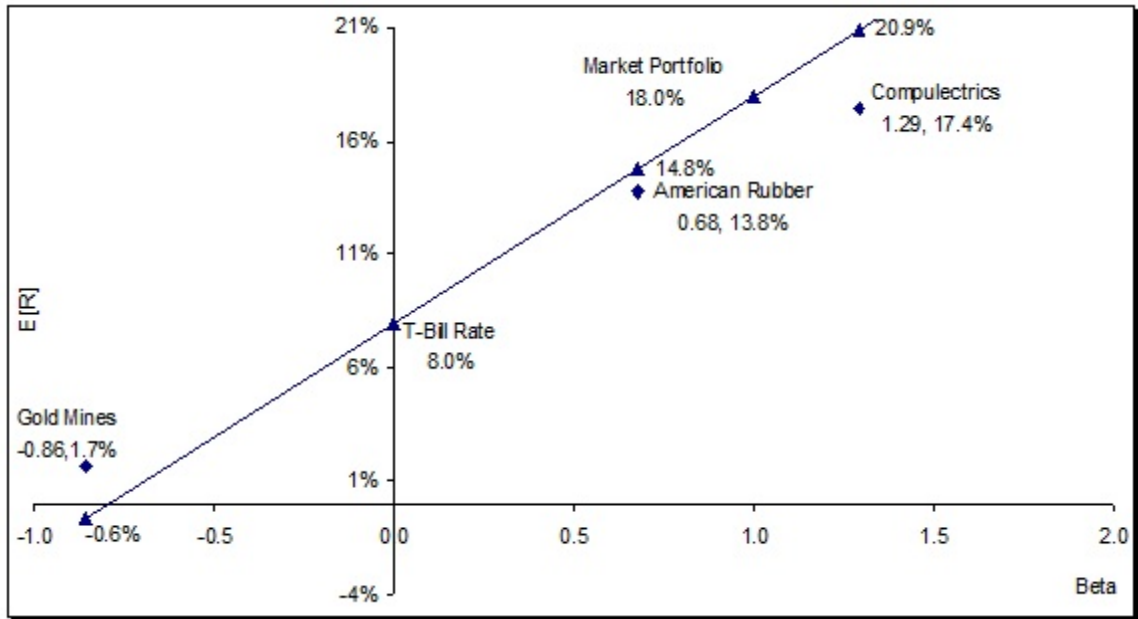
- H. If inflation expectations rise by 3% then the risk free rate increases from 8% to 11% and the market rate increases from 15% to 18%, generating the equation $R = 11.0 + \beta (7.0)$. All required returns increase by 3 percentage points. Note that the expected returns will also change. Run back to the investment guru and have his department regenerate the expected returns on these stocks. Note that the market risk premium remains at 7%.
- I. When investors' risk aversion increases, the market risk premium increases, in this case from 7% (15.0 - 8.0 = 7%) to 10% (18.0 - 8.0 = 10%). The required rate of return will increase sharply on high risk (high β) stocks, but will increase only marginally on low risk stocks.

J.

	CAPM: required Return	E[R]	
Compulectrics	$8.0 + 1.29 (10.0) = 20.9\%$	17.4	overvalued (sell)
American Rubber	$8.0 + 0.68 (10.0) = 14.8\%$	13.8%	overvalued (sell)
Gold Mines	$8.0 - 0.86 (10.0) = -0.6\%$	1.7%	undervalued (buy)
Portfolio $\beta = 0.215 [\beta = .5(1.29) + .5(-0.86)]$	$8.0 + .215 (10.0) = 10.15\%$	9.6%	overvalued (sell)

Gold Mines has a negative required return. Investing in a stock that has a negative required return is just like buying an insurance policy; insurance also has a negative expected return. A negative beta stock is insurance against a bear market.

Note how American Rubber and Compulectrics changes from a buy to a sell and Gold Mines changes from a sell to a buy when the risk premium changes from 7% to 10%.



5 - 5:

- A. $E[R] = 3.3\% + \beta [12.3\% - 3.3\%] = 3.3\% + \beta 9.0\%$
- B. Intercept = 3.3% and slope = 9.0% (Graphed below)
- C. $E[R] = 3.3\% + (0.8) 9.0\% = 10.5\%$
- D. Abnormal Return = Actual Return - Normal Return = 14.8% - 10.5% = +4.3%

E. We calculate the Sharpe and Treynor measures as follows:

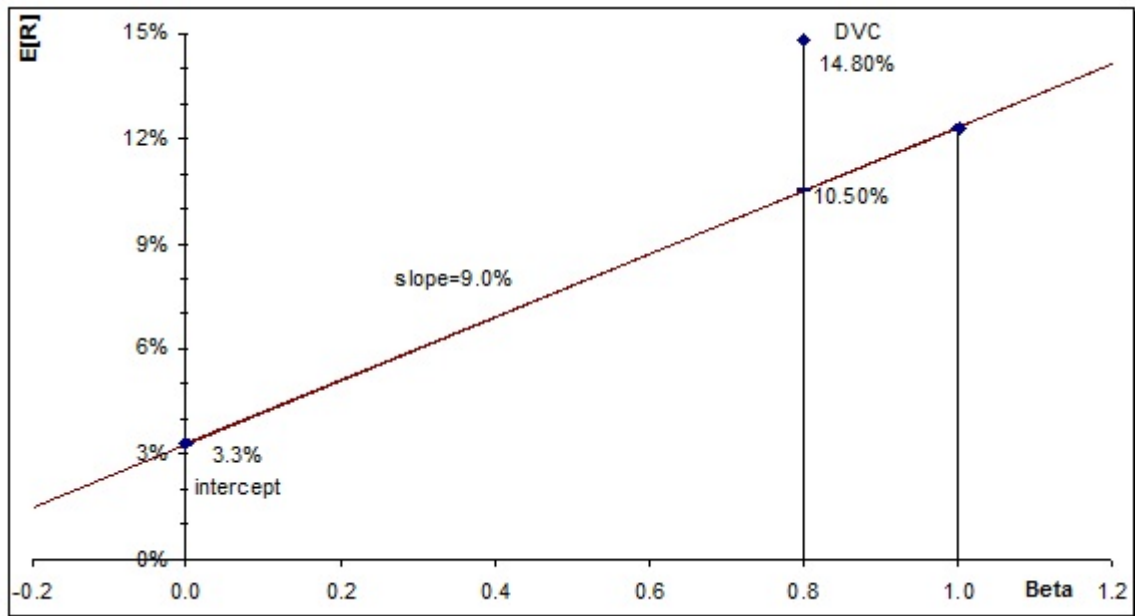
$$R_{Sharpe} = \frac{R_p - r_f}{\sigma_p} \qquad R_{Treynor} = \frac{R_p - r_f}{\beta_p}$$

T-Bills	$R_S = \frac{3.3\% - 3.3\%}{0.0\%} = 0$	$R_T = \frac{3.3\% - 3.3\%}{0} = 0\%$
Market Index Fund	$R_S = \frac{12.3\% - 3.3\%}{15.0\%} = 0.6$	$R_T = \frac{12.3\% - 3.3\%}{1.0} = 9.0\%$
Discovery Café	$R_S = \frac{14.8\% - 3.3\%}{27.3\%} = 0.421$	$R_T = \frac{14.8\% - 3.3\%}{0.8} = 14.375\%$

The Treynor measure for the equity index will always be equal to the risk premium whereas the Sharpe measure will capture the standard deviation of the market itself.

If the Treynor measure for Discovery Café is equal to the risk premium of 9.0% then Discovery Café adds the same risk and return to the portfolio as does the market index itself. If the Treynor measure is greater than the risk premium then Discovery Café adds more return than risk; if less than the risk premium then it adds more risk than return.

To see this calculate what the return on Discovery Café must be to get a Treynor measure equal to 9.0%. What must it be to get a measure higher than 9.0%? Lower than 9.0%?





5 - 6:

- A. Remember to round the market value in each month to two decimal places before continuing.

	S&P 500	IBM		GM	
			\$10,000.00		\$10,000.00
2001	-9.9%	-1.4%	\$9,860.00	-2.6%	\$9,740.00
2002	11.3%	13.9%	\$11,230.54	31.2%	\$12,778.88
2003	7.5%	34.3%	\$15,082.62	29.7%	\$16,574.21
2004	12.9%	-14.4%	\$12,910.72	16.9%	\$19,375.25
2005	-1.3%	2.5%	\$13,233.49	-38.4%	\$11,935.15

- B. Use the [data] and [stat] in your business calculator to calculate the statistics on IBM. Enter the observations on IBM into X and the observations on the index into Y.

IBM

1. $E[R] = 6.98\%$
2. $\sigma = 16.38\%$
3. $s = 18.31$
4. $\alpha = 3.60$
5. $\beta = 0.07$

- C. Use [data] to enter the GM stats into X.

GM

1. $E[R] = 7.36\%$
2. $\sigma = 25.89\%$
3. $s = 28.95\%$
4. $\alpha = 2.52$
5. $\beta = 0.21$

- D. Use [data] to enter the IBM stats into Y. You can double check your data entry by verifying the answers to 1, 2, and 3 in questions B. and C.

$$\rho = 0.405 \text{ (40.5\%)}$$



5 - 7:

Factor	Risk Free rate	Market Index	GDP	Expected Return
Security	F_1	F_2	F_3	
IBM (IBM)	1.0	1.1	0.5	22.0%
Compaq (CPQ)	1.0	0.9	0.4	18.8%
AT&T (T)	1.0	0.2	0.5	13.0%
Motorola (MOT)	1.0	0.8	0.5	12.0%
Ford (F)	1.0	0.2	0.6	14.2%

A. $F_1 = 5; F_2 = 10; F_3 = 12$

For each stock $R = \beta_1 F_1 + \beta_2 F_2 + \beta_3 F_3$. Using the information given in the question we can generate three equations in three unknowns:

$$\begin{array}{rclcl} \text{IBM} & 1.0 F_1 & + 1.1 F_2 & + 0.5 F_3 & = & 22.0\% \\ \text{Compaq} & 1.0 F_1 & + 0.9 F_2 & + 0.4 F_3 & = & 18.8\% \\ \text{AT\&T} & 1.0 F_1 & + 0.2 F_2 & + 0.5 F_3 & = & 13.0\% \end{array}$$

There are several ways to solve the system of equations. One simple way is

1) subtract AT&T from IBM

$$\begin{array}{rclcl} \text{IBM} & F_1 & + 1.1 F_2 & + 0.5 F_3 & = & 22.0 \\ \text{AT\&T} & - F_1 & - 0.2 F_2 & - 0.5 F_3 & = & - 13.0 \\ \hline & & 0.9 F_2 & & = & 9.0 \\ & & F_2 & & = & 10.0 \end{array}$$

2) substitute for F_2 and subtract Compaq from IBM

$$\begin{array}{rclcl} \text{IBM} & F_1 & & + 0.5 F_3 & = & 11.0 \\ \text{Compaq} & - F_1 & & - 0.4 F_3 & = & - 9.8 \\ \hline & & & .1 F_3 & = & 1.2 \\ & & & F_3 & = & 12.0 \end{array}$$

3) substitute in any equation to get $F_1 = 5.0$

B. Motorola: $1.0 (5.0) + 0.8 (10.0) + 0.5 (12.0) = 19.0\%$

Motorola is overpriced because we would require a return of 19.0% to compensate for the risk exposure but, at the current price, we expect to generate only a 12.0% return.

C. Put X% in IBM, Y% in Compaq, and Z% in AT&T. The resulting portfolio will then have a beta for each factor equal to the weighted average beta for that factor. The weights are then adjusted so that the portfolio beta equals Motorola's beta for each factor.

$$\begin{array}{rclcl} & & \text{IBM} & \text{Compaq} & \text{AT\&T} & \\ \text{Risk Free Rate} & (F_1) & 1.0 X & + 1.0 Y & + 1.0 Z & = & 1.0 \\ \text{Market Index} & (F_2) & 1.1 X & + 0.9 Y & + 0.2 Z & = & 0.8 \\ \text{GDP} & (F_3) & 0.5 X & + 0.4 Y & + 0.5 Z & = & 0.5 \end{array}$$

1) subtract $2 \cdot F_3$ from F_1

$$\begin{array}{rclcl} (F_1) & 1.0 X & + 1.0 Y & + 1.0 Z & = & 1.0 \\ (2F_3) & - 1.0 X & - 0.8 Y & - 1.0 Z & = & -1.0 \\ \hline & & 0.2 Y & & = & 0.0 \\ & & Y & & = & 0.0 \end{array}$$

2) substitute for Y and subtract $5 \cdot F_2$ from F_1

$$\begin{array}{rclcl} (F_1) & 1.0 X & & + 1.0 Z & = & 1.0 \\ (5F_2) & - 5.5 X & & - 1.0 Z & = & - 4.0 \\ \hline & - 4.5 X & & & = & - 3.0 \\ & X & & & = & \frac{2}{3} \end{array}$$

3) substitute in F_1 to get $Z = \frac{1}{3}$

Construct a portfolio of $\frac{2}{3}$ IBM and $\frac{1}{3}$ AT&T. The portfolio's betas will be 1.0, 0.8, and 0.5 (identical to Motorola's) but the expected return is 19.0%.

$$\frac{\$666,666.67(22.0\%) + \$333,333.33(13.0\%)}{\$1,000,000} = 19.0\%$$

The portfolio at an expected return of 19.0% is a better investment than Motorola at 12.0% since both investments have the same market risk.

- D. Ford: $1.0(5.0) + 0.2(10.0) + 0.6(12.0) = 14.2\%$

Ford is correctly priced.

- E. Put X% in IBM, Y% in Compaq, and Z% in AT&T. The resulting portfolio will then have a beta for each factor equal to the weighted average beta for that factor. The weights are then adjusted so that the portfolio beta equals Ford's beta for each factor.

$$\begin{array}{rcll} \text{Risk Free Rate (F}_1\text{)} & 1.0 X & + 1.0 Y & + 1.0 Z = 1.0 \\ \text{Market Index (F}_2\text{)} & 1.1 X & + 0.9 Y & + 0.2 Z = 0.2 \\ \text{GDP (F}_3\text{)} & 0.5 X & + 0.4 Y & + 0.5 Z = 0.6 \end{array}$$

- 1) subtract $2F_3$ from F_1

$$\begin{array}{rcll} (F_1) & 1.0 X & + 1.0 Y & + 1.0 Z = 1.0 \\ (2F_3) & - 1.0 X & - 0.8 Y & - 1.0 Z = -1.2 \\ \hline & & 0.2 Y & = -0.2 \\ & & Y & = -1.0 \end{array}$$

- 2) substitute for Y and subtract $5F_2$ from F_1

$$\begin{array}{rcll} (F_1) & 1.0 X & & + 1.0 Z = 2.0 \\ (5F_2) & - 5.5 X & & - 1.0 Z = -5.5 \\ \hline & - 4.5 X & & = -3.5 \\ & X & & = \frac{7}{9} \end{array}$$

- 3) substitute in F_1 to get $Z = \frac{11}{9}$

Construct a portfolio of $\frac{7}{9}$ IBM, $\frac{11}{9}$ AT&T and short sell Compaq. The portfolio's betas will also be 1.0, 0.8, and 0.5 and the expected return is also 14.2%.

$$\frac{\$777,777.78(22.0\%) + \$1,222,222.22(13.0\%) - \$1,000,000.00(18.8\%)}{\$1,000,000} = 14.2\%$$

The portfolio and Ford have the same betas and the same expected return. Ford is priced correctly.