Selling a Product Line Through a Retailer When Demand is Stochastic: Analysis of Price-Only Contracts

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Abstract

To expand sales, many manufacturers try to develop and sell product lines. Frequently, however, the operations of distributing a product line creates tension between manufacturers and retailers as they do not necessarily agree on which product versions included in the product line should be sold to consumers. To mitigate this tension, previous literature has shown that if a manufacturer (he) wants to sell his product line through a retailer (she) who faces deterministic demand, then he needs to adjust his product qualities according to her requirements; otherwise she will not carry the entire line. In contrast, this paper shows that if demand is stochastic, then a manufacturer can mitigate the same tension merely by re-allocating inventory risk in the supply chain. This strategy can be so effective that it is possible to find cases where the equilibrium product line is actually longer in a decentralized supply chain than in the direct-selling case. To understand the tradeoff, we consider a supply chain with a manufacturer capable of producing multiple product designs and a retailer who faces stochastic consumer demand. The manufacturer sells his output through the retailer using one of the following variations on the classical wholesale contract: push (PH), pull (PL), or instantaneous fulfillment (IF). With PH and PL (IF), wholesale prices and quantities are decided before (after) demand is revealed. Retail prices are always set after demand is revealed. With PH (PL) the retailer (manufacturer) carries retail inventory. Taking the manufacturer’s point of view, we characterize the equilibrium product line length and equilibrium contracting strategy. Our answers are determined by three important drivers: demand variability, product substitutability, and the retailer’s outside option. Low outside option and low (high) substitutability imply that the manufacturer maximizes his expected profit offering the retailer longer (shorter) product line using the IF contract. As outside option increases, the equilibrium contract will be either PH or PL. High demand variability and low substitutability imply that the manufacturer should be expected to sell a longer product line with a PH contract. Low demand variability and high substitutability imply that the manufacturer should be expected to sell a shorter product line with a PL contract.
1 Introduction

To facilitate sales growth and to induce cross-selling, many manufacturers design and produce products in product lines rather than in single product variants. Implementing a successful product line strategy, however, can become challenging for a manufacturer who sells his output through an independent retailer because such a manufacturer does not control the assortment and stocking decisions at the retail location. While he may attempt to influence these decisions, e.g., through pricing, they both are ultimately made by the retailer who cares about her own profits. Such a retailer may choose to carry not more than a subset of the manufacturer’s product line and stock smaller quantities of each product version than what the manufacturer would prefer. This occurrence is widespread. To illustrate, consider Oberweis Dairy, which serves the Greater St. Louis, MO-IL metropolitan area. Oberweis has a long product line, which includes four types of milk, five flavors of ice-cream cakes, over forty flavors of ice cream, and several flavors of yogurt. Although the full product line is available at retail locations, which Oberweis operates, independent grocery chains tend to carry only one or two Oberweis’ ice cream flavors and a limited selection of Oberweis’ milk. Further examples from popular business press mention manufacturers who want to extend their product lines in order to increase sales and retailers who are only interested in offering limited selections of “best sellers.” The grocery chain Kroger Co., for example, stripped about 30% of its cereal varieties made by companies like Kelloggs and General Mills (Brat et al., 2009). To motivate retailers to carry their products, manufacturers run sales promotions (Nijs et al., 2010; Yuan et al., 2013) or even tailor product designs to the needs of individual retailers (Villas-Boas, 1998). However, empirical research reveals that the effectiveness of some of these practices – the practice of sales promotions in particular – is not completely clear (Kumar et al., 2001; Nijs et al., 2010).

Recent research in marketing and economics has done an excellent job of formally documenting the aforementioned tension in a setting where retail demand is deterministic and the manufacturer delivers his output to the retailer with zero lead time (Villas-Boas, 1998; Liu and Cui, 2010). Taken together, these assumptions are conducive to studying the product line length (or assortment) question without having to consider the retailer’s stocking decisions. (Since the retailer stocks what she

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1**Definition:** A product line is a group of products that are closely related to each other by function and price. In offering a product line, manufacturers often use common components or building blocks to create derivative offerings that target various consumer preferences.
sells.) One of the central findings in this literature is that if a manufacturer wants to sell a product line through a retailer, then he needs to adjust his product qualities according to her requirements; otherwise the retailer will drop some products included in the product line (see Villas-Boas, 1998; Liu and Cui, 2010).

In contrast, the inventory and supply chain literature studied the previously mentioned tension in a setting where the manufacturer produces a single product design, delivery lead times are strictly positive, retail demand is stochastic, and retail prices are fixed. Taken together, these assumptions are fitting for studying the stocking question, but are not necessarily conducive to studying the questions of product variety and the retailer’s incentive to carry a product line. One of the important findings in this literature is that the choice of a supply contract type (supply contract types differ in pricing, lead times, and inventory risk allocation) affects the retailer’s stocking decision (see Cachon, 2003, for a survey of this large literature).

However, when product assortment decisions are combined with inventory decisions in the presence of uncertain demand, existing theories do not necessarily lend themselves to answering the overarching question of how to engage a retailer who chooses not only how much to sell but also what to sell. Would the manufacturer offer the retailer a product line or just a single product? Would the manufacturer deliver the goods with zero lead time (as it is assumed in the product line literature) or would the manufacturer quote the retailer a longer lead time, giving rise to inventory?2 If quoting long lead time is the optimal choice for the manufacturer, how would the inventory risk be optimally allocated and how would that affect the product line length?

The thesis behind this paper is that answering these questions requires a new model – one that combines stochastic demand, lead time choice, endogenous pricing, and assortment planning at both ends of the supply chain. In this paper, we propose a stylized version of one such model. In particular, our model is a bilateral supply chain where a manufacturer sells up to two differentiated product designs through a retailer to consumers. Consumer demand is linear and stochastic. At time 0, only the distribution of future demand is known. At time 1, demand outcome is known, and is controlled by the retail price set by the retailer.

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2Note that zero delivery lead times allow both firms to wait for demand to be revealed before committing to prices and quantities. However, an argument given in Iyer et al. (2007) suggests that a manufacturer selling a single product design may benefit from exposing the retailer to inventory risk. Therefore it is unclear that zero lead time is always preferable to the manufacturer.
We consider a supply chain with an upstream manufacturer (he) capable of producing multiple product designs and a downstream retailer (she) who faces stochastic demand. Each product design that the manufacturer decides to produce requires a costly setup and can be delivered to the retail location with zero lead time or depending on the manufacturer’s delivery lead time choice. The manufacturer makes a “take-it-or-leave-it” wholesale contract offer to the retailer, subject to leaving her with some minimum acceptable expected payoff. This payoff may be seen as a reservation utility that the retailer could achieve by pursuing another opportunity, e.g., by carrying a different manufacturer’s product line. For this reason, we refer to this as the retailer’s outside option. (For an empirical manifestation of outside options, see Ailawadi, 2001; Bloom and Perry, 2001.)

We consider three variations of the wholesale contract: push, pull, or instantaneous fulfillment. Throughout the paper, prices, quantities, and product line length are all endogenous and product qualities are exogenous. This is because our main interest is in the oper of distributing a product line rather than in designing one.

The wholesale contract variations differ in the timing of quantity and pricing decisions: Retail prices are always set after demand is revealed. Wholesale prices, product line length, and stocking quantities are decided before demand is revealed if the contract is push or pull; with instantaneous fulfillment, they are all set after demand is revealed. Push and pull, however, differ in how they allocate inventory risk. With push (pull), inventory risk is with the retailer (manufacturer). Finally, in terms of lead times, with push and pull, the production lead time is 1 and with instantaneous fulfillment, the production lead time is 0.

All three contracts are simple because each has a single wholesale price per product version. It is plausible that firms may restrict attention to these simple contracts, especially because more complex alternatives such as buy-backs do not coordinate in our setting even when the product line length is reduced to a single product. Those contracts run into trouble because the incentive they provide to coordinate the retailer’s quantity action distorts the retailer’s pricing decision (see Cachon, 2003, §3).

Using our model, we illustrate how optimal inventory decisions interact with optimal product line length in the presence of stochastic demand. Surprisingly, our results reveal that the manufacturer can incentivize the retailer to carry a longer product line not necessarily by designing products according to the retailer’s requirements (a recommendation put forth in the literature with determin-
istic retail demand) but by re-allocating inventory risk in the supply chain. Moreover, this strategy can be so effective that it is possible to find cases where the equilibrium product line is actually longer in a decentralized supply chain than in the direct-selling case.

To begin the investigation, we imagine a world in which the supply chain is vertically integrated, and refer to this setting as integrated supply chain. We show that in such a world, the manufacturer’s most preferred mode of operation is to utilize his zero lead time capability, wait for demand realization and then decide which designs to produce and set the production quantity and the retail price of each design. Waiting gives the manufacturer better information about demand, which has a direct, positive effect on his profits due to improved efficiency of matching assortment and quantity with demand. In particular, waiting eliminates cases in which the manufacturer incurs a setup cost for products, which appear profitable ex ante, but turn out to be unprofitable ex post. We find a range of setup costs for which the manufacturer operating under the integrated supply chain wants to produce and sell a greater variety of products so as to achieve increased sales without having to cut retail price significantly.

When facing the same setup cost in a decentralized supply chain, the manufacturer can again choose to wait for demand to be revealed by utilizing the instantaneous fulfillment contract. However, there is also an indirect, strategic effect of waiting that negatively affects the manufacturer’s profits as it can lead to too high double-marginalization and too high retail prices. We find that the high double-marginalization makes the instantaneous fulfillment contract particularly unappealing to a retailer with an attractive outside option. Therefore the manufacturer would not always want to utilize the instantaneous fulfillment even if producing and delivering goods with zero lead time could be done costlessly.

When facing a retailer with an attractive outside option, the manufacturer will either announce the number of product versions and their wholesale prices before demand is revealed and let the retailer pull the products she decides to sell from his inventory after demand is revealed (pull contract). Or, the manufacturer will announce the number of product versions and their wholesale prices before demand is revealed and ask the retailer to respond with strictly positive order quantities for all products that she decides to carry (push contract). We find that the former is a high-price-low-quantity strategy whereas the latter is a low-price-high-quantity strategy; the former is dominated by the latter as demand becomes more variable, in which case the wholesale demand is relatively
more elastic (i.e., the retailer becomes more responsive to the manufacturer’s wholesale price cut under the push contract).

By committing to product line before demand uncertainty is resolved, push and pull contracts respectively expose the retailer and the manufacturer to the inventory risk. Thus, the bearer of the inventory risk has an incentive to have a longer product line which can serve as a hedge against the demand uncertainty. We show that under demand uncertainty the equilibrium product line may become longer in a decentralized supply chain than in a centralized one. This result complements previous findings of a shorter product line reported in studies that assumed deterministic demand (Villas-Boas, 1998; Netessine and Taylor, 2007).

Finally, we find that the length of the product line not only depends on product substitutability but also on contract choices, when substitutability is in the mid-range. Everything else being equal, lower substitutability and higher demand variability point to a longer product line offered under push contract; higher substitutability and lower demand variability point to a shorter product line offered under pull contract.

We conclude our study by constructing a simple numerical example that illustrates how much better the manufacturer can do by being able to adjust his product line length and to change the contract type. The data shows that the manufacturer’s expected payoff can increase by more than 60%, which illustrates that both decisions generate first-order effects.

2 Related Literature

There are three areas of literature related to this paper: The literature on supply chain management with price-only contracts, the literature on inventory models with demand substitution, and the literature on product line design in distribution channels.

In supply chains managed with price-only contracts, manufacturers sell their output to retailers who stock the manufacturer’s output and re-sell it to consumers; manufacturers retain the wholesale revenue while retailers keep the retail revenue. Lariviere and Porteus (2001); Cachon (2004) study price-only contracts in the context of the newsvendor problem. Because the retailers in the newsvendor setting are unable to adjust either retail prices or the retail assortment, managing profits at the retail location becomes about setting an order quantity that matches supply and demand: Too much
supply (demand) leads to lower profits due to excessive inventory costs (excessive opportunity costs of lost margins). In contrast, the retailer in our paper manages her expected profits by optimizing not only over order quantities but also over assortment and retail prices. Endogenous assortment decision and retail pricing introduce new forces into the manufacturer-retailer interaction. First, unlike a newsvendor whose revenue would increase by having more inventory, a retailer with price-sensitive demand benefits from a higher inventory when the retail demand is elastic (necessities tend to have inelastic demand while luxuries are more elastic). Second, a manufacturer selling to a newsvendor fares best as demand variability approaches zero, because double-marginalization reduces as the demand variability decreases (Lariviere and Porteus, 2001). In contrast, a manufacturer selling to a price-setting retailer would not necessarily want to have a supply chain where demand uncertainty can be completely avoided even if adopting such a supply chain would be costless (see our explanation in Section 1; additional explanation can be found in Iyer et al., 2007). Third, adjusting assortment gives the retailer another lever to manage demand uncertainty: Everything else being equal, retail assortment increases with the variability of demand, because a larger product variety can serve as a hedge against not knowing, which designs will be in high demand. Finally, building upon Cachon (2004), which studies how the allocation of inventory risk impacts both supply chain inventory and supply chain efficiency, we find that the allocation of inventory risk affects not only inventory levels but also retail assortment. That is, push contract and pull contract can result in different retail assortment.

Several literatures build on the above newsvendor supply chain models. The literature on inventory financing, for example, combines the analysis of inventory risk with financing consideration (e.g., see Caldentey and Chen, 2012; Yang and Birge, 2013; Kouvelis and Zhao, 2012; Babich and Tang, 2012; Chod, 2015). We are related to this literature because the contracts we study can also be viewed as financing contracts. Namely, with the push contract, the retailer pays the manufacturer before demand is revealed whereas in the pull contract, the retailer pays the manufacturer after demand is revealed. Therefore the pull contract could be interpreted as a financing contract. Similarly, the contracts we study are also broadly related to the literature on demand forecasting. In particular, by choosing the push contract, the firms can be seen as giving up a demand signal. In contrast, by choosing the either PL or IF contract, either the retailer or both firms make quantity and pricing decisions after receiving a perfect demand signal. Thus by avoiding the push contract, the manufacturer
institutes an information-enabled supply chain. The incentives to invest into information-enabled supply chain is one of the central questions studied in the literature on demand forecasting (e.g., see Taylor, 2006; Taylor and Xiao, 2008; Shin and Tunca, 2010; Guo and Iyer, 2010). We differ from this literature by focusing on the joint assortment and stocking problem.

In addition to the single-item newsvendor model, there is a long literature on assortment planning, multi-item inventory systems, and product line design (e.g. van Ryzin and Mahajan, 1999; Gaur and Honhon, 2006; Netessine and Taylor, 2007; Aydin and Porteus, 2008; Liu and Cui, 2010). Netessine and Taylor (2007), for example, study the effect of production technology on a firm’s product line design strategies with an Economic Order Quantity model; Aydin and Porteus (2008) analyze a newsvendor’s optimal price and inventory choice of a given assortment; and Alptekinoglu and Corbett (2010) analyze a firm’s integrated product line design problem that involves variety, leadtime (or inventory), and pricing decision. Some of the earlier research (Cachon et al., 2005) is reviewed in Kök et al. (2009). Unlike in these papers, however, the manufacturer in our model operates in a decentralized supply chain. Everything else being equal, we find that the manufacturer may choose a shorter or a longer product line when he sells through a retailer, and product line decision and contract decision interact.

Finally, the analysis in this paper is also related to the literature on product line design for distribution channels. Villas-Boas (1998) and Liu and Cui (2010) study this problem in settings where demand is deterministic and delivery lead times are zero. They find that the length of the manufacturer’s optimal product line changes (shortens in most cases) when going from a centralized channel to a decentralized one. Villas-Boas (1998) also argues that to expand the product line, the manufacturer’s best strategy is to re-design it. In this paper, we find that with stochastic demand, product line expansion can be achieved by re-allocating demand risk in the channel, which the manufacturer achieves by switching between the instantaneous, push, and pull systems. Interestingly, we report that the retailer might actually choose to carry a longer product line in the decentralized channel than what the manufacturer choose to sell in the centralized channel (for a detailed explanation of this result, see our results in Section 5.2).

To summarize, although there is a rich literature on single product distribution in supply chains with stochastic demand, the incentive problems that arise in distributing a product line have been understudied and it is something that we do in this paper. Taking the manufacturer’s point of
view, we characterize the equilibrium product line length and equilibrium contracting strategy. Our answers are determined by three important drivers: demand variability, product substitutability, and the retailer’s outside option. Lengthening product line, at its core, is about increasing sales without having to cut price significantly. With uncertain demand, low product substitutability incentivizes the manufacturer to supply a longer product line which can serve as a hedge against demand uncertainty; so much so that the manufacturer may produce a longer product line when he sells through an independent retailer than when he sells directly to consumers. In the contracting space, low outside option makes it more likely that the equilibrium contract will be instantaneous fulfillment. As outside option increases, the equilibrium contract will be either push or pull; push is preferred over pull when the manufacturer faces elastic wholesale demand, which can be driven by increasing variability in retail demand.

The rest of the paper is organized as follows. The next section describes the model. Sections 4 characterizes the equilibrium in a world in which the manufacturer is able to sell directly to consumers or is able to achieve a first-best vertically integrated supply chain. Section 5 describes the solution in a world where the supply chain is decentralized and characterizes the subgame-perfect Nash equilibrium (SPNE) in a game in which the manufacturer can choose which contract to adopt and the retailer can choose which products to carry. Section 6 concludes.

3 Model

Consider a supply chain where a manufacturer (he) has the capability to produce up to two different product designs, which he sells through a retailer (she) to consumers. Consumer demand at retail prices \( p = (p_1, p_2) \) will be denoted by \( s(p, A) \). When viewed from time 0, \( A = (A_1, A_2) \) takes on one of the four possible values, \((1 + u, 1 + u), (1 + u, 1 - u), (1 - u, 1 + u), (1 - u, 1 - u)\), with equal probability. Thus \( A \) is a proxy for demand uncertainty due to factors such as consumer valuation or market size, and \( u \) measures the demand variation. The value of \( A \) is fully resolved when consumer demand clears, which is at time 1. The direct demand \( s(p, A) \) has an inverse \( p(s, A) \) such that

\[
p(s, A) = (A_1 - \alpha s_1 - \beta s_2, A_2 - \beta s_1 - \alpha s_2).
\] (1)
Equation (1) is a familiar inverse demand model proposed by Singh and Vives (1984); variants of this model have been used by many authors. The term $\alpha \geq 0$ measures how own outputs affect prices while the term $\beta \geq 0$ measures product substitutability. The goods are substitutes or independent according to whether $\beta > 0$ or $\beta = 0$. $^3$ The substitutability effects in this system are identical across the inverse demand curves. (For a careful derivation of (1) from a consumer utility model, see Singh and Vives, 1984, p.547 or Ingene and Parry, 2004, p.23.) To ensure that both products are viable, the substitutability effect cannot be too large. Therefore we adopt the following Assumption 1.

**Assumption 1.** $0 \leq \beta < \alpha \frac{1-u}{1+u}$, where $\alpha > 0$ and $0 \leq u \leq 1$.

The manufacturer can choose between offering zero, one, or two product versions. To add a product version to his product line, the manufacturer has to invest a fixed cost $f \geq 0$, independent of the amount produced. The fixed cost, $f$, captures costly production setups$^4$ (e.g., Netessine and Taylor, 2007; Liu and Cui, 2010; Nasser et al., 2013), or marketing costs associated with adding a new product$^5$ (e.g., Copple, 2002). The marginal production cost for the manufacturer is normalized to zero without further loss of generality. The manufacturer’s output can be delivered to the retail location with zero lead time at no additional cost, depending on the manufacturer’s lead time choice.

We analyze two supply chains: An integrated supply chain and a decentralized one, where the former serves as a benchmark for the latter. In the integrated supply chain, the manufacturer behaves as if he were able to sell directly to consumers and take downstream retail actions (including pricing and stocking decisions). In the decentralized supply chain, one can interpret the firms as playing a game over two dates, labeled “time 0” and “time 1.” At time 0, the manufacturer makes a “take-it-or-leave-it” wholesale contract offer to the retailer, subject to leaving her with some minimum expected payoff, $\pi^0$. $^6$

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$^3$$\beta < 0$ means that the products are complements, which is a case that we disregard in this paper.

$^4$One can think of a world in which the manufacturer produces the product designs in batches – one batch per design while facing fixed setup cost $f \geq 0$ for each batch.

$^5$Frequently, retailers expect manufacturers to bear a portion of marketing expenditures associated with selling a product.

$^6$The retailer’s reservation payoff may be seen as a reservation utility that the retailer could achieve by pursuing another opportunity (e.g., see Lariviere and Porteus, 2001, p.302). In practice, retailers often guarantee themselves payoffs by either imposing restrictions on pricing, by requiring slotting allowances, or by using some combination of both (Ailawadi, 2001; Bloom and Perry, 2001). Detailed modeling of these institutional details, however, does not add important insights to our analysis.
We consider three variations on the wholesale contract: (1) instantaneous fulfillment contract (IF), (2) pull contract (PL), and (3) push contract (PH).

With IF, the manufacturer and the retailer enter into the contract at time 0, but quantities and prices (both wholesale and retail) decisions are postponed to time 1. With PL, the manufacturer announces the available inventory (including product line length) and wholesale prices at time 0; at time 1, the retailer pulls the products that she decides to sell from the manufacturer’s inventory and sets retail prices. With PH, the manufacturer announces the number of product versions and their wholesale prices at time 0 at which time the retailer responds with strictly positive order quantities for all products that she decides to carry; retail prices are set by the retailer at time 1.

The production and delivery logistics behind each contract are as follows: With instantaneous fulfillment contract, the production and delivery lead times are 0. With the push and pull contracts, the production lead time is 1 and delivery lead time is 0. Production lead time acts as an assortment and quantity constraint since the retailer cannot sell more than what the manufacturer produced for at time 0. Recall, however, that our assumed cost structure does not assign any cost premium to the zero lead time operations. Adding such premium would not eliminate any of our main results and would not add any new important insights.

Figure 1 summarizes the sequence of events for each contract.

**Notation Summary.** Our notation is as follows: Let $x$ be a placeholder for variables ($w$=wholesale price, $q$=order quantity, $s$=quantity sold, $p$=retail price, $\Pi$=manufacturer’s profit, $\pi$=retailer’s profit). As such, $x_j = (x_{j,1}, x_{j,2})$ is a vector of prices, quantities, or payoffs and $x_{j,k}$ is either a price or a quantity of product $k = 1, 2$ for a contract type $j \in \{IF, PH, PL\}$. Alternatively, $x_{j,k}$ could be a payoff from selling product $k = 1, 2$ under a contract type $j \in \{IF, PH, PL\}$. If $N = 1$, then $x_j$ indicates association with contract type $j \in \{IF, PH, PL\}$. Finally, $\Pi_j^{(N)}$ and $\pi_j^{(N)}$ denote respectively the manufacturer’s and the retailer’s profits from selling $N = 1, 2$ product designs under contract $j \in \{IF, PH, PL\}$.

### 4 Integrated Supply Chain Solution

Imagine first that the manufacturer can achieve vertical integration, which allows him to choose his most preferred product line, stocking quantities at the retail location, and retail prices. We open
with this construct because it provides a useful yardstick for the equilibrium product line length, quantities, and prices that we observe in the decentralized supply chain, which is something that we will derive later in the paper.

Our analysis begins with a preliminary result that deals with optimal timing of events in the integrated supply chain.

**Observation 1.** *In the integrated supply chain, the manufacturer finds it optimal to use his zero delivery lead time capability, wait for demand to be revealed at time 1 and then choose how many product versions to sell and the selling quantity and retail price of each product version.*

This observation reflects the fact that by choosing assortment, output, and prices before demand is realized, the manufacturer can match supply with demand only on average. In contrast, by making these choices after demand is realized, the manufacturer can match supply with demand for each
optimal retail prices will be. These yield a payoff \( \Pi^{s} \), quantities will be \( s^{*} \) and \( p^{*}(A) \) that determine the optimal product line length, \( N^{*} \).

Table 1: Manufacturer’s Ex Post Optimal Decisions in a Market Without Retailer

<table>
<thead>
<tr>
<th>( \mathbf{A} )</th>
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<td>((1 + u, 1 - u))</td>
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<td>((1 - u, 1 + u))</td>
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With \( N = 1 \), the manufacturer will sell product \( i = 1, 2 \) if \( A_{i} \geq A_{j} \), where \( j = 1, 2 \) and \( i \neq j \). From (1), the inverse demand for the product \( i \) will be \( pA_{i} - \alpha s \), where \( s \) is the sales quantity of product \( i \).

The manufacturer’s most preferred sales quantity will be \( s^{*} = \arg \max_{s} (A_{i} - \alpha s) s - f = \frac{A_{i}}{2\alpha} \), which yields an optimal retail price of \( p^{*} = \frac{A_{i}}{2 \alpha} \). The corresponding payoff will be \( \Pi^{1*}_{\text{no retailer}} = \left( \frac{A_{i}^{2}}{4\alpha} - f \right) \).

With \( N = 2 \), the inverse demands are given by (1). The manufacturer’s most preferred sales quantities will be \( s^{*} = \arg \max_{s} (A_{1} - \alpha s_{1} - \beta s_{2}) s_{1} + (A_{2} - \alpha s_{2} - \beta s_{1}) s_{2} - 2f \). The corresponding optimal retail prices will be \( p^{*} = \left( \frac{A_{1}}{2}, \frac{A_{2}}{2} \right) \), which induce a level of sales \( s^{*} = \left( \frac{\alpha A_{1} - \beta A_{2}}{2(\alpha^{2} - \beta^{2})}, \frac{\alpha A_{2} - \beta A_{1}}{2(\alpha^{2} - \beta^{2})} \right) \). These yield a payoff \( \Pi^{2*}_{\text{no retailer}} = \frac{\alpha(A_{1}^{2} + A_{2}^{2}) - 2\beta A_{1}A_{2}}{4(\alpha^{2} - \beta^{2})} - 2f \). Note that \( s_{1}^{*}(A) < s^{*}(A_{1}) \), producing and selling in greater variety reduces the sales of the original version. The next Proposition 1 compares the manufacturer’s profits, \( \Pi^{1*}_{\text{no retailer}} \) and \( \Pi^{2*}_{\text{no retailer}} \), to determine his optimal product line length, \( N^{*} \).

Proposition 1. 1. If \( f < \frac{(\alpha(1-u) - \beta(1+u))^{2}}{4\alpha(\alpha^{2} - \beta^{2})} \), \( N^{*} = 2 \) for all realizations of \( A \).
2. If \( \frac{(\alpha(1-u) - \beta(1+u))^{2}}{4\alpha(\alpha^{2} - \beta^{2})} < f < \frac{(1-u)^{2}(\alpha - \beta)}{4\alpha(\alpha + \beta)} \), \( N^{*} = 2 \) when \( A = (1 + u, 1 + u) \) or \( (1 - u, 1 - u) \); \( N^{*} = 1 \) when \( A = (1 + u, 1 - u) \) or \( (1 - u, 1 + u) \).
3. If \( \frac{(1-u)^{2}(\alpha - \beta)}{4\alpha(\alpha + \beta)} < f < \frac{(1+u)^{2}(\alpha - \beta)}{4\alpha(\alpha + \beta)} \), \( N^{*} = 2 \) when \( A = (1 + u, 1 + u) \); \( N^{*} = 1 \) when \( A =
(1 + u, 1 - u) or \( A = (1 - u, 1 + u) \); \( N^* \leq 1 \) when \( A = (1 - u, 1 - u) \).

4. If \( f > \frac{(1+u)^2(\alpha-\beta)}{4\alpha(\alpha+\beta)} \), \( N^* \leq 1 \) for all realizations of \( A \).

To develop the intuition for the above results and to illustrate the forces at work, we re-write the manufacturer’s payoff when \( N = 2 \) as a sum of three functions: (1) \( \Pi^{(1)*}_{\text{no retailer}} \), which is the manufacturer’s payoff when \( N = 1 \); (2) expansion payoff (\( EXP \)), which is defined as the manufacturer’s payoff from selling the second product, i.e., \( EXP = p_j^*(A)s_j^*(A) > 0 \); and (3) cannibalization payoff (\( CAN \)), which is defined as the difference between the manufacturer’s payoffs with \( N = 1 \) and \( N = 2 \) from selling the original product, i.e., \( CAN = p_i^*(A)s_i^*(A) - p_i(A)s_i(A) \leq 0, \ i \in \{1, 2\}, \ i \neq j \). With these definitions, it easy to check that \( \Pi^{(2)*}_{\text{no retailer}} = \Pi^{(1)*}_{\text{no retailer}} + EXP + CAN - f \), and \( \Pi^{(2)*}_{\text{no retailer}} > \Pi^{(1)*}_{\text{no retailer}} \) if and only if \( EXP + CAN - f > 0 \). Therefore the necessary and sufficient condition for the profit-maximizing manufacturer to sell the full product line is that the expansion effect must exceed the cannibalization effect and the fixed cost \( f \) required to add a second product version.

By substituting the optimal prices and outputs from Table 1, we can see that

\[
EXP = \frac{A_j(\alpha A_j - \beta A_i)}{4(\alpha^2 - \beta^2)} > 0 \quad \text{and} \quad CAN = -\frac{\beta A_i(\alpha A_j - \beta A_i)}{4\alpha(\alpha^2 - \beta^2)} < 0. \tag{2}
\]

It follows that \( N^* = 2 \) whenever

\[
0 \leq EXP + CAN - f = \frac{(\alpha A_j - \beta A_i)^2}{4\alpha(\alpha^2 - \beta^2)} - f. \tag{3}
\]

Thus, as the Proposition 1 shows if \( f \leq \frac{(1+u)^2(\alpha-\beta)}{4\alpha(\alpha+\beta)} \), the manufacturer will produce two products for at least some realizations of \( A \), in particular, when market valuation for both products is high.

Moreover, we find that \( \frac{(1+u)^2(\alpha-\beta)}{4\alpha(\alpha+\beta)} \) decreases in \( \beta \) (this is confirmed by differentiating the right side of the above equation with respect to \( \beta \) and making use of Assumption 1, which requires \( \beta \leq \alpha \frac{1-u}{1+u} \)).

This makes sense: Since \( \beta \) measures the degree of product differentiation, the monotonicity property says that adding a second product is always more appealing when the second product is sufficiently differentiated from the first one. By substituting \( \beta = \alpha \frac{1-u}{1+u} \) (Assumption 1) into this expression, we obtain a critical level of fixed cost, \( f \), below which the second product becomes viable in the integrated supply chain (for at least some outcome of \( A \)). We assume that the critical fixed cost level is in force in the remainder of the paper.

**Assumption 2.** \( f \leq \frac{u(1+u)^2}{4\alpha} \).
Another way to understand the Inequality (3) is to see what adding a second product does to sales: While in the single-product case, the manufacturer’s sales are \( s^* = \frac{A_i}{2\alpha} \), by adding a second product, his total sales increase to \( s^*_i + s^*_j = \frac{A_i + A_j}{2(\alpha + \beta)} > \frac{A_i}{2\alpha} = s^* \). Although a naïve manufacturer could achieve the same level of sales, namely \( s^*_i + s^*_j \), by selling only a single product, such level of sales would require setting a retail price of \( \frac{\alpha A_i - \alpha A_j + 2A_j\beta}{2(\alpha + \beta)} \), which is much lower than the manufacturer’s optimal retail prices of \( \frac{A_i}{2\alpha} \). Therefore lengthening the product line is fundamentally about increasing sales without having to cut retail prices significantly.

5 Decentralized Supply Chain Solution

Recall from Section 4 that if (1) \( f \leq \frac{u(1+u)^2}{4\alpha} \), and (2) \( 0 \leq \beta < \alpha \frac{1-u}{1+u} \), then there exists a value of \( A \) for which the manufacturer will produce and sell both product designs.

Building on this analysis, in this section, we introduce an independent, optimizing retailer who only cares about her own profits and who is in control of all decision-making at the retail location. In this setting, the manufacturer can no longer choose his most preferred retail assortment, retail stocking quantities, and retail prices (something that he did in Section 4) because these decisions are now in the retailer’s domain. The manufacturer, however, can influence the retailer’s decisions by offering her his most preferred variation of the wholesale price contract. Two questions immediately arise: How many product designs should the manufacturer launch? Which wholesale contract variation \( j \in \{IF, PH, PL\} \) should the manufacturer pick?

For expositional clarity, we derive the equilibrium contract terms in Sections 5.1, 5.2, and 5.3 conditional on the contract type choice \( j \in \{IF, PH, PL\} \) of Stage 1 (see Figure 1). Then in Section 5.4 we explore the question of which contract type the manufacturer should choose. The equilibrium is derived via backward induction to guarantee subgame perfection.

5.1 Instantaneous Fulfillment Contract

With instantaneous fulfillment contract, the negotiations are conducted at time 0, but the physical transaction happens at time 1 (see Figure 1): At time 1, after demand is revealed, the manufacturer announces \( N, w_{IF}, \) and \( q_{IF} \). The retailer responds by announcing the retail prices for all products that she wants to sell, \( p_{IF} \). Finally, the representative consumer makes purchase decisions, given
Proposition 2. For a given the value of, $A$, and a given vector of wholesale prices, $w_{IF}$, the retailer’s most preferred retail prices are

$$p_{IF}^* = \arg \max_{p \geq w_{IF}} (p - w_{IF}) \cdot s(p, A) \text{ s.t. } s(p, A) \leq q_{IF} = \begin{cases} \frac{A_1 + w_{IF} \cdot 1}{2}, & \text{if } N = 1 \text{ and } A_1 \geq A_2, \\ (\infty, \frac{A_2 + w_{IF} \cdot 2}{2}), & \text{if } N = 1 \text{ and } A_1 \leq A_2, \\ \frac{A_1 + w_{IF} \cdot 1}{2}, & \text{if } N = 2, \end{cases}$$

which induce a level of sales $s_{IF,i}^* = \frac{\alpha(A_i - w_{IF,i}) - \beta(A_i - w_{IF,i})}{2 \alpha^2 - 2 \beta^2}$ when $N = 2$ and $s_{IF,i}^* = \frac{A_i - w_{IF,i}}{2 \alpha}$ when $N = 1$ and $A_i \geq A_j$, where $i, j = 1, 2, i \neq j$.

Wholesale Pricing and Product Line Length Decisions. Correctly anticipating the retailer’s response, the manufacturer’s most preferred wholesale prices are

$$(w_{IF}^*, q) = \arg \max_{q, w_{IF} \geq 0} w_{IF} \cdot s(p_{IF}^*, A) - N f = \begin{cases} (\frac{A_1}{2}, \infty, s(p_{IF}^*, A)), & \text{if } N = 1 \text{ and } A_1 \geq A_2, \\ (\infty, \frac{A_2}{2}, s(p_{IF}^*, A)), & \text{if } N = 1 \text{ and } A_1 \leq A_2, \\ (\frac{A_1}{2}, \frac{A_2}{2}, s(p_{IF}^*, A)), & \text{if } N = 2, \end{cases}$$

where $p_{IF}^*$ comes from the retail pricing stage. The manufacturer’s production quantity decision, $q$, is particularly simple: since his marginal cost is zero, then, given $N$, his optimal production quantities are $s(p_{IF}^*, A)$. The profits for the manufacturer, $\Pi_{IF}^{(N)}$, and the retailer, $\pi_{IF}^{(N)}$, are

$$\Pi_{IF}^{(N)}(A) = \begin{cases} \frac{A_1^2}{8 \alpha} - f, & \text{if } N = 1 \text{ and } A_1 \geq A_2, \\ \frac{A_2^2}{8 \alpha} - f, & \text{if } N = 1 \text{ and } A_1 \leq A_2, \\ \frac{\alpha(A_1^2 + A_2^2) - 2\beta A_1 A_2}{8 \alpha^2 - 2 \beta^2} - 2f, & \text{if } N = 2, \end{cases}$$

$$\pi_{IF}^{(N)}(A) = \begin{cases} \frac{A_1^2}{16 \alpha}, & \text{if } N = 1 \text{ and } A_1 \geq A_2, \\ \frac{A_2^2}{16 \alpha}, & \text{if } N = 1 \text{ and } A_1 \leq A_2, \\ \frac{\alpha(A_1^2 + A_2^2) - 2\beta A_1 A_2}{16 \alpha^2 - 16 \beta^2}, & \text{if } N = 2. \end{cases}$$

Finally, to determine the optimal product line length in the integrated supply chain we compare manufacturer’s profits across $N$. Our results are summarized in the following Proposition 2.

Proposition 2. 1. If $f \leq \frac{(\alpha(1-u)-\beta(1+u))^2}{8 \alpha(\alpha^2-\beta^2)}$, $N^* = 2$ for all realizations of $A$.

2. If $\frac{(\alpha(1-u)-\beta(1+u))^2}{8 \alpha(\alpha^2-\beta^2)} < f \leq \frac{(1-u)^2(\alpha-\beta)}{8 \alpha(\alpha+\beta)}$, $N^* = 2$ when $A = (1 + u, 1 + u)$ or $(1 - u, 1 - u)$.

3. If $\frac{(1-u)^2(\alpha-\beta)}{8 \alpha(\alpha+\beta)} < f \leq \frac{(1-u)^2(\alpha-\beta)}{8 \alpha(\alpha+\beta)}$, $N^* = 2$ when $A = (1 + u, 1 + u)$; $N^* = 1$ when $A = (1 + u, 1 - u)$ or $A = (1 - u, 1 + u)$; $N^* \leq 1$ when $A = (1 - u, 1 - u)$.

4. If $f > \frac{(1-u)^2(\alpha-\beta)}{8 \alpha(\alpha+\beta)}$, $N^* \leq 1$ for all realizations of $A$. 16
Discussion. Proposition 2 is a counterpart of Proposition 1. In both results, as the setup cost, \( f \), increases, the manufacturer’s incentive to provide a longer product line diminishes. Given the same setup cost, however, the product line that the manufacturer offers to the retailer with the IF contract is either the same length or shorter than what the manufacturer sells in the centralized supply chain. To see this, it is again helpful to re-write the manufacturer’s profits with \( N = 2 \) as a function of expansion (\( EXP \)) and cannibalization (\( CAN \)) (see Section 4). With the IF contract, we have: \( EXP_{IF} = w_{IF,2}^*q_{IF,2}^* > 0 \) and \( CAN_{IF} = w_{IF,1}^*q_{IF,1}^* - w_{IF}^*q_{IF}^* < 0 \). After substituting for prices, which are included in the appendix, we can see that

\[
EXP_{IF} = \frac{A_j(\alpha A_j - \beta A_i)}{8(\alpha^2 - \beta^2)} > 0 \quad \text{and} \quad CAN_{IF} = -\frac{\beta A_i(\alpha A_j - \beta A_i)}{8\alpha(\alpha^2 - \beta^2)} < 0.
\]

Due to the double-marginalization,

\[
EXP_{IF}(A) + CAN_{IF}(A) < EXP(A) + CAN(A),
\]

where \( EXP \) and \( CAN \) are given by (2). We conclude that for a manufacturer offering a IF contract, the benefit of introducing a second product is lower than that in the integrated supply chain, which implies a (weakly) shorter product line.

It is worth mentioning, that alternatively, we could have presented Proposition 2 in terms of substitution coefficient, \( \beta \) (for a given \( f \)), instead of \( f \) (for a given \( \beta \)). Similar story would have emerged: For high values of \( \beta \) (i.e., \( \beta \geq \hat{\beta}_IF \equiv \frac{\alpha (1+2u+u^2-8f\alpha)}{1+2u+u^2+8f\alpha} \)), \( N^* \leq 1 \); for low values of \( \beta \) (i.e., \( \beta \leq \hat{\beta}_IF \equiv \frac{\alpha (1-u^2-4\sqrt{2f\alpha(u+2f\alpha)})}{1+2u+u^2+8f\alpha} \)), \( N^* = 2 \); for values in the mid-range, \( N^* = 1 \) or \( N^* = 2 \), depending on the outcome of \( A \). This is seen graphically in Figure 2a. Interestingly, Figure 2a also shows graphically a dashed curve \( \hat{\pi}_{IF} \equiv E[\pi_{IF}^{(N^*)}(A)] \). To the right of this dashed curve lies the un-shaded area, where \( E[\pi_{IF}^{(N^*)}(A)] < \pi^0 \), which is where the IF contract is infeasible. To see why, recall that the manufacturer must guarantee the retailer a minimum level of payoff, \( \pi^0 \), only when the firms enter into the supply agreement, which is the time 0. With the IF contract, however, wholesale prices are announced at time 1 at which time \( \pi^0 \) is no longer a constraint. Retailers with an attractive outside option are therefore not interested in the IF contract.

5.2 Push Contract

With the push contract, the retailer chooses the number of product versions, \( N \), pre-books inventory, \( q_{PH} \), and pays \( w_{PH} \cdot q_{PH} \) to the manufacturer at time 0, where \( w_{PH} \) is the vector of wholesale prices.
set by the manufacturer. The manufacturer produces exactly what the retailer pre-books. At time 1, the retailer sets retail prices that maximize the sales proceeds from the inventory that she brings to the market. Figure 1 shows the sequence of events in the push contract subgame graphically.

**Retail Pricing Decision.** In Stage 4, after the state of the world, $A$, is revealed, the retailer correctly anticipates the representative consumer’s purchase quantities, $s(p, A)$, and chooses retail prices that maximize her sales revenue with the proviso that she cannot sell more than what she ordered for in Stage 3: 

$$\max_{p \geq 0} p \cdot s(p, A) \quad \text{s.t.} \quad s(p, A) \leq q_{PH}. \quad (5)$$

If $N = 2$, the profit maximizing $p = p^*_{PH} = (p^*_{PH,1}, p^*_{PH,2})$ is given by:

$$p^*_{PH,i} = A_i - \alpha s^*_{PH,i} - \beta s^*_{PH,j}, \quad \text{where}$$

$$s^*_{PH,i} = \min \left\{ q_{PH,i}, \max \left\{ \frac{A_i - 2\beta q_{PH,j}}{2\alpha}, \frac{\alpha A_i - \beta A_j}{2(\alpha^2 - \beta^2)} \right\} \right\}, \ i, j = 1, 2, \ i \neq j. \quad (6a)$$

If $N = 1$, then $p = p^*_{PH} = p^*_{PH,1}$, where $p_{PH,1}$ is recovered from (6a) by setting $s^*_{PH,1} = \min \{q_{PH,1}, \frac{A_1}{2\alpha} \}$ and $q_{PH,2} = s^*_{PH,2} = 0$ in (6), and the value of $p^*_{PH,2}$ becomes irrelevant.

---

7Note that the inventory procurement cost, $w_{PH} \cdot q_{PH}$, is sunk in Stage 4 and thus it does not affect the prices the retailer sets.
The program (5) and (6) can be thought of as a dumpster theory of pricing (see Kreps, 1990, Ch.9): If the retailer’s stock of product version \( i = 1, 2 \), \( q_{PH,i} \), is sufficiently low, then the constraint in (5) is *active* and her equilibrium strategy is to set the highest retail price for product \( i \) that clears her inventory of product \( i \), \( q_{PH,i} \). In such a case, we can say that the retailer is *under-stocked* in product \( i \). In contrast, if \( q_{PH,i} \) is sufficiently high, then the constraint in (5) is *inactive*, the retailer throws any excess stock and sets a retail price for product \( i \) that maximizes her revenue from the sales of product \( i \). In such a case, we can say that the retailer is *over-stocked* in product \( i \). Table 2 makes the process even more explicit for different combinations of market potential outcomes \( A \) and inventory vectors \( q_{PH} \).

<table>
<thead>
<tr>
<th>( A )</th>
<th>PH.1: High Inventory</th>
<th>PH.2: Moderate Inventory</th>
<th>PH.3 Low Inventory</th>
</tr>
</thead>
<tbody>
<tr>
<td>((1 + u, 1 + u))</td>
<td>( s_{PH}^{PH} = q_{PH} )</td>
<td>( s_{PH}^{PH} = q_{PH} )</td>
<td>( s_{PH}^{PH} = q_{PH} )</td>
</tr>
<tr>
<td>((1 + u, 1 - u))</td>
<td>( s_{PH}^{PH} = q_{PH,1}^{PH}, s_{PH}^{PH,2} = q_{PH,2} )</td>
<td>( s_{PH}^{PH} = q_{PH,1}^{PH}, s_{PH}^{PH,2} = q_{PH,2} )</td>
<td>( s_{PH}^{PH} = q_{PH} )</td>
</tr>
<tr>
<td>((1 - u, 1 + u))</td>
<td>( s_{PH}^{PH} = q_{PH,1}^{PH}, s_{PH}^{PH,2} = q_{PH,2} )</td>
<td>( s_{PH}^{PH} = q_{PH,1}^{PH}, s_{PH}^{PH,2} = q_{PH,2} )</td>
<td>( s_{PH}^{PH} = q_{PH} )</td>
</tr>
<tr>
<td>((1 - u, 1 - u))</td>
<td>( s_{PH}^{PH} = q_{PH} )</td>
<td>( s_{PH}^{PH} = q_{PH} )</td>
<td>( s_{PH}^{PH} = q_{PH} )</td>
</tr>
</tbody>
</table>

**Table 2: Effect of Inventory on Sales under PH contract**

**Note.** \( s_{PH,i} = q_{PH,i}^{PH} (s_{PH,i} < q_{PH,i}) \) means that the retailer understocked (overstocked) in product \( i = 1, 2 \).

**Stocking Decision.** In Stage 3, before the baseline valuation, \( A \) is revealed and conditional on wholesale prices \( w_{PH} = (w_{PH,1}, w_{PH,2}) \), the retailer sets her stocking quantity, \( q_{PH} \), with a clear expectation of how her choice will affect future retail prices. To determine her optimal stocking levels, the retailer will solve the following program

\[
\max_{q_{PH} \geq 0} E[p_{PH}^* \cdot s_{PH}^*] - w_{PH} \cdot q_{PH},
\]

where \( p_{PH}^* \) and \( s_{PH}^* \) are given by (6a) and (6b).

If \( N = 2 \), then profit maximization is achieved by taking \( q_{PH}^* = (q_{PH,1}^*, q_{PH,2}^*) \), where:

\[
q_{PH,i}^*(w_{PH}) = \begin{cases} 
\frac{\alpha(1+u) - \beta(1-u) - 4\alpha w_{PH,i}}{2(\alpha^2 - 2\alpha u)} & \text{if } w_{PH,i} \leq \frac{\beta u}{2\alpha} \\
\frac{\alpha(1+u) - \beta(1-u) - 4\alpha w_{PH,i}}{4\alpha^2 + 2\alpha \beta - 2\beta^2} & \text{if } \frac{\beta u}{2\alpha} < w_{PH,i} \leq u, \quad i = 1, 2 \\
\frac{1-w_{PH,i}}{2(\alpha + \beta)} & \text{if } u < w_{PH,i} \leq 1
\end{cases}
\]

\(^8\)As will be seen in Proposition 3, when \( N = 2 \) wholesale prices \( w_{PH,1}^* = w_{PH,2}^* \), to which the retailer responds by ordering \( q_{PH,1} = q_{PH,2} \). Since the cases when \( q_{PH,1} < q_{PH,2} \) or \( q_{PH,1} > q_{PH,2} \) are not on the equilibrium path, in the interest of space, we omit the presentation of these cases.
There are three stocking decision scenarios that the manufacturer’s wholesale price vector, \( w_{PH} \), can induce in equilibrium:

- A low wholesale price \( \left( w_{PH,i} \leq \frac{\beta u}{2a} \right) \) induces a high stocking quantity \( q_{PH}^* \) (i.e., see Scenario PH.1 in Table 2) such that the retailer will be over-stocked even when \( A = (1 + u, 1 + u) \).
- A moderate wholesale price \( \left( \frac{\beta u}{2a} < w_{PH,i} \leq u \right) \) induces a moderate stocking quantity \( q_{PH}^* \) (i.e., see Scenario PH.2 in Table 2) such that the retailer will be over-stocked in product \( i = 1, 2 \) when \( A_i = 1 - u \).
- A high wholesale price \( (u < w_{PH,i} \leq 1) \) induces a low stocking quantity \( q_{PH}^* \) (i.e., see Scenario PH.3 in Table 2) such that the retailer will be under-stocked for all realizations of \( A \).

If \( N = 1 \), the retailer stocks

\[
q_{PH}^* = \begin{cases} 
1 + \frac{u - 2w_{PH}}{2a}, & \text{if } w_{PH} \leq u \\
1 - \frac{w_{PH}}{2a}, & \text{if } u < w_{PH} \leq 1
\end{cases}
\]  

(7b)

In (7b), the low \( w_{PH} \) results in over-stocking only in the realization of \( A = 1 - u \), and the high \( w_{PH} \) results in under-stocking for all realizations of \( A \).

Which of the above scenarios play out in the equilibrium will be ultimately decided by the wholesale price, \( w_{PH} \), which is set by the manufacturer in Stage 2.

**Wholesale Pricing and Product Line Length Decisions.** In Stage 2, the manufacturer chooses wholesale prices, \( w_{PH} \), and the number of products, \( N \), that are optimal in

\[
\max_{w_{PH}, N} \Pi^{(N)}_{PH}(w_{PH}) = w_{PH} \cdot q_{PH}^*(w_{PH}) - Nf
\]  

(8a)

\[
\text{s.t. } \pi_{PH}(w_{PH}) = \mathbb{E}[p_{PH}^* \cdot s_{PH}^*] - w_{PH} \cdot q_{PH}^*(w_{PH}) \geq \pi^0,
\]  

(8b)

where \( q_{PH}^* \) is the retailer’s stocking decision of Stage 3 and (8b) is the retailer’s participation constraint. The next Proposition 3 depicts the solution to (8) and identifies two thresholds for the PH contract that help us predict the number of product version the manufacturer will sell and the prices he will set:

\( \hat{\pi}_{PH} \): This is a threshold for \( \pi^0 \) above which the manufacturer cannot sell through the retailer if he offers only a single product.

\( \hat{\beta}_{PH}(\pi^0) \): This is a threshold for \( \beta \) above which the manufacturer finds it not optimal to sell two
Both thresholds can be seen graphically in Figure 2b.

**Proposition 3.** Suppose the manufacturer offers the retailer a PH contract. There exist \( \hat{\beta}_{PH}(\pi^0) \in [0, \alpha \frac{1-u}{1+u}] \) and \( \hat{\pi}_{PH} \geq 0 \) such that if

1. \( 0 \leq \beta \leq \hat{\beta}_{PH}(\pi^0) \), then the manufacturer’s optimal strategy is to sell two products and set wholesale prices of

\[
\begin{align*}
\pi_{PH,1}^* = \pi_{PH,2}^* & = \begin{cases} 
\pi_{PH,hv}^{(2)}, & \text{if } u \geq \frac{2\alpha(2\alpha-\beta)+\beta-2\alpha}{2\alpha+\beta}, \\
\pi_{PH,mv}^{(2)}, & \text{if } \frac{2\alpha-\beta}{6\alpha-\beta} \leq u < \frac{2\alpha(2\alpha-\beta)+\beta-2\alpha}{2\alpha+\beta}, \\
\pi_{PH,lv}^{(2)}, & \text{if } 0 \leq u < \frac{2\alpha-\beta}{6\alpha-\beta},
\end{cases}
\end{align*}
\]

where \( \pi_{PH,hv}^{(2)}, \pi_{PH,mv}^{(2)} \) and \( \pi_{PH,lv}^{(2)} \) are included in the appendix.

2. \( \pi^0 \leq \hat{\pi}_{PH} \) and \( \beta > \hat{\beta}_{PH}(\pi^0) \), then the manufacturer’s optimal strategy is to sell a single product and set a wholesale price of

\[
\pi_{PH}^* = \begin{cases} 
\pi_{PH,hv}, & \text{if } u \geq \sqrt{2} - 1, \\
\pi_{PH,lv}, & \text{if } u < \sqrt{2} - 1,
\end{cases}
\]

where \( \pi_{PH,lv} \) and \( \pi_{PH,hv} \) are included in the appendix.

3. \( \pi^0 > \hat{\pi}_{PH} \) and \( \beta > \hat{\beta}_{PH}(\pi^0) \), then PH contract is infeasible to sell any products.

**Discussion.** In Stage 2, the forward looking manufacturer correctly anticipates how the retailer orders in Stage 3 and decides how many product versions to sell, \( N^* \). If \( \pi^0 > \hat{\pi}_{PH} \) and \( \beta > \hat{\beta}_{PH}(\pi^0) \), then the manufacturer sells nothing because the retailer’s reservation level, \( \pi^0 \), is so high that positive selling quantities would require wholesale prices that yield negative profits; thus, the PH contract becomes infeasible. (The area where the PH contract is infeasible is shown graphically as the unshaded area in Figure 2b.)

Note that in Figure 2(b) for \( \pi^0 > \hat{\pi}_{PH} \), the manufacturer will sell two product versions when \( \beta \leq \hat{\beta}_{PH}(\pi^0) \). That is, product line extension enables the manufacturer to engage a retailer who has an attractive outside option. To engage such a retailer with only one product version would require a very low wholesale price (too low to make the contract attractive for the manufacturer). In contrast, an extended product line appeals to the retailer for reasons that we already describe in Section 4: a longer product line allows the retailer to increase sales without having to cut retail prices. Thus an
extended product line can dominate her outside option while allowing the manufacturer to charge wholesale prices that make the PH contract appealing to him as well.

If \( \pi^0 \leq \hat{\pi}_{PH} \), then the PH contract is feasible and the manufacturer must decide between offering the retailer one or two product versions. (Note that \( \pi^0 \leq \hat{\pi}_{PH} \) is the shaded area left of \( \hat{\pi}_{PH} \) in Figure 2b.) To understand the intuition behind the manufacturer’s optimal product length, \( N^* \), it is (again) helpful to write the manufacturer’s profits from selling two product versions \( \Pi_{PH}^{(2)*} \) as a function of expansion \( EXP_{PH} \) and cannibalization \( CAN_{PH} \). Using the wholesale prices given in Proposition 3:

\[
\begin{align*}
\Pi_{PH}^{(2)*} &= \Pi_{PH}^{(1)*} + EXP_{PH} + CAN_{PH} - f, \tag{9}
\end{align*}
\]

where \( \Pi_{PH}^{(2)*} = w_{PH} \cdot q_{PH}, \) \( \Pi_{PH}^{(1)*} = w_{PH} q_{PH}', \) \( EXP_{PH} = w_{PH,2} q_{PH,2} > 0, \) and \( CAN_{PH} = w_{PH,1} q_{PH,1} - w_{PH} q_{PH} < 0. \) From (9), it is readily seen that the second product will be added whenever \( EXP_{PH} + CAN_{PH} - f > 0, \) for then we have \( \Pi_{PH}^{(2)*} > \Pi_{PH}^{(1)*} \). The threshold \( \hat{\beta}_{PH} \) for substitution coefficient \( \beta \) is obtained by setting \( \Pi_{PH}^{(2)*} = \Pi_{PH}^{(1)*} \).

With regard to the pricing decision, if the retailer faces demand with high variation (when \( u > \frac{2\sqrt{\alpha(2\alpha-\beta)+\beta-2\alpha}}{2\alpha+\beta} \)), then the manufacturer sets wholesale prices to induce moderate stocking levels at the retail location (corresponding to stocking scenario PH.2 in Table 2) when \( \pi^0 \) is small and high stocking levels (corresponding to stocking scenario PH.1 in Table 2) when \( \pi^0 \) is large. If she faces demand with low variation (when \( u \leq \frac{2\sqrt{\alpha(2\alpha-\beta)+\beta-2\alpha}}{2\alpha+\beta} \)), then he sets wholesale prices so as to induce low stocking levels (corresponding to scenario PH.3 in Table 2) when \( \pi^0 \) is small, and to induce medium or high stocking levels (corresponding to scenarios PH.1 and PH.2 in Table 2) when \( \pi^0 \) is large.

Interestingly, the next Proposition 4 asserts that the manufacturer in a decentralized supply chain can provide a longer product line with the PH contract than that in the centralized system of Section 4.

**Proposition 4.** There exists \( \hat{f}_{PH}(\beta, \pi^0) \) such that if \( \frac{(\alpha(1-u) - \beta(1+u))^2}{4\alpha(\alpha^2-\beta^2)} < f < \hat{f}_{PH}(\beta, \pi^0) \), then the manufacturer will always sell two product versions with the PH contract, but he will only sell a single product version in the centralized supply chain whenever the realized market potential is \( A = (1+u, 1-u) \) or \( A = (1-u, 1+u) \).
about by supply chain decentralization always shortens product lines (Villas-Boas, 1998; Netessine and Taylor, 2007). That result is obtained for deterministic demand environment, where the timing of quantity commitment has no effect on the product line decision. Proposition 4 shows that timing matters when demand is uncertain: a longer product line can arise purely because the quantity commitment is made under demand uncertainty. The result can be understood as follows. When selling in the centralized channel, the manufacturer first waits for demand to be fully revealed and then decides, which product designs to produce; thus, the manufacturer only produces what he sells. In contrast, with the PH contract, the retailer must decide both product variety and order quantities before demand is revealed, with the restriction that she can only sell what she ordered for. Thus a longer product line offers the retailer greater flexibility when she maximizes her sales revenue after demand is revealed. By ordering a longer product line, the retailer hedges against outcomes in which she bets on product $i \in \{1, 2\}$ at time 0 and then ends up seeing low demand for product $i$ and high demand for product $j \in \{1, 2\}, j \neq i$, at time 1. The forward-looking manufacturer understands this and has therefore a greater incentive to extend his product line to engage the retailer in the bet on two products.

We conclude by providing comparative statics on the thresholds $\hat{\pi}_{PH}$ and $\hat{\beta}_{PH}$ with respect to $\pi^0$ (retailer’s outside option), $u$ (demand variation), and $f$ (fixed setup cost).

**Corollary 1.**
1. $\hat{\pi}_{PH}$ decreases in $f$ and increases in $u$.
2. When $\pi^0 > \hat{\pi}_{PH}$, $\hat{\beta}_{PH}(\pi^0)$ decreases in $\pi^0$ and increases in $u$; when $\pi^0 \leq \hat{\pi}_{PH}$ and $u > \sqrt{2} - 1$, $\hat{\beta}_{PH}(\pi^0)$ increases in $\pi^0$.
3. $\hat{\beta}_{PH}(\pi^0)$ decreases in $f$.

Corollary 1 reveals that increasing $\pi^0$ and increasing $u$ gives the manufacturer a greater incentive to introduce a longer product line. The former follows because the manufacturer needs to offer sales increase opportunity to the retailer without compromising wholesale and retail prices. The latter follows because as $u$ increases, the retailer becomes more likely to order multiple product versions as a hedge against not knowing, which design(s) will be in high demand. Increasing setup cost $f$ has the opposite effect in that it gives the manufacturer a greater incentive to shorten the product line.
5.3 Pull Contract

With the PL contract, the manufacturer announces the number of product versions, $N$, wholesale prices, $w_{PL}$, and the production quantity, $q_{PL}$, at time 0. At time 1, the retailer sets retail prices, $p_{PL}$, that maximize her payoff, $(p_{PL} - w_{PL}) \cdot s_{PL}$, with the proviso that she cannot sell more than what the manufacturer produced at time 0, i.e., $s_{PL} \leq q_{PL}$. Figure 1 shows the sequence of events graphically and backward induction steps solve the PL subgame.

Retail Pricing and Retail Assortment Decision. In Stage 4, after the state of the world, $A$, is revealed, the retailer chooses a vector of retail prices, $p$, which determine her sales, $s(p, A)$. Then using her sales revenue, $p \cdot s(p, A)$, she pays $w_{PL} \cdot s(p, A)$ to the manufacturer. Thus, with the PL contract, the retailer chooses a vector of retail prices, $p$, so as to maximize her profit subject to an inventory constraint:

$$\max_{p \geq w_{PL}} (p - w_{PL}) \cdot s(p, A) \quad \text{s.t.} \quad s(p, A) \leq q_{PL}. \quad (10)$$

Notice that (10) differs from (5), because with the PL contract, wholesale prices are not sunk. As a result, after $A$ is revealed, the retailer may choose to drop one of the products in the manufacturer’s product line, which is something that we do not observe with the PH contract (we refer to this as retail assortment decision). The equilibrium retail assortment decisions are illustrated in Table 3. Note that $s_{PL,i}^* = 0$ means that the retailer drops product $i = 1, 2$ from the retail assortment. Solution to (10) is included in the appendix.

<table>
<thead>
<tr>
<th>A</th>
<th>PL.1: Low W. Price</th>
<th>PL.2: Moderate W. Price</th>
<th>PL.3: High W. Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(1 + u, 1 + u)$</td>
<td>$s_{PL}^* &gt; 0$</td>
<td>$s_{PL}^* &gt; 0$</td>
<td>$s_{PL}^* &gt; 0$</td>
</tr>
<tr>
<td>$(1 + u, 1 - u)$</td>
<td>$s_{PL}^* &gt; 0$</td>
<td>$s_{PL,1}^* &gt; 0, s_{PL,2}^* = 0$</td>
<td>$s_{PL,1}^* &gt; 0, s_{PL,2}^* = 0$</td>
</tr>
<tr>
<td>$(1 - u, 1 + u)$</td>
<td>$s_{PL}^* &gt; 0$</td>
<td>$s_{PL,1}^* = 0, s_{PL,2}^* &gt; 0$</td>
<td>$s_{PL,1}^* = 0, s_{PL,2}^* &gt; 0$</td>
</tr>
<tr>
<td>$(1 - u, 1 - u)$</td>
<td>$s_{PL}^* &gt; 0$</td>
<td>$s_{PL}^* &gt; 0$</td>
<td>$s_{PL}^* = 0$</td>
</tr>
</tbody>
</table>

Table 3: Effect of Wholesale Price on Sales under PL contract

Wholesale Pricing, Stocking and Product Line Length Decisions. In Stage 2, the manufacturer chooses wholesale prices, $w_{PL}$, production quantities, $q_{PL}$, and the number of products, $N,$
that are optimal in:

\[
\max_{q_{PL} \text{,} w_{PL} \text{,} N} \Pi_{PL}^{(N)} (w_{PL}) = w_{PL} \cdot E[s_{PL}^*] - Nf \tag{11a}
\]

\[
\text{s.t. } \pi_{PL} (w_{PL}) = E[(p_{PL} - w_{PL}) \cdot s_{PL}^*] \geq \pi^0. \tag{11b}
\]

where \( p_{PL}^* \) is the retailer’s price decision of Stage 3 and (11b) is the retailer’s participation constraint.

The next Proposition 5, depicts the solution to (11).

**Proposition 5.** Suppose the manufacturer offers the retailer a PL contract. There exist \( \hat{\beta}_{PL}(\pi^0) \leq \frac{1-u}{1+u} \) and \( \hat{\pi}_{PL} \) such that if

1. \( 0 \leq \beta \leq \hat{\beta}_{PL}(\pi^0) \), then the manufacturer’s optimal strategy is to sell two product versions and to set wholesale prices of

\[
w_{PL,1}^* = w_{PL,2}^* \begin{cases} 
  w_{PL,hv}, & \text{if } u \geq u_1(\beta) \\
  w_{PL,mv}, & \text{if } u_2(\beta) < u < u_1(\beta) \\
  w_{PL,lv}, & \text{if } 0 \leq u \leq u_2(\beta), 
\end{cases}
\]

where \( w_{PL,hv}^*, w_{PL,mv}^*, w_{PL,lv}^* \), \( u_1(\beta) \), and \( u_2(\beta) \) are included in the appendix. The equilibrium production quantity \( q_{PL,i}^* \):

\[
q_{PL,i}^* = \frac{\alpha(1+u-w_{PL,i}^*)-\beta(1-u-w_{PL,i})}{2\alpha^2-2\beta^2}.
\]

2. \( \pi^0 \leq \hat{\pi}_{PL} \) and \( \beta > \hat{\beta}_{PL}(\pi^0) \), then the manufacturer’s optimal strategy is to sell a single product and to set a wholesale price of

\[
w_{PL}^* = \begin{cases} 
  w_{PL,hv}, & \text{if } u > \sqrt{2} - 1, \\
  w_{PL,lv}, & \text{if } u \leq \sqrt{2} - 1, 
\end{cases}
\]

where \( w_{PL,hv} \) and \( w_{PL,lv} \) are included in the appendix. The equilibrium production quantity \( q_{PL}^* = \frac{1+u-w_{PL}^*}{2\alpha} \).

3. \( \pi^0 > \hat{\pi}_{PL} \) and \( \beta > \hat{\beta}_{PL}(\pi^0) \), then PL contract is infeasible to sell any products.

**Discussion.** The PL contract shares many similarities with the PH contract of Section 5.2. Since the intuition behind equilibrium product line design and pricing is similar to that of the PH contract, we skip the discussion so as to avoid repetition.\(^9\) The biggest difference is in the inventory risk allocation. The PH contract imposes the inventory risk onto the retailer, as the retailer placing order \textit{before} demand is revealed and the manufacturer producing the exact amount. The PL contract leaves

\(^9\)Counterparts of Proposition 4 and Corollary 1 hold for the PL contract. We omit them for brevity.
the inventory risk to the manufacturer, as the manufacturer decides and produces the product line (incurring fixed cost) without knowing for sure whether the retailer will sell every product version in the line (Table 3 shows the retailer drops product in some demand scenarios). Such difference decides that the manufacturer’s preference over the two contracts can be influenced by the level of demand uncertainty faced by the supply chain. For example, as we will show in the Section 5.4, the manufacturer might prefer PH over PL when demand variation is high.

5.4 Push, Pull, or Instantaneous Fulfillment?

SPNE. In Sections 5.1 through 5.3, we examined the equilibrium product lines and prices conditional on the supply contract type \( j \in \{IF, PH, PL\} \). Now we turn to Stage 1 of the game, where the forward-looking manufacturer decides which contract type to offer to the retailer. Proposition 6 details the SPNE and the remainder of this section describes the intuition behind the SPNE.

**Proposition 6.** There exist \( \hat{\pi}, \hat{\beta}(\pi^0), \underline{\beta}_{PHPL}(\pi^0), \) and \( \bar{\beta}_{PHPL}(\pi^0) \) such that if

1. \( 0 \leq \pi^0 \leq \hat{\pi}_{IF} \), then the manufacturer’s optimal strategy is to offer the retailer an IF contract.
2. \( \pi^0 > \hat{\pi}_{IF} \) and \( \beta \leq \hat{\beta}(\pi^0) \), then the manufacturer’s optimal strategy is to sell two product versions and offer the retailer a PH contract whenever \( \underline{\beta}_{PHPL}(\pi^0) < \beta < \min\{ \bar{\beta}_{PHPL}(\pi^0), \hat{\beta}(\pi^0) \} \) and a PL contract otherwise.
3. \( \hat{\pi}_{IF} < \pi^0 \leq \hat{\pi} \) and \( \beta > \hat{\beta}(\pi^0) \), where \( \hat{\pi} = \max\{ \hat{\pi}_{PH}, \hat{\pi}_{PL} \} \), then the manufacturer’s optimal strategy is to sell a single product version and offer the retailer a PH contract whenever \( \sqrt{2} - 1 \leq u \leq 1 \) and a PL contract whenever \( 0 \leq u \leq \sqrt{2} - 1 \).
4. \( \pi^0 > \max\{ \hat{\pi}, \hat{\pi}_{IF} \} \) and \( \beta > \hat{\beta}(\pi^0) \), then all three contracts are infeasible.

The SPNE that we advance in Proposition 6 is driven by three parameters. The first parameter, coefficient of substitution, \( \beta \), measures product substitutability (see Equation 1). The second parameter, retailer’s reservation level, \( \pi^0 \), is a proxy for the retailer’s outside option. The third parameter, the coefficient, \( u \), drives both the variability of retail demand and the elasticity of wholesale demand, the latter is a metric that we define later in this section (see 12a). Figure 3 illustrates the SPNE graphically in the parameter space of \( \beta \) and \( \pi^0 \).

Having three parameters, we generally find that sketching the SPNE using simple, intuitive rules is by and large difficult to do. The task, however, becomes more manageable if one breaks up the
parameter space into smaller regions for which separate intuition can be developed. In the next Corollary 2 we identify three such regions.

Corollary 2. Let

\[
\beta(\pi^0) = \begin{cases} 
\min\{\hat{\beta}_{PH}(\pi^0), \hat{\beta}_{PL}(\pi^0), \hat{\beta}_{IF}\} & \text{and} \\
\min\{\hat{\beta}_{PH}(\pi^0), \hat{\beta}_{PL}(\pi^0)\} & \hat{\beta}(\pi^0) = \begin{cases} 
\max\{\hat{\beta}_{PH}(\pi^0), \hat{\beta}_{PL}(\pi^0), \hat{\beta}_{IF}\}, & \text{if } \pi^0 \leq \hat{\pi}_{IF} \\
\max\{\hat{\beta}_{PH}(\pi^0), \hat{\beta}_{PL}(\pi^0)\}, & \text{if } \pi^0 > \hat{\pi}_{IF} 
\end{cases}
\end{cases}
\]

With these definitions, we identify the following regions.

(Low-\(\beta\)). If \(\beta \leq \beta(\pi^0)\), then the manufacturer will sell two product versions, whatever the contract.

(Medium-\(\beta\)). If \(\beta(\pi^0) < \beta < \hat{\beta}(\pi^0)\), then the product line length will vary across contracts.

(High-\(\beta\)). If \(\beta \geq \hat{\beta}(\pi^0)\), then the manufacturer will at most sell a single product version, whatever the contract.

Contracting on Quantity (Low-\(\beta\) Case and High-\(\beta\) Case). In the low- and high-\(\beta\) regions identified in Corollary 2, the coefficient of substitution is either so low or so high that the manufacturer’s contract choice at most affects how much the retailer orders; it does not, however, affect the
product line length. This is because the strong market expansion (more precisely, the net effect of market expansion, cannibalization, and fixed cost, $f$) generates a first order effect that trumps any influence the contract choice might have on the product line length. The manufacturer’s equilibrium choices are settled in the following two corollaries, which are established using the results given in Proposition 6.

**Corollary 3** (Low-$\beta$ Region). If $0 \leq \beta \leq \beta(\pi^0)$, then the manufacturer’s optimal strategy is to sell two product versions and offer the retailer

1. IF contract whenever $0 \leq \pi^0 \leq \hat{\pi}_{IF}$.
2. PH contract whenever $\beta_{PHPL}(\pi^0) < \beta < \min\{\beta_{PHPL}(\pi^0), \hat{\beta}(\pi^0)\}$ and $\pi^0 > \hat{\pi}_{IF}$, with $\beta_{PHPL}(\pi^0) = 0$ for $\sqrt{2} - 1 < u \leq 1$ and $\beta_{PHPL}(\pi^0) > 0$ for $0 \leq u < \sqrt{2} - 1$.
3. PL contract whenever the conditions in 1 and 2 fail to hold.

**Corollary 4** (High-$\beta$ Region). If $\beta > \beta(\pi^0)$ and $\pi^0 < \hat{\pi}$, then the manufacturer’s optimal strategy is to sell a single product version and offer the retailer

1. IF contract whenever $0 \leq \pi^0 \leq \hat{\pi}_{IF}$.
2. PH contract whenever $\sqrt{2} - 1 < u \leq 1$ and $\hat{\pi}_{IF} < \pi^0 \leq \hat{\pi}$.
3. PL contract whenever $0 \leq u < \sqrt{2} - 1$ and $\hat{\pi}_{IF} < \pi^0 \leq \hat{\pi}$.

**Discussion.** As we explain in Section 5.1, the IF contract has a positive effect on reducing wasteful ordering, it can also have a (negative) strategic effect in increasing retail prices and reducing retail profits. The IF contract is therefore unappealing to retailers with high $\pi^0$. To engage the retailer, the manufacturer optimally chooses either the PH or the PL contract.

He does that by comparing PH and PL contract profits. In general, for a given product line length $N$, the manufacturer will prefer a PH contract over a PL contract if and only if $\Delta_{PL,PH}^{(N)} \equiv \Pi_{PH}^{(N)*} - \Pi_{PL}^{(N)*} > 0$, where $\Pi_{PH}^{(N)*} = \sum_{i=1}^{N} (w_{PH,i}^* q_{PH,i}^* - f)$ and $\Pi_{PL}^{(N)*} = \sum_{i=1}^{N} (w_{PL,i}^* E[s_{PL,i}^*] - f)$ (see 8a and 11a).

Establishing whether or not $\Delta_{PL,PH}^{(N)} > 0$, is particularly easy if $w_{PH}^* \leq w_{PL}^*$ and $q_{PH}^* \geq E[s_{PL}^*]$. In equilibrium, however, we find that $w_{PH}^* \leq w_{PL}^*$ and $q_{PH}^* \geq E[s_{PL}^*]$. In other words, PH contract represents a low-price-high-quantity strategy, and PL contract represents a high-price-low-quantity strategy. To understand the ordering of prices and quantities under both contracts, note that the PH contract exposes the retailer to inventory risk. To bear this inventory risk, the retailer is offered
a lower wholesale price with the PH contract than with the PL contract. The retailer responds to this lower wholesale price of PH contract by ordering more than average sales (so as to avoid not having enough stock when demand is high). In contrast, with the PL contract, the retailer matches supply with demand by ordering the exact amount she wants to sell.

Anticipating the equilibrium outcome of the PH contract and the equilibrium outcome of the PL contract, the manufacturer faces a discrete, wholesale demand curve with two demand scenarios, \((w^*_\text{PH}, q^*_\text{PH})\) and \((w^*_\text{PL}, \mathbb{E}[s^*_\text{PL}])\). Then, the manufacturer must decide the net effect of choosing between the low-price-high-quantity PH contract and the high-price-low-quantity PL contract. The concept of demand elasticity helps to provide the answer. We can rewrite \(\Delta^{(N)}_{\text{PH},\text{PL}}\) as

\[
\Delta^{(N)}_{\text{PL},\text{PH}} = \sum_{i=1}^{N} \mathbb{E}[s^*_\text{PL,i}] \left( w^*_\text{PL,i} - w^*_\text{PH,i} \right) \left( \eta_{\text{PL},\text{PH},i} - 1 \right)
\]

where

\[
\eta_{\text{PL},\text{PH},i} \equiv \frac{q^*_\text{PH,i} - \mathbb{E}[s^*_\text{PL,i}]}{\mathbb{E}[s^*_\text{PL,i}] w^*_\text{PH,i}} \geq 0.
\]

Because \(w^*_\text{PL} > w^*_\text{PH}\),

\[
\Delta^{(N)}_{\text{PL},\text{PH}} \geq 0 \iff \eta_{\text{PL},\text{PH},i} \geq 1, \quad \forall i \in \{1, N\}.
\]

The measure \(\eta_{\text{PL},\text{PH},i}\) represents the manufacturer’s demand elasticity when making a discrete move from one point on the wholesale demand curve, \((w^*_\text{PL}, \mathbb{E}[s^*_\text{PL}])\), to another point, \((w^*_\text{PH}, q^*_\text{PH})\). The numerator represents percentage in (expected) quantity gain associated with choosing contract PH over contract PL. The denominator represents the percentage in price gain associated with choosing contract PL over PH. When \(\eta_{\text{PL},\text{PH},i} > (\leq)1, \forall i \in \{1, N\}\), the percentage change in quantity demanded is greater (smaller) than that in price. We say that in such a case wholesale demand is elastic (inelastic). Consequently the manufacturer will prefer PH to PL (PL to PH) when demand is elastic (inelastic).

In the single-product case (Corollary 4, high-\(\beta\) case), wholesale demand becomes elastic as \(u\) increases. This is because high \(u\) indicates to the retailer high market potential, which increases her willingness to stock high quantities under the PH contract. Thus, high demand variation \(u\) increases

\[\text{Recall from Propositions 3 and 5 that } w_{j,1} = w_{j,2} \text{ and } q_{j,1} = q_{j,2}, \ j \in \{\text{PH, PL}\}. \text{ Therefore } \eta_{\text{PL},\text{PH},i} > 1 \text{ for all } i = 1, 2 \text{ or } \eta_{\text{PL},\text{PH},i} \leq 1 \text{ for all } i = 1, 2.\]
the manufacturer’s incentive to choose PH over PL.

In the two-product case (Corollary 3, low-β case), the coefficient of substitution, β, has an added role to play. When products are highly differentiable (low β), the second product brings strong market expansion and little cannibalization. Distributing the product line is similar to distributing two independent products, and our intuition from the single-product case largely applies: PH contract is preferred when \( u \) is high, and PL contract is preferred when \( u \) is low. As products become more substitutable (increasing β), due to the increasing market cannibalization, ordering more inventory for one product version has weaker effect on increasing sales. Thus, a wholesale price cut does not induce a significant increase in the retailer’s order, and wholesale demand becomes less elastic. PL contract becomes more attractive as products becomes more substitutable. For moderately substitutable products, PH and PL are close choices, and the level of the retailer’s reservation payoff, \( \pi^0 \), can affect the contract choice decision in one direction or the other. In particular, when \( \pi^0 \) is very high, it is impossible for the manufacturer to engage the retailer by charging high wholesale prices; instead, high \( \pi^0 \) forces the manufacturer to play his low-price-high-quantity strategy via PH contract.

**Contracting on Both Quantity and Product Line Length (Medium-β Case)** In the medium-β region identified in Corollary 2, the coefficient of substitution is at an intermediate level and the manufacturer’s contract choice affects both the retailer’s order quantities as well as the retail assortment. As such, it is a region where all the following forces are in play: (1) Retailer’s reservation payoff, which drives the optimality of the IF contract. (2) Market expansion, which decides the equilibrium product line length. We have seen this force in the subgames, where we were studying pricing and product line length decisions conditional on the contract type. (3) Demand elasticity, which affects the manufacturer’s preference over the PH and PL contracts. We have seen this force in our preceding discussion, where the manufacturer chooses his most preferred contract type conditional on pre-determined values of product line length \( N \).

The role of \( \pi^0 \) in the medium-β region is analogous to the role \( \pi^0 \) plays in the low-β and the high-β regions. Therefore when \( \pi^0 \) is sufficiently high, the manufacturer ends up choosing between the PH and PL contracts and his choice is again driven by elasticity and market expansion effects. What makes the medium-β region different from the low-β and the high-β regions is that the elasticity
and expansion effects interact—PH contract and PL contract lead to different product line lengths. Fortunately, the manner in which these effects interact is additive and much of the intuition we have given so far remains valid in this region as well. To see that, note that without loss of generality, the profit difference between two contracts under different lengths of product line can be written as:

\[
\Pi^{(2)*}_k - \Pi^{(1)*}_j = \left( \Pi^{(2)*}_k - \Pi^{(2)*}_j \right) + \left( \Pi^{(2)*}_j - \Pi^{(1)*}_j \right) = \Delta^{(2)}_{j,k} + EXP_j + CAN_j - f,
\]

where

\[
\Pi^{(2)*}_j - \Pi^{(1)*}_j = EXP_j + CAN_j - f, \quad j \in \{PH, PL\}
\]

\[
\Delta^{(2)}_{j,k} \equiv \Pi^{(2)*}_k - \Pi^{(2)*}_j, \quad j, k \in \{PH, PL\}, \quad k \neq j.
\]

To illustrate how the additivity property drives the manufacturer’s SPNE choices, let us consider a scenario in which the manufacturer prefers to offer two versions of product under PH contract and only a single version under the PL contract. When comparing \(\Pi^{(2)*}_{PH}\) and \(\Pi^{(1)*}_{PL}\),

\[
\Pi^{(2)*}_{PH} - \Pi^{(1)*}_{PL} = \Delta^{(2)}_{PL,PH} + EXP_{PL} + CAN_{PL} - f,
\]

because \(N^* = 1\) with the PL contract, then \(EXP_{PL} + CAN_{PL} - f < 0\). If the manufacturer’s wholesale demand is elastic (recall, that is, when \(\eta_{PL,PH,i} > 1, \forall i \in \{1, 2\}\), then \(\Delta^{(2)}_{PL,PH} > 0\). When this positive effect from elastic demand is strong enough to “correct” the negative market expansion under the PL contract, the manufacturer would prefer to sell two product designs and only the PH contract enables him to deliver them. If the manufacturer’s wholesale demand is inelastic (i.e., when \(\eta_{PL,PH,i} < 1, \forall i \in \{1, 2\}\), then \(\Delta^{(2)}_{PL,PH} < 0\), which means offering two product designs under the PH contract is dominated by a single-version PL contract.

Given the additive nature of the market expansion and demand elasticity forces, based on the intuition that we have given, we conclude that as \(\pi^0\) and \(u\) increase and \(\beta\) decreases, offers of longer product lines sold through some PH contract should be expected (Figure 3a). On the other hand, as \(\pi^0\) and \(u\) decrease and \(\beta\) increases, offers of a shorter product line sold under the PL contract should be expected (Figure 3b). Although this provides basic intuition, with so many parameters in play, exact outcomes, in general, can be only determined by a careful comparisons of expected payoffs, as we have done in Proposition 6.
Example 1. Table 4 presents data on how much better the manufacturer can do by being able to adjust his product line length and to change the contract type. In computing these results, we take PH contract with a single product as our benchmark and then investigate how much the manufacturer’s profit increases above the benchmark if he switches to the SPNE choice identified in Proposition 6. The data shows that the manufacturer’s expected payoff can increase by more than 60% (this is seen in the last column of the table). This illustrates that choosing the right product line length and the right contracting strategy are first-order effects.

Table 4: Managing Contracts and the Product Line Length

<table>
<thead>
<tr>
<th>u</th>
<th>f</th>
<th>β</th>
<th>π₀</th>
<th>Equilibrium Contract Type</th>
<th>N*</th>
<th>( \frac{\Pi^* - \Pi^{(1)<em>}_{PH}}{\Pi^{(1)</em>}_{PH}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.08</td>
<td>0.5</td>
<td>0.05</td>
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<td>0.5</td>
<td>0.13</td>
<td>PL</td>
<td>1</td>
<td>7.3%</td>
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<td>0.1</td>
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<td>1 or 2</td>
<td>42.1%</td>
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<td>33.4%</td>
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<td>0.13</td>
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</tr>
<tr>
<td>0.44</td>
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6 Conclusion

One market expansion strategy that many manufacturers seem to follow involves developing a product line with several product variants designed to meet different customer needs. However, as it turns out, independent retailers often give manufacturers a pushback on how many product variants should
be included in the product line and in what quantities the variants should be stocked. The point of this paper is to explore how a manufacturer who has designed a product line can use simple, price-only supply chain contracts to engage a retailer in his pursuit of profit maximization.

To do this, we consider a supply chain with an upstream manufacturer capable of producing multiple product designs and instantaneous fulfillment, and a downstream retailer who faces stochastic demand. The manufacturer distributes his output through the retailer using one of the following variations of the classical wholesale contract: push, pull, or instantaneous fulfillment. The contracts differ in when wholesale prices are set and in how inventory risk is allocated. With push and pull (instantaneous fulfillment), wholesale prices are set before (after) demand is revealed; in terms of inventory risk, the push (pull) contract exposes the retailer (manufacturer); with instantaneous fulfillment, both firms can avoid inventory risk.

Whereas the classic newsvendor models focus on maximizing profit through matching supply with demand, this paper explores how firms can exploit multiple levers – quantity, price, and product line length – and timing of those decisions to achieve profit maximization.

For the benchmark case of a manufacturer selling directly to consumers (i.e., an integrated supply chain), we find that lengthening product line is a profitable strategy for a range of setup costs. This result verifies the intuition that lengthening the product line is fundamentally about increasing sales without having to cut retail price significantly.

In a decentralized supply chain, where the manufacturer selling through a retailer, we find that the equilibrium product length and the equilibrium contracting strategy are determined by three important drivers: the retailer’s outside option, demand variability, and product substitutability.

First, we demonstrate that instantaneous fulfillment, although an optimal strategy to follow in an integrated supply chain, leads to high double-marginalization in the decentralized chain. Thus the manufacturer would not want to utilize his instantaneous fulfillment capability when the retailer has an attractive outside option. Instead, the manufacturer will choose to make commitment on product line length, quantity, and wholesale prices before demand uncertainty is resolved, by offering a PL contract; alternatively, he will make product line and wholesale price commitment and ask the retailer to make quantity commitment before demand uncertainty is resolved, by offering a PH contract. The PH (PL) contract exposes the retailer (manufacturer) to inventory risk.

Second, the choice between PH contract and PL contract is driven by the wholesale demand
elasticity induced by the two contracts, and the product line length decision is driven by the market effect of introducing a second product (i.e., the sum of market expansion and market cannibalization net the fixed cost). When products are highly differentiated (substitutable), the market effect is of first order, and the firm would offer a full (shorter) product line under either PH or PL contract. In distributing a given product line, PH contract represents a low-price-high-quantity strategy, and PL contract represents a high-price-low-quantity strategy. The PH (PL) contract is more effective when the manufacturer’s wholesale demand is elastic (inelastic), i.e., when the retailer’s quantity increase in response to the manufacturer’s wholesale price cut is much more (less) significant under PH than under PL. We find that the wholesale demand becomes elastic as retail demand becomes more variable. Interestingly, both contracts, because of the exposure to inventory risk, may incentivize the manufacturer to supply a product line longer than what he would when selling directly to consumers, because a longer product line can serve as a hedge against demand uncertainty. (Note that in the centralized supply chain, the manufacturer does not need to hedge against demand uncertainty because he waits for demand to be revealed before making any decisions.)

Finally, when products are moderately differentiated, the manufacturer’s contract choices matter in his product line length decision because the market effect of the second product greatly varies, depending on demand variability and supply contract type. When product differentiation is moderately high and demand variability is relatively high, we find that the manufacturer uses the PH contract to offer a longer product line (which PL contract does not allow); when product differentiation is moderately low and demand variability is relatively low, the manufacturer choose the PL contract to sell a shorter product line (which PH contract does not allow). Our numerical examples show that adjusting product line length and contract type offered to the retailer, if done smartly, can improve the manufacturer’s profit significantly while keeping the retailer engaged.

References


A Proof of Proposition 1

Proof. If the integrated firm decides to produce one product version, then given the realized market potential \( A \) with \( A_i \geq A_j \) for \( i, j = 1, 2 \) and \( i \neq j \), the integrated firm will produce product \( i \). The integrated firm’s problem is \( \max_{p \geq 0} p(s, A_i) - f \). This is equivalent to \( \max_{s \geq 0} p(s, A_i) s - f \), where \( p(s, A_i) = A_i - \alpha s \). The first order condition leads to \( s^* = \frac{A_i}{2\alpha} \) and \( p^* = \frac{A_i}{2} \). The integrated firm’s optimal profit with a single product version is \( \Pi^{(1)*}_{\text{no retailer}}(A) = \frac{A_i^2}{4\alpha} - f \).

If the integrated firm decides to produce two product versions, then given the realized market potential \( A \), the integrated firm’s problem is \( \max_{p \geq 0} p \cdot s(p, A) - 2f \). This is equivalent to \( \max_{s \geq 0} p(s, A) \cdot s - 2f \), where \( p(s, A) = (A_1 - \alpha s_1 - \beta s_2, A_2 - \alpha s_2 - \beta s_1) \). The first order condition leads to \( s^* = \left( \frac{\alpha A_1 - \beta A_2}{2(\alpha^2 - \beta^2)}, \frac{\beta A_1 - \alpha A_2}{2(\alpha^2 - \beta^2)} \right) \) and \( p^* = \left( \frac{A_1}{2}, \frac{A_2}{2} \right) \). The integrated firm’s optimal profit with two product versions is \( \Pi^{(2)*}_{\text{no retailer}}(A) = \frac{\alpha(A_1^2 + A_2^2) - 2\beta A_1 A_2}{4(\alpha^2 - \beta^2)} - 2f \).

A comparison of the firm’s profit with one and two product versions reveals

\[
\Pi^{(2)*}_{\text{no retailer}}(A) - \Pi^{(1)*}_{\text{no retailer}}(A) = \frac{(\alpha A_1 - \beta A_2)^2}{4\alpha(\alpha^2 - \beta^2)} - f.
\]

Thus, if \( f \leq \frac{(\alpha A_1 - \beta A_2)^2}{4\alpha(\alpha^2 - \beta^2)} \), then \( N = 2 \) is optimal; otherwise \( N = 1 \) is optimal. Therefore,

\( a \) if \( f \leq \frac{(\alpha(1-u) - \beta(1+u))^2}{4\alpha(\alpha^2 - \beta^2)} \), then \( N^* = 2 \) for all realizations of \( A \);

\( b \) if \( \frac{(\alpha(1-u) - \beta(1+u))^2}{4\alpha(\alpha^2 - \beta^2)} < f \leq \frac{(1-u)^2(\alpha - \beta)}{4\alpha(\alpha + \beta)} \), then \( N^* = 2 \) whenever \( A = (1+u, 1+u) \) or \( (1-u, 1-u) \), \( N^* = 1 \) whenever \( A = (1+u, 1-u) \) or \( (1-u, 1+u) \);

\( c \) if \( \frac{(1-u)^2(\alpha - \beta)}{4\alpha(\alpha + \beta)} < f \leq \frac{(1+u)^2(\alpha - \beta)}{4\alpha(\alpha + \beta)} \), then \( N^* = 2 \) whenever \( A = (1+u, 1+u) \), \( N^* \leq 1 \) otherwise;

and

\( d \) if \( f > \frac{(1+u)^2(\alpha - \beta)}{4\alpha(\alpha + \beta)} \), then \( N^* \leq 1 \) for all realizations of \( A \).

\( \square \)

B Proof of Proposition 2

Proof. The equilibrium is derived using backward induction.

Retailer’s Pricing Problem. Given the wholesale prices \( w_{IF} \) and the realized market potential \( A \) the retailer sets retail prices so as to achieve

\[
\max_{p \geq w_{IF}} (p - w_{IF}) \cdot s(p, A) \quad \text{s.t.} \quad s(p, A) \leq q_{IF}.
\]
and $N$ is large enough due to zero production cost so the constraint $s(p, A) \leq q_{IF}$ is never binding in equilibrium. The first-order condition leads to

$$p^*_i = \begin{cases} 
\left(\frac{A_1+w_{IF,1}}{2}, \infty\right) & \text{if } N = 1 \text{ and } A_1 \geq A_2 \\
\left(\frac{A_2+w_{IF,2}}{2}, \infty\right) & \text{if } N = 1 \text{ and } A_1 \leq A_2 \\
\left(\frac{A_1+w_{IF,1}}{2}, \frac{A_2+w_{IF,2}}{2}\right) & \text{if } N = 2
\end{cases}$$

which induce a level of sales $s^*_{IF,i} = \frac{\alpha(A_i-w_{IF,i})-\beta(A_i-w_{IF,i})}{2\alpha^2-2\beta^2}$ when $N = 2$ and $s^*_{IF,i} = \frac{A_i-w_{IF,i}}{2\alpha}$ when $N = 1$ and $A_i \geq A_j$, where $i, j = 1, 2, i \neq j$.

**Manufacturer’s Pricing Problem.** If the manufacturer decides to offer a single product version, then he will offer product $i$ whenever $A_i \geq A_j$ for $i, j = 1, 2$ and $i \neq j$. The manufacturer’s problem is to find wholesale prices that are optimal in

$$\max_{w \geq 0} w s^*_{IF,i}(w) - f = w \frac{A_i-w}{2\alpha} - f.$$ 

The first order condition leads to $w^*_{IF} = A_i/2$. The manufacturer’s optimal production quantity will equal his sales, $\frac{A_i}{4\alpha}$. Thus, given $A_i \geq A_j$, the manufacturer’s optimal profit with a single product version is

$$\Pi^{(1)*}_{IF}(A) = \frac{A_i^2}{8\alpha} - f.$$ 

The retailer earns

$$\pi^{(1)*}_{IF}(A) = \frac{A_i^2}{16\alpha}.$$ 

If the manufacturer decides to offer two product versions, his problem is

$$\max_{w \geq 0} ws^*_{IF}(w) - 2f.$$ 

The first order condition leads to $w^*_{IF,1} = A_1/2$ and $w^*_{IF,2} = A_2/2$. The product quantities will be $s^*_{IF,i}$, $i = 1, 2$. The firms’ profits are

$$\Pi^{(2)*}_{IF}(A) = \frac{\alpha(A_1^2 + A_2^2) - 2\beta A_1 A_2}{8\alpha^2 - 8\beta^2} - 2f \quad \text{and} \quad \pi^{(2)*}_{IF}(A) = \frac{\alpha(A_1^2 + A_2^2) - 2\beta A_1 A_2}{16\alpha^2 - 16\beta^2}.$$ 

A comparison of the manufacturer’s profit with one and two product versions reveals

$$\Pi^{(2)*}_{IF}(A) - \Pi^{(1)*}_{IF}(A) = \frac{(\alpha A_i - \beta A_j)^2}{8\alpha^2 - 8\beta^2} - f.$$ 

Thus $\Pi^{(2)*}_{IF}(A) > \Pi^{(1)*}_{IF}(A)$ if and only if $f < \frac{(\alpha A_i - \beta A_j)^2}{8\alpha^2 - 8\beta^2}$. Therefore,

(a) if $f < \frac{(\alpha(1-u)-\beta(1+u))^2}{8\alpha(\alpha^2-\beta^2)}$, then $N^* = 2$ for all realizations of $A$;

(b) if $\frac{(\alpha(1-u)-\beta(1+u))^2}{8\alpha(\alpha^2-\beta^2)} < f < \frac{(1-u)^2(\alpha-\beta)}{8\alpha(\alpha+\beta)}$, then $N^* = 2$ whenever $A = (1+u, 1+u)$ or $(1-u, 1-u)$, and $N^* = 1$ whenever $A = (1+u, 1-u)$ or $(1-u, 1+u)$;

(c) if $\frac{(1-u)^2(\alpha-\beta)}{8\alpha(\alpha+\beta)} < f < \frac{(1+u)^2(\alpha-\beta)}{8\alpha(\alpha+\beta)}$, then $N^* = 2$ whenever $A = (1+u, 1+u)$ and $N^* = 1$;

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The equilibrium is derived using backward induction. Then we obtain the equilibrium outcome when $N^* \leq 1$ for all realizations of $A$.

The manufacturer's expected profit $E[\Pi^{(N^*)}_{IF}(A)] = E[\max\{\Pi^{(1^*)}_{IF}(A), \Pi^{(2^*)}_{IF}(A2)\}]$ is

$$E[\Pi^{(N^*)}_{IF}(A)] = \begin{cases} \frac{\alpha(1+u^2) - 8u^2f - \beta + 8u^2f}{4(\alpha^2 - \beta^2)}, & \text{if } f \leq \frac{(1-u)(u+u^2)}{8u(\alpha^2 - \beta^2)} \\ \frac{\alpha(3+2u+u^2) - 4\beta f + 5u^2 + 6u + 5}{16\alpha(\alpha + \beta)}, & \text{if } \frac{(1-u)(u+u^2)}{8u(\alpha^2 - \beta^2)} < f \leq \frac{(1-u^2)(\alpha - \beta)}{8u(\alpha^2 - \beta^2)} \\ \frac{-40a^2f + \alpha(-40\beta f + 5a^2 + 6u + 5)}{16\alpha(\alpha + \beta)}, & \text{if } \frac{(1-u^2)(\alpha - \beta)}{8u(\alpha^2 - \beta^2)} < f \leq \min\{\frac{(1-u^2)(\alpha - \beta)}{8u(\alpha^2 - \beta^2)}, \frac{(1-u^2)}{8u}\} \\ \frac{-8a^2f + 2a^2 - 8f + 5a^2 + 6u + 5 + \beta(u+1)^2}{16a(\alpha + \beta)}, & \text{if } \min\{\frac{(1-u^2)(\alpha - \beta)}{8u(\alpha^2 - \beta^2)}, \frac{(1-u^2)}{8u}\} < f \leq \frac{(1-u)(u+u^2)}{8u} \\ \frac{\frac{3}{4} \left(\frac{(u+1)^2}{8u} - f\right)}{8a}, & \text{if } f > \frac{(1-u)(u+u^2)}{8u} \end{cases}$$

The retailer's corresponding expected profit is

$$E[\pi^{(N^*)}_{IF}(A)] = \begin{cases} \frac{\alpha(1+u^2) - \beta}{8(\alpha - \beta)^2}, & \text{if } f \leq \frac{(1-u)(u+u^2)}{8u(\alpha^2 - \beta^2)} \\ \frac{\alpha(3+2u+u^2) - 4\beta f + 5u^2 + 6u + 5}{16a(\alpha + \beta)}, & \text{if } \frac{(1-u)(u+u^2)}{8u(\alpha^2 - \beta^2)} < f \leq \frac{(1-u^2)(\alpha - \beta)}{8u(\alpha^2 - \beta^2)} \\ \frac{\alpha(5u^2 + 6u + 5 + \beta(u+1)^2)}{16a(\alpha + \beta)}, & \text{if } \frac{(1-u^2)(\alpha - \beta)}{8u(\alpha^2 - \beta^2)} < f \leq \min\{\frac{(1-u^2)(\alpha - \beta)}{8u(\alpha^2 - \beta^2)}, \frac{(1-u^2)}{8u}\} \\ \frac{(u+1)^2 + 2a^2 - 2u^2}{8a}, & \text{if } \min\{\frac{(1-u^2)(\alpha - \beta)}{8u(\alpha^2 - \beta^2)}, \frac{(1-u^2)}{8u}\} < f \leq \frac{(1-u)(u+u^2)}{8u} \\ 0, & \text{if } f > \frac{(1-u)(u+u^2)}{8u} \end{cases}$$

We define $\hat{\pi}_{IF} = E[\pi^{(N^*)}_{IF}(A)]$. When $\pi^0 > \hat{\pi}_{IF}$ or $f > \frac{(1-u)^2}{8u}$, IF is not feasible.

**B.1 Proof of Proposition 3**

We first derive the equilibrium outcome when $N = 1$ (we label this case using a mnemonic PH1); then we obtain the equilibrium outcome when $N = 2$ (labeled PH2). Finally, we arrive at the result in Proposition 3 by comparing profits across both cases.

**B.1.1 PH1**

*Proof.* The equilibrium is derived using backward induction.

**Retail Pricing Decision.** After demand is revealed, the retailer sets retail price so as to achieve

$$\max_{p \geq 0} p \cdot s(p, A) \quad \text{s.t.} \quad s(p, A) \leq q_{PH},$$

which is equivalent to

$$\max_{0 \leq s \leq q_{PH}} p(s, A) \cdot s,$$
where \( p(s, A) = A - \alpha s \). The optimal solution is given by

\[
s^*_PH = \min \left\{ \frac{A}{2\alpha}, q^*_PH \right\} \quad \text{and} \quad \pi^*_PH = \max \left\{ \frac{A}{2}, A - \alpha q^*_PH \right\}.
\]

**Stocking Quantity Decision.** Before demand is revealed, the retailer sets the stocking quantity to achieve

\[
\max_{q^*_PH \geq 0} \mathbb{E}[\pi^*_PH \cdot s^*_PH] - w^*_PH \cdot q^*_PH,
\]

If \( q^*_PH > \frac{1+u}{2\alpha} \), then \( s^*_PH = \frac{A}{2\alpha} \) for \( A = 1 - u \) or \( 1 + u \). Thus the retailer’s expected profit can be simplified as \( \frac{1+u^2}{4\alpha} - w^*_PH \cdot q^*_PH \), which is decreasing in \( q^*_PH \). Thus \( q^*_PH > \frac{1+u}{2\alpha} \) is not optimal.

If \( \frac{1-u}{2\alpha} \leq q^*_PH \leq \frac{1+u}{2\alpha} \), then \( s^*_PH = \frac{1-u}{2} \) when \( A = 1 - u \) and \( s^*_PH = q^*_PH \) when \( A = 1 + u \). Thus the retailer’s expected profit can be simplified as \( \frac{q^*_PH(1+u-\alpha q^*_PH)}{2} + \frac{(1-u)^2}{8\alpha} - w^*_PH \cdot q^*_PH \). The first order condition leads to \( q^*_PH = \frac{1+u-2w^*_PH}{2\alpha} \) when \( w^*_PH \leq u \).

If \( q^*_PH < \frac{1-u}{2\alpha} \), then \( s^*_PH = q^*_PH \) when \( A = 1 - u \) or \( 1 + u \). Thus the retailer’s expected profit can be simplified as \( q^*_PH - \alpha q^2_PH - w^*_PH \cdot q^*_PH \). The first order condition leads to \( q^*_PH = \frac{1-w^*_PH}{2\alpha} \) when \( u < w^*_PH \leq 1 \) and when \( w^*_PH > 1 \), \( q^*_PH = 0 \). Thus the optimal stocking quantity is

\[
q^*_PH(w^*_PH) = \begin{cases} \frac{1+u-2w^*_PH}{2\alpha}, & \text{if } w^*_PH \leq u \\ \frac{1-w^*_PH}{2\alpha}, & \text{if } u < w^*_PH \leq 1 \\ 0, & \text{if } w^*_PH > 1 \end{cases}
\]

**Wholesale Pricing Decision.** As a function of wholesale price, the firms’ expected profits are

\[
\Pi^{(1)}_{PH}(w^*_PH) = -f + \begin{cases} \frac{1+u-2w^*_PH}{2\alpha} w^*_PH, & \text{if } w^*_PH \leq u \\ \frac{1-w^*_PH}{2\alpha} w^*_PH, & \text{if } u < w^*_PH \leq 1 \\ 0, & \text{if } w^*_PH > 1 \end{cases}
\]

\[
\pi^{(1)}_{PH}(w^*_PH) = \begin{cases} \frac{u^2-2uw^*_PH+2w^2_PH-2w^*_PH+1}{4\alpha}, & \text{if } w^*_PH \leq u \\ \frac{(1-w^*_PH)^2}{4\alpha}, & \text{if } u < w^*_PH \leq 1 \\ 0, & \text{if } w^*_PH > 1 \end{cases}
\]

The manufacturer’s problem is set wholesale prices so as to achieve

\[
\max_{w^*_PH \geq 0} \Pi^{(1)}_{PH}(w^*_PH) = w^*_PH \cdot q^*_PH(w^*_PH) - f \tag{B.4a}
\]

s.t. \( \pi^{(1)}_{PH}(w^*_PH) \geq \pi^0 \), \tag{B.4b}

where (B.4b) is the retailer’s participation constraint.

For any given \( \pi^0 \), let \( w^*_PL(\pi^0) \), \( \Pi^{(1)*}_{PL}(\pi^0) \), and \( \pi^{(1)*}_{PL}(\pi^0) \) denote the manufacturer’s optimal wholesale price, the manufacturer’s corresponding profit, and the retailer’s corresponding profit, respectively. We first solve the unconstrained problem with \( \pi^0 = 0 \). We just need to optimize \( \Pi^{(1)}_{PH}(w^*_PH) \). If \( w^*_PH \leq u \), then the manufacturer’s profit can be simplified as \( \Pi^{(1)}_{PH}(w^*_PH) = w^*_PH \frac{1+u-2w^*_PH}{2\alpha} - f \).
The first-order condition leads to \( w_{PH} = \frac{1+u}{4} \) if \( u > \frac{1}{3} \), and the manufacturer’s corresponding optimal profit is \( \frac{(1+u)^2}{16\alpha} \). If \( u < w_{PH} \leq 1 \), then the manufacturer’s profit can be simplified as \( \Pi_{PH}(w_{PH}) = w_{PH} \frac{1-w_{PH}}{2a} - f \). The first-order condition leads to \( w_{PH} = \frac{1}{2} \) if \( u < \frac{1}{2} \), and the manufacturer’s corresponding optimal profit is \( \frac{1}{8a} \). When \( \frac{1}{3} < u < \frac{1}{2} \), both \( w_{PH} = \frac{1+u}{4} \) and \( w_{PH} = \frac{1}{2} \) are the local optima. A comparison of the manufacturer’s profit at \( w_{PH} = \frac{1+u}{4} \) and \( w_{PH} = \frac{1}{2} \) reveals that \( w_{PH} = \frac{1+u}{4} \) is optimal when \( u > \sqrt{2} - 1 \) and \( w_{PH} = \frac{1}{2} \) is optimal when \( u < \sqrt{2} - 1 \). Thus the optimal wholesale price of the unconstrained problem (i.e., \( \pi^0 = 0 \)) is

\[
w_{PH}^*(0) = \begin{cases} \frac{1+u}{4}, & \text{if } u \geq \sqrt{2} - 1 \\ \frac{1}{2}, & \text{otherwise} \end{cases}.
\]

The manufacturer’s and retailer’s corresponding profits are

\[
\Pi_{PH}^{(1)}(0) = \Pi_{PH}^{(1)}(w_{PH}^*(0)) = -f + \begin{cases} \frac{(1+u)^2}{16a}, & \text{if } u \geq \sqrt{2} - 1 \\ \frac{1}{8a}, & \text{otherwise} \end{cases}
\]

\[
\pi_{PH}^{(1)}(0) = \pi_{PH}^{(1)}(w_{PH}^*(0)) = \begin{cases} \frac{5-6u+5u^2}{32a}, & \text{if } u \geq \sqrt{2} - 1 \\ \frac{-\alpha}{16a}, & \text{otherwise} \end{cases}.
\]

When \( \pi^0 \leq \pi_{PH}^{(1)*}(0) \), the manufacturer faces the unconstrained problem and the solution is the same as \( \pi^0 = 0 \). When \( \pi^0 > \pi_{PH}^{(1)*}(0) \), in order to satisfy the retailer’s reservation constraint B.4b, the manufacturer has to set wholesale price \( w_{PH} < w_{PH}^*(0) \) since the retailer’s profit decreases in \( w_{PH} \). The manufacturer’s and retailer’s profits can be shown in Figure B.1. If \( u \geq \sqrt{2} - 1 \) or \( u < 1/3 \) (see Figure B.1(a) and B.1(c)), then \( \Pi_{PH}(w_{PH}) \) increases in \( w_{PH} \) for \( w_{PH} < w_{PH}^*(0) \). Thus, if \( \pi^0 > \pi_{PH}^{(1)*}(0) \), then the retailer’s reservation constraint (B.4b) will be active and the optimal wholesale price will be solved from \( \pi_{PH}^{(1)}(w_{PH}^*) = \pi^0 \). If \( 1/3 < u < \sqrt{2} - 1 \) (see Figure B.1(b) ), then \( \Pi_{PH}(w_{PH}) \) first increases, then decreases, and then increases in \( w_{PH} \) for \( w_{PH} < w_{PH}^*(0) \). We define \( \tilde{w}_{PH} \) as \( \Pi_{PH}(\tilde{w}_{PH}) = \Pi_{PH}(\frac{1+u}{4}) \). When \( \pi_{PH}^{(1)}(0) < \pi^0 < \pi_{PH}^{(1)}(\tilde{w}_{PH}) \), the retailer’s reservation constraint will be binding and the optimal wholesale price is solved from \( \pi_{PH}^{(1)}(w_{PH}) = \pi^0 \). When \( \pi_{PH}^{(1)}(\tilde{w}_{PH}) \leq \pi^0 \leq \pi_{PH}^{(1)}(\frac{1+u}{4}) \), the manufacturer’s optimal wholesale price is \( w_{PH} = \frac{1+u}{4} \). Finally, when \( \pi^0 > \pi_{PH}^{(1)}(\frac{1+u}{4}) \), the retailer’s reservation constraint will again be binding.

Thus the optimal wholesale price is

\[
w_{PH}^*(\pi^0) = \begin{cases} w_{PH,hv}(\pi^0), & \text{if } u \geq \sqrt{2} - 1 \\ w_{PH,lv}(\pi^0), & \text{if } u < \sqrt{2} - 1 \end{cases},
\]

where

\[
w_{PH,hv}(\pi^0) = \begin{cases} \frac{1+u}{4}, & \text{if } 0 \leq \pi^0 \leq \frac{5-6u+5u^2}{32a} \\ \frac{1+u}{4} - \sqrt{\frac{8a\pi^0 - u^2 + 2u - 1}{2}}, & \text{if } \frac{5-6u+5u^2}{32a} < \pi^0 \leq \frac{1+u^2}{4a} \\ 0, & \text{if } \pi^0 > \frac{1+u^2}{4a} \end{cases}.
\]
Figure B.1: Firms’ Ex-ante Profit Function under PH1

(a) $u \geq \sqrt{2} - 1$ and $w_{PH}^*(0) = \frac{1+u}{4}$

(b) $1/3 \leq u < \sqrt{2} - 1$ and $w_{PH}^*(0) = 1/2$

(c) $u < 1/3$ and $w_{PH}^*(0) = 1/2$

and

$$w_{PH,lv}(\pi^0) = \begin{cases} 
1/2, & \text{if } 0 \leq \pi^0 \leq \frac{1}{16\alpha} \\
1 - 2\sqrt{\alpha \pi^0}, & \text{if } \frac{1}{16\alpha} < \pi^0 \leq \frac{1}{16\alpha} + \frac{3-2u-u^2}{32\alpha} \\
1+u, & \text{if } 1/3 < u < \sqrt{2} - 1 \\
\frac{1+u-u^2}{2}, & \text{if } 1/3 < u < \sqrt{2} - 1 \leq \pi^0 \leq \frac{5-6u+5u^2}{16\alpha} \\
0, & \text{if } \pi^0 > \frac{1+u^2}{4\alpha}.
\end{cases}$$

The manufacturer’s and retailer’s corresponding profits are
(a) If \( u \geq \sqrt{2} - 1 \), then

\[
\Pi_{PH}^{(1)\ast}(\pi^0) = -f + \begin{cases} 
\frac{(1+u)^2}{4\pi\alpha} 
\sqrt{8\alpha\pi u^2 + u^4 - 1(1+u - \sqrt{8\alpha\pi u^2 + u^4 - 2u^2})}, & \text{if } 0 \leq \pi^0 \leq \frac{5-6u+5u^2}{32\alpha} \\
0, & \text{if } \frac{5-6u+5u^2}{32\alpha} < \pi^0 \leq \frac{1+u^2}{4\alpha} \quad \text{and} \\
\pi^0, & \text{if } \pi^0 > \frac{1+u^2}{4\alpha} \end{cases}
\]

\[
\pi_{PH}^{(1)\ast}(\pi^0) = \begin{cases} 
\frac{5-6u+5u^2}{32\alpha}, & \text{if } 0 \leq \pi^0 \leq \frac{5-6u+5u^2}{32\alpha} \\
\pi^0, & \text{if } \frac{5-6u+5u^2}{32\alpha} < \pi^0 \leq \frac{1+u^2}{4\alpha} \quad \text{and} \\
0, & \text{if } \pi^0 > \frac{1+u^2}{4\alpha} \end{cases}
\]

(b) If \( u < \sqrt{2} - 1 \), then

\[
\Pi_{PH}^{(1)\ast}(\pi^0) = -f + \begin{cases} 
\frac{1}{8\alpha^2}, & \text{if } 0 \leq \pi^0 \leq \frac{1}{16\alpha} \\
\sqrt{\frac{\pi^0}{\alpha}} - 2\pi^0, & \text{if } \frac{1}{16\alpha} < \pi^0 \leq \frac{1-2u+u^2}{4\alpha} \quad \text{if } u \leq 1/3 \\
\frac{(1+u)^2}{16\alpha^2}, & \text{if } 1/3 < u < \sqrt{2} - 1 \quad \text{and} \\
\frac{1-2u+u^2}{4\alpha}, & \text{if } 1/3 < u < \sqrt{2} - 1 \quad \text{and} \\
X_1, & \text{if } 1/3 < u < \sqrt{2} - 1 < \pi^0 \leq \frac{1+u^2}{4\alpha} \\
0, & \text{if } \pi^0 > \frac{1+u^2}{4\alpha} 
\end{cases}
\]

and

\[
\pi_{PH}^{(1)\ast}(\pi^0) = \begin{cases} 
\frac{1}{16\alpha^2}, & \text{if } 0 \leq \pi^0 \leq \frac{1}{16\alpha} \\
\pi^0, & \text{if } \frac{1}{16\alpha} < \pi^0 \leq \frac{1-2u+u^2}{4\alpha} \quad \text{if } u \leq 1/3 \\
\frac{5-6u+5u^2}{32\alpha}, & \text{if } 1/3 < u < \sqrt{2} - 1 \quad \text{and} \\
\pi^0, & \text{if } 1/3 < u < \sqrt{2} - 1 \quad \text{and} \\
\frac{1-2u+u^2}{4\alpha}, & \text{if } 1/3 < u < \sqrt{2} - 1 \quad \text{and} \\
\pi^0, & \text{if } 1/3 < u < \sqrt{2} - 1 < \pi^0 \leq \frac{1+u^2}{4\alpha} \\
0, & \text{if } \pi^0 > \frac{1+u^2}{4\alpha} 
\end{cases}
\]

where \( X_1 = \sqrt{8\alpha\pi u^2 + u^4 - 1(1+u - \sqrt{8\alpha\pi u^2 + u^4 - 2u^2})} \).

\( \Pi_{PH}^{(1)\ast}(\pi^0) \) decreases in \( \pi^0 \) for \( 0 \leq \pi^0 \leq \frac{1+u^2}{4\alpha} \). Thus there exists \( 0 \leq \hat{\pi}_{PH} \leq \frac{1+u^2}{4\alpha} \) such that \( \Pi_{PH}^{(1)\ast}(\pi^0) > 0 \) when \( \pi^0 < \hat{\pi}_{PH} \) and \( \Pi_{PH}^{(1)\ast}(\pi^0) < 0 \) when \( \pi^0 > \hat{\pi}_{PH} \). Therefore when \( \pi^0 > \hat{\pi}_{PH} \), PH1 is not feasible.

**B.1.2 PH2**

**Proof.** As in PH1, the equilibrium is derived using backward induction.

**Retail Pricing Decision.** After demand is revealed, the retailer sets retail prices so as to achieve

\[
\max_{p \geq 0} p \cdot s(p, A) \quad \text{s.t.} \quad s(p, A) \leq q_{PH},
\]

(B.5)
which is equivalent to
\[
\max_{0 \leq s \leq q_{PH}} \mathbf{p}(s, A) \cdot s,
\]
(B.6)
where \( \mathbf{p}(s, A) = (A_1 - \alpha s_1 - \beta s_2, A_2 - \alpha s_2 - \beta s_1) \). The optimal solution is given by
\[
s_{PH,i}^* = \min \left\{ q_{PH,i}, \max \left\{ \frac{A_i - 2\beta q_{PH,j}}{2\alpha}, \frac{\alpha A_i - \beta A_j}{2(\alpha^2 - \beta^2)} \right\} \right\}
\quad \text{and} \quad p_{PH,i}^* = A_i - \alpha s_{PH,i}^* - \beta s_{PH,j}^*,
\]
where \( i, j = 1, 2 \) and \( i \neq j \).

**Stocking Quantity Decision.** Before demand is revealed, the retailer sets the stocking quantity to achieve
\[
\max_{q_{PH} \geq 0} \mathbb{E}[\mathbf{p}_{PH}^* \cdot s_{PH}^*] - w_{PH} \cdot q_{PH}.
\]
We just need to focus on the symmetric solutions \( q_{PH,1} = q_{PH,2} \) when \( w_{PH,1} = w_{PH,2} \). This is due to the following Lemma B.1.

**Lemma B.1.** If \( N = 2 \) and \( w_{PH,1} = w_{PH,2} \), then neither \( q_{PH,1} > q_{PH,2} > 0 \) nor \( 0 < q_{PH,1} < q_{PH,2} \) can be supported as an equilibrium with the PH contract.

**Proof of Lemma B.1.** Given wholesale prices \( w_{PH} = (w_{PH,1}, w_{PH,2}) \) and \( q_{PH} = (q_{PH,1}, q_{PH,2}) \), the retailer’s profit can be written as \( \pi_{PH}^{(2)}(q_{PH}) = \mathbb{E}[\mathbf{p}_{PH}^*(q_{PH}) \cdot s_{PH}^*(q_{PH})] - w_{PH} \cdot q_{PH} \). The retailer chooses optimal \( q_{PH} \) to maximize her expected profit.

We will show that given \( w_{PH,1} = w_{PH,2} = w_{PH}^{(2)} \geq 0 \), any \( q_{PH,1} > q_{PH,2} > 0 \) is not optimal for the retailer. We just need to show that there exists small enough \( 0 < \delta q < (q_{PH,1} - q_{PH,2})/2 \), such that \( q_{PH} = (q_{PH,1} - \delta q, q_{PH,2} + \delta q) \) dominates \( q_{PH} \), i.e., \( \pi_{PH}^{(2)}(q_{PH}) > \pi_{PH}^{(2)}(q_{PH}) \).

Define \( R_{PH}(q_{PH}) = \mathbb{E}[\mathbf{p}_{PH}^*(q_{PH}) \cdot s_{PH}^*(q_{PH})] \). Since \( \pi_{PH}^{(2)}(q_{PH}) - \pi_{PH}^{(2)}(q_{PH}) = R_{PH}(q_{PH}) - R_{PH}(q_{PH}) \), we just need to show \( R_{PH}(q_{PH}) > R_{PH}(q_{PH}) \).

Taking the one-side derivative of \( R_{PH}(q_{PH}) \) with respective to \( q_{PH,1} \) and \( q_{PH,2} \), we have
\[
\frac{\partial R_{PH}(q_{PH})}{\partial q_{PH,1}} = \lim_{\delta q \to 0^+} \frac{R_{PH}(q_{PH,1} - \delta q, q_{PH,2}) - R_{PH}(q_{PH})}{-\delta q} > 0 \quad \text{and} \quad \frac{\partial R_{PH}(q_{PH})}{\partial q_{PH,2}} = \lim_{\delta q \to 0^+} \frac{R_{PH}(q_{PH,1}, q_{PH,2} + \delta q) - R_{PH}(q_{PH})}{\delta q} > 0.
\]

We can verify that \( \frac{\partial R_{PH}(q_{PH})}{\partial q_{PH,2}} > \frac{\partial R_{PH}(q_{PH})}{\partial q_{PH,1}} \), that is the marginal value of increasing \( q_{PH,2} \) is greater than that of \( q_{PH,1} \) for the retailer.

For small enough \( \delta q > 0 \), the Taylor series of \( R_{PH}(q_{PH}) \) can be written as
\[
R_{PH}(q_{PH}) = R_{PH}(q_{PH}) - \delta q \frac{\partial R_{PH}(q_{PH})}{\partial q_{PH,1}} + \delta q \frac{\partial R_{PH}(q_{PH})}{\partial q_{PH,2}} + o(\delta q)
\]
\[
= R_{PH}(q_{PH}) + \delta q(\frac{\partial R_{PH}(q_{PH})}{\partial q_{PH,2}} - \frac{\partial R_{PH}(q_{PH})}{\partial q_{PH,1}}) + o(\delta q) > R_{PH}(q_{PH}).
\]

Thus we have shown that for given \( w_{PH,1} = w_{PH,2} > 0 \), any \( q_{PH} \) with \( q_{PH,1} > q_{PH,2} > 0 \) is not
optimal for the retailer. Similarly we can show that any \( q_{PH} \) with \( 0 < q_{PH,1} < q_{PH,2} \) is not optimal either. Thus the retailer’s optimal order quantities must satisfy \( q_{PH,1} = q_{PH,2} \).

We will now show that in the PH2 equilibrium, \( w_{PH,1} = w_{PH,2} \) holds in Lemma B.2.

**Lemma B.2.** If \( N = 2 \), then neither \( w_{PH,1} < w_{PH,2} \) nor \( w_{PH,1} > w_{PH,2} \) can be supported as an equilibrium with the PH contract.

**Proof of Lemma B.2.** We show that given \( N = 2 \) under PH, any \( w_{PH} = (w_{PH,1}, w_{PH,2}) \) with \( w_{PH,1} < w_{PH,2} \) is not optimal for the manufacturer. We just need to show that there exist \( 0 \leq \delta w_1, \delta w_2 < (w_{PH,2} - w_{PH,1})/2 \), such that \( \tilde{w}_{PH} = (w_{PH,1} + \delta w_1, w_{PH,2} - \delta w_2) \) dominates \( w_{PH} \), i.e., \( \Pi^{(2)}_{PH}(\tilde{w}_{PH}) > \Pi^{(2)}_{PH}(w_{PH}) \) and \( \pi^{(2)}_{PH}(\tilde{w}_{PH}) > \pi^{(2)}_{PH}(w_{PH}) \).

The manufacturer’s and retailer’s expected profits can be written as

\[
\Pi^{(2)}_{PH}(w_{PH}) = w_{PH} \cdot q^*_P(w_{PH}) - 2f \quad \text{and} \quad \pi^{(2)}_{PH}(w_{PH}) = E[q^*_P(w_{PH}) \cdot s^*_P(q^*_P(w_{PH}))] - w_{PH} \cdot q^*_P(w_{PH}).
\]

For small enough \( \delta w_1 \) and \( \delta w_2 \), we have

\[
\Pi^{(2)}_{PH}(\tilde{w}_{PH}) = \Pi^{(2)}_{PH}(w_{PH}) + \delta w_1 \frac{\partial \Pi^{(2)}_{PH}(w_{PH})}{\partial w^+_PH} - \delta w_2 \frac{\partial \Pi^{(2)}_{PH}(w_{PH})}{\partial w^-PH} + o(\delta w_1) + o(\delta w_2),
\]

\[
\pi^{(2)}_{PH}(\tilde{w}_{PH}) = \pi^{(2)}_{PH}(w_{PH}) + \delta w_1 \frac{\partial \pi^{(2)}_{PH}(w_{PH})}{\partial w^+_PH} - \delta w_2 \frac{\partial \pi^{(2)}_{PH}(w_{PH})}{\partial w^-PH} + o(\delta w_1) + o(\delta w_2).
\]

To show \( \Pi^{(2)}_{PH}(\tilde{w}_{PH}) > \Pi^{(2)}_{PH}(w_{PH}) \) and \( \pi^{(2)}_{PH}(\tilde{w}_{PH}) > \pi^{(2)}_{PH}(w_{PH}) \), we just need to show that

\[
\delta w_1 \frac{\partial \Pi^{(2)}_{PH}(w_{PH})}{\partial w^+_PH} - \delta w_2 \frac{\partial \Pi^{(2)}_{PH}(w_{PH})}{\partial w^-PH} > 0 \quad \text{and} \quad \delta w_1 \frac{\partial \pi^{(2)}_{PH}(w_{PH})}{\partial w^+_PH} - \delta w_2 \frac{\partial \pi^{(2)}_{PH}(w_{PH})}{\partial w^-PH} > 0.
\]

One can easily verify that \( \frac{\partial \pi^{(2)}_{PH}(w_{PH})}{\partial w^+_PH} < \frac{\partial \pi^{(2)}_{PH}(w_{PH})}{\partial w^-PH} < 0 \), that is the retailer’s profit decreases in the wholesale prices and is more sensitive to the lower wholesale price \( w_{PH,1} \) than the higher wholesale price \( w_{PH,2} \).

If \( \frac{\partial \Pi^{(2)}_{PH}(w_{PH})}{\partial w^+_PH} < 0 \), then we just need to take \( \delta w_1 = 0 \) and small enough \( \delta w_2 > 0 \). Then we have

\[
\delta w_1 \frac{\partial \Pi^{(2)}_{PH}(w_{PH})}{\partial w^+_PH} - \delta w_2 \frac{\partial \Pi^{(2)}_{PH}(w_{PH})}{\partial w^-PH} = -\delta w_2 \frac{\partial \Pi^{(2)}_{PH}(w_{PH})}{\partial w^-PH} > 0 \quad \text{and} \quad \delta w_1 \frac{\partial \pi^{(2)}_{PH}(w_{PH})}{\partial w^+_PH} - \delta w_2 \frac{\partial \pi^{(2)}_{PH}(w_{PH})}{\partial w^-PH} = -\delta w_2 \frac{\partial \pi^{(2)}_{PH}(w_{PH})}{\partial w^-PH} > 0. \]

Thus \( \Pi^{(2)}_{PH}(\tilde{w}_{PH}) > \Pi^{(2)}_{PH}(w_{PH}) \) and \( \pi^{(2)}_{PH}(\tilde{w}_{PH}) > \pi^{(2)}_{PH}(w_{PH}) \).

If \( \frac{\partial \Pi^{(2)}_{PH}(w_{PH})}{\partial w^+_PH} \geq 0 \), then one can verify that \( \frac{\partial \Pi^{(2)}_{PH}(w_{PH})}{\partial w^+_PH} > \frac{\partial \Pi^{(2)}_{PH}(w_{PH})}{\partial w^-PH} \geq 0 \), that is the manufacturer’s profit increases in the wholesale prices and is more sensitive to the lower wholesale price \( w_{PH,1} \) than the higher wholesale price \( w_{PH,2} \). And we can also show that \( 0 \leq \frac{\partial \Pi^{(2)}_{PH}(w_{PH})}{\partial w^-PH} \frac{\partial \pi^{(2)}_{PH}(w_{PH})}{\partial w^+_PH} < \frac{\partial \pi^{(2)}_{PH}(w_{PH})}{\partial w^+_PH} \). Therefore we can find small enough \( \delta w_1 > 0 \) and \( \delta w_2 > 0 \) that satisfy
\[
\frac{\partial \Pi^{(2)}_{PH}(w_{PH})}{\partial w_{PH,2}} / \frac{\partial \Pi^{(2)}_{PH}(w_{PH})}{\partial w_{PH,1}} < \frac{\delta w_1}{\delta w_2} < \frac{\partial \Pi^{(2)}_{PH}(w_{PH})}{\partial w_{PH,2}} / \frac{\partial \Pi^{(2)}_{PH}(w_{PH})}{\partial w_{PH,1}},
\]
which will lead to
\[
\delta w_1 \frac{\partial \Pi^{(2)}_{PH}(w_{PH})}{\partial w_{PH,1}} - \delta w_2 \frac{\partial \Pi^{(2)}_{PH}(w_{PH})}{\partial w_{PH,2}} > 0 \quad \text{and} \quad \delta w_1 \frac{\partial \Pi^{(2)}_{PH}(w_{PH})}{\partial w_{PH,1}} - \delta w_2 \frac{\partial \Pi^{(2)}_{PH}(w_{PH})}{\partial w_{PH,2}} > 0.
\]

Therefore there exist \( \bar{w}_{PH} \) such that \( \Pi^{(2)}_{PH}(w_{PH}) > \Pi^{(2)}_{PH}( \bar{w}_{PH} ) \) and \( \pi^{(2)}_{PH}( \bar{w}_{PH} ) > \pi^{(2)}_{PH}(w_{PH}) \). We have shown that any \( w_{PH} = (w_{PH,1}, w_{PH,2}) \) with \( w_{PH,1} < w_{PH,2} \) can not be supported as an equilibrium. Similarly, we can show that any \( w_{PH} = (w_{PH,1}, w_{PH,2}) \) with \( w_{PH,1} > w_{PH,2} \) can not be supported as an equilibrium, either. Therefore, given \( N = 2 \) under PH, we must have \( w_{PH,1} = w_{PH,2} \) in an equilibrium.

Due to Lemma B.1 and B.2, we just need to consider the symmetric solutions. Let \( w_{PH,1} = w_{PH,2} = w^{(2)}_{PH} \) and \( q_{PH,1} = q_{PH,2} = q^{(2)}_{PH} \).

If \( q^{(2)}_{PH} > \frac{\alpha(1+u) - \beta(1-u)}{2(\alpha^2 - \beta^2)} \), then the retailer’s expected profit can be simplified as
\[
\frac{-\alpha^3 q^{(2)}_{PH}^2 + \alpha \left( 4 \beta^2 q^{(2)}_{PH} + u(8 \beta q^{(2)}_{PH} - 2) + 3u^2 + 3 \right) + 4 \alpha^2 q^{(2)}_{PH}( -\beta q^{(2)}_{PH} + u + 1) + \beta(2 \beta q^{(2)}_{PH} + u - 1)^2 - 2 \alpha^2 q^{(2)}_{PH}}{8 \alpha (\alpha + \beta)}
\]

The first order condition leads to
\[
q^{(2)}_{PH} = \frac{\alpha - \beta + \alpha \alpha - 4 \alpha w^{(2)}_{PH} + 4}{2 \alpha^2 - 2 \beta^2}
\]
when \( w^{(2)}_{PH} \leq w^\beta_{PH} \).

If \( \frac{1}{2} < \frac{2}{\alpha + \beta} \leq q^{(2)}_{PH} \leq \frac{1}{2} + \frac{u}{\alpha + \beta} \), then the expected profit can be written as
\[
\frac{-8 \alpha^3 q^{(2)}_{PH}^2 + 2 \alpha \left( 2 \beta q^{(2)}_{PH} + u(6 \beta q^{(2)}_{PH} - 2) + u^2 + 1 \right) + 4 \alpha^2 q^{(2)}_{PH}( -3 \beta q^{(2)}_{PH} + 2u + 2) + \beta(2 \beta q^{(2)}_{PH} + u - 1)^2 - 2 \alpha^2 q^{(2)}_{PH}}{8 \alpha (\alpha + \beta)}
\]

The first order condition leads to
\[
q^{(2)}_{PH} = \frac{\beta(u-1) + 2 \alpha (u - 2 w^{(2)}_{PH} + 1)}{4 \alpha^2 + 4 \alpha \beta - 2 \beta^2}
\]
when \( \frac{u \beta}{2 \alpha} < w^{(2)}_{PH} \leq u \).

If \( q^{(2)}_{PH} < \frac{1}{2} + \frac{u}{\alpha + \beta} \), then the retailer’s expected profit can be simplified as
\[
\frac{2 q^{(2)}_{PH}(1 - q_{PH}(\alpha + \beta)) - 2 q^{(2)}_{PH}}{2(\alpha + \beta)}
\]
The first order condition leads to
\[
q^{(2)}_{PH} = \frac{1 - w^{(2)}_{PH}}{2(\alpha + \beta)}
\]
when \( u < w^{(2)}_{PH} \leq 1 \). And when \( w^{(2)}_{PH} > 1 \),
\[
q^{(2)}_{PH} = 0.
\]

Given \( w_{PH,1} = w_{PH,2} = w^{(2)}_{PH} \), the optimal stocking quantities are
\[
q^{(2)}_{PH,1}(w_{PH}) = q^{(2)}_{PH,2}(w_{PH}) = q^{(2)}_{PH}(w_{PH}) = \left\{ \begin{array}{ll}
\frac{\alpha - \beta + \alpha \alpha - 4 \alpha w^{(2)}_{PH}}{2 \alpha^2 - 2 \beta^2}, & \text{if } w^{(2)}_{PH} \leq \frac{u \beta}{2 \alpha} \\
\frac{\beta(u-1) + 2 \alpha (u - 2 w^{(2)}_{PH} + 1)}{4 \alpha^2 + 4 \alpha \beta - 2 \beta^2}, & \text{if } \frac{u \beta}{2 \alpha} < w^{(2)}_{PH} \leq u \\
\frac{1 - w^{(2)}_{PH}}{2(\alpha + \beta)}, & \text{if } u < w^{(2)}_{PH} \leq 1 \\
0, & \text{if } w^{(2)}_{PH} > 1
\end{array} \right.
\]

**Wholesale Pricing Decision.** In this stage, the manufacturer chooses wholesale prices that are optimal in
\[
\max_{w_{PH} \geq 0} \Pi^{(2)}_{PH}(w_{PH}) = w_{PH} \cdot q^*_PH(w_{PH}) - 2f
\]
Above (B.7b) is the retailer’s participation constraint.

Using Lemma B.2, which implies \( w_{PH,1} = w_{PH,2} = w^{(2)}_{PH} \), the manufacturer’s and retailer’s expected profits are

\[
\Pi^{(2)}_{PH}(w^{(2)}_{PH}) = -2f + \begin{cases} \\
\frac{\alpha - \beta + \alpha u + \beta u - 4\alpha w^{(2)}_{PH} \alpha^2}{\alpha^2 - \beta^2} w^{(2)}_{PH}, & \text{if } w^{(2)}_{PH} \leq \frac{u \beta}{2 \alpha} \\
\frac{\beta(u-1) + 2\alpha(u-2w^{(2)}_{PH} + 1)}{2\alpha u^2 + \alpha^2 - \beta^2} w^{(2)}_{PH}, & \text{if } \frac{u \beta}{2 \alpha} < w^{(2)}_{PH} \leq u \\
\frac{1 - w^{(2)}_{PH}}{\alpha + \beta} w^{(2)}_{PH}, & \text{if } u < w^{(2)}_{PH} \leq 1 \\
0, & \text{if } w^{(2)}_{PH} > 1
\end{cases}
\]

and

\[
\pi^{(2)}_{PH}(w^{(2)}_{PH}) = \begin{cases} \\
\frac{\alpha (w^2 - 2uw^{(2)}_{PH} + 4w^{(2)}_{PH}^2 - 2w^{(2)}_{PH} + 1) + \beta(1-u)(w^{(2)}_{PH} - 1)}{2(\alpha^2 - \beta^2)}, & \text{if } w^{(2)}_{PH} \leq \frac{u \beta}{2 \alpha} \\
\frac{2\alpha (w^2 - 2uw^{(2)}_{PH} + 2w^{(2)}_{PH}^2 - 2w^{(2)}_{PH} + 1) + \beta(1-u)(2w^{(2)}_{PH} - u - 1)}{(2\alpha^2 - \beta)(\alpha + \beta)}, & \text{if } \frac{u \beta}{2 \alpha} < w^{(2)}_{PH} \leq u \\
\frac{(1 - w^{(2)}_{PH})^2}{2(\alpha + \beta)}, & \text{if } u < w^{(2)}_{PH} \leq 1 \\
0, & \text{if } w^{(2)}_{PH} > 1
\end{cases}
\]

For any given \( \pi^0 \), let \( w^{(2)*}_{PH}(\pi^0) \), \( \Pi^{(2)*}_{PH}(\pi^0) \), and \( \pi^{(2)*}_{PH}(\pi^0) \) denote the manufacturer’s optimal wholesale price, the manufacturer’s corresponding profit, and the retailer’s corresponding profit, respectively. We first solve the unconstrained problem with \( \pi^0 = 0 \). Similar to the analysis in PH1, we just need to optimize \( \Pi^{(2)}_{PH}(w^{(2)}_{PH}) \). The optimal wholesale price of the unconstrained problem is

\[
w^{(2)*}_{PH}(0) = \begin{cases} \\
\frac{2\alpha(1+u) - \beta(1-u)}{8\alpha}, & \text{if } u \geq \frac{2\sqrt{T^* - \beta + \beta^2 - 2}}{2 + \beta} \\
\frac{1}{2}, & \text{otherwise}
\end{cases}
\]

The manufacturer’s and retailer’s corresponding profits are

\[
\Pi^{(2)*}_{PH}(0) = \Pi^{(2)*}_{PH}(w^{(2)*}_{PH}(0)) = -2f + \begin{cases} \\
\frac{2\alpha(1+u) + \beta(u - 1))^2}{16\alpha(2\alpha - \beta)(\alpha + \beta)}, & \text{if } u \geq \frac{2\sqrt{T^* - \beta + \beta^2 - 2}}{2 + \beta} \\
\frac{1}{4(\alpha + \beta)}, & \text{otherwise}
\end{cases}
\]

and

\[
\pi^{(2)*}_{PH}(0) = \pi^{(2)*}_{PH}(w^{(2)*}_{PH}(0)) = \begin{cases} \\
\frac{4\alpha^2(5\alpha^2 - 6\alpha + 5) + 4\alpha \beta(u^2 - 1) - 3\beta^2(u - 1)^2}{32\alpha(2\alpha - \beta)(\alpha + \beta)}, & \text{if } u \geq \frac{2\sqrt{T^* - \beta + \beta^2 - 2}}{2 + \beta} \\
\frac{1}{8(\alpha + \beta)}, & \text{otherwise}
\end{cases}
\]

When \( \pi^0 \leq \pi^{(2)*}_{PH}(0) \), the manufacturer faces the unconstrained problem and the solution is the same as \( \pi^0 = 0 \). When \( \pi^0 > \pi^{(2)*}_{PH}(0) \), in order to satisfy the retailer’s reservation constraint B.7b, the manufacturer has to set wholesale price \( w_{PH} < w^{(2)*}_{PH}(0) \) since the retailer’s profit decreases in \( w^{(2)}_{PH} \). The manufacturer’s and retailer’s profits can be shown in Figure B.2. If \( u \geq \frac{2\sqrt{T^* - \beta + \beta^2 - 2}}{2 + \beta} \) or \( u < \frac{2 - \beta}{6 - \beta} \) (see Figure B.2(a) and B.2(c)), then \( \Pi^{(2)}_{PH}(w^{(2)}_{PH}) \) increases in \( w^{(2)}_{PH} \) for \( w^{(2)}_{PH} < w^{(2)*}_{PH}(0) \). Thus if \( \pi^0 > \pi^{(2)*}_{PH}(0) \), then the retailer’s reservation constraint (B.7b) will be active and the optimal wholesale price is solved from \( \pi^{(2)}_{PH}(w^{(2)}_{PH}) = \pi^0 \).
If $\frac{2-\beta}{6-\beta} \leq u < \frac{2\sqrt{2-\beta+\beta-2}}{2+\beta}$ (see Figure B.2(a)), then $\Pi_{PH}^{(2)}(w_{PH}^{(2)})$ first increases, then decreases, and then increases in $w_{PH}^{(2)}$ for $w_{PH}^{(2)} < w_{PH}^{(2)*}$. We define $w_{PH}^{(2)} \in [u, 1/2]$ as $\Pi_{PH}^{(2)}(w_{PH}^{(2)}) = \Pi_{PH}^{(2)}(\frac{2(1+u)\alpha-(1-u)\beta}{8\alpha})$. When $\pi^{(2)}_{PH}(0) < \pi^{0} < \pi^{(2)}_{PH}(\tilde{w}_{PH}^{(2)})$, the retailer’s reservation constraint will be binding. When $\pi^{(2)}_{PH}(\tilde{w}_{PH}^{(2)}) < \pi^{0} \leq \pi^{(2)}_{PH}(\frac{2(1+u)\alpha-(1-u)\beta}{8\alpha})$, the manufacturer’s optimal wholesale price will be $w_{PH}^{(2)} = \frac{2(1+u)\alpha-(1-u)\beta}{8\alpha}$. And when $\pi^{0} > \pi^{(2)}_{PH}(\frac{2(1+u)\alpha-(1-u)\beta}{8\alpha})$, the retailer’s reservation constraint will be binding again.

Figure B.2: Firms’ Ex-ante Profit Function under PH2

<table>
<thead>
<tr>
<th>Condition</th>
<th>Graph Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) $u \geq \frac{2\sqrt{2-\beta+\beta-2}}{2+\beta}$ and $w_{PH}^{(2)*}(0) = w_L$</td>
<td><img src="image1" alt="Graph (a)" /></td>
</tr>
<tr>
<td>(b) $\frac{2-\beta}{6-\beta} \leq u &lt; \frac{2\sqrt{2-\beta+\beta-2}}{2+\beta}$ and $w_{PH}^{(2)*}(0) = 1/2$</td>
<td><img src="image2" alt="Graph (b)" /></td>
</tr>
<tr>
<td>(c) $u &lt; \frac{2-\beta}{6-\beta}$ and $w_{PH}^{(2)*}(0) = 1/2$</td>
<td><img src="image3" alt="Graph (c)" /></td>
</tr>
</tbody>
</table>

Note. $w_L = \frac{2(1+u)\alpha-(1-u)\beta}{8\alpha}$.

Thus the optimal wholesale price is

$$w_{PH}^{(2)*}(\pi^{0}) = \begin{cases} 
    w_{PH,hv}^{(2)}(\pi^{0}), & \text{if } u \geq \frac{2\sqrt{2-\beta+\beta-2}}{2+\beta} \\
    w_{PH,lv}^{(2)}(\pi^{0}), & \text{if } \frac{2-\beta}{6-\beta} \leq u < \frac{2\sqrt{2-\beta+\beta-2}}{2+\beta} \\
    w_{PH,lv}^{(2)}(\pi^{0}), & \text{if } u < \frac{2-\beta}{6-\beta}
\end{cases}$$
where

\[ w_{PH, h,v}^{(2)}(\pi^0) = \begin{cases} 
\frac{2(1+\alpha)-(1-\alpha)\beta}{8\alpha}, & \text{if } \pi^0 \leq \frac{4\alpha^2(5u^2-6u+5)+4\alpha\beta(u^2-1)-3\beta^2(u^2-1)^2}{32\alpha(2\alpha-\beta)(\alpha+\beta)} \\
\frac{1}{2}, & \text{if } \frac{4\alpha^2(5u^2-6u+5)+4\alpha\beta(u^2-1)-3\beta^2(u^2-1)^2}{32\alpha(2\alpha-\beta)(\alpha+\beta)} < \pi^0 \leq \frac{\alpha+\alpha u^2}{2\alpha^2-2\beta^2} \\
1/2, & \text{if } \frac{\alpha+\alpha u^2}{2\alpha^2-2\beta^2} < \pi^0 \leq \frac{\alpha-\beta+\alpha u^2}{2\alpha^2-2\beta^2} \\
0, & \text{if } \frac{\alpha-\beta+\alpha u^2}{2\alpha^2-2\beta^2} > \pi^0 
\end{cases} \]

and

\[ w_{PH, m,v}^{(2)}(\pi^0) = \begin{cases} 
\frac{1}{2}, & \text{if } \pi^0 \leq \frac{1}{8(\alpha+\beta)} \\
\frac{2(1+\alpha)-(1-\alpha)\beta}{8\alpha}, & \text{if } \frac{1}{8(\alpha+\beta)} < \pi^0 \leq W_1 \\
\frac{1}{2}, & \text{if } W_1 < \pi^0 \leq \frac{4\alpha^2(5u^2-6u+5)+4\alpha\beta(u^2-1)-3\beta^2(u^2-1)^2}{32\alpha(2\alpha-\beta)(\alpha+\beta)} \\
\frac{4\alpha^2(5u^2-6u+5)+4\alpha\beta(u^2-1)-3\beta^2(u^2-1)^2}{32\alpha(2\alpha-\beta)(\alpha+\beta)}, & \text{if } \frac{4\alpha^2(5u^2-6u+5)+4\alpha\beta(u^2-1)-3\beta^2(u^2-1)^2}{32\alpha(2\alpha-\beta)(\alpha+\beta)} < \pi^0 \leq \frac{\alpha+\alpha u^2}{2\alpha^2-2\beta^2} \\
0, & \text{if } \frac{\alpha+\alpha u^2}{2\alpha^2-2\beta^2} < \pi^0 \leq \frac{\alpha-\beta+\alpha u^2}{2\alpha^2-2\beta^2} \\
0, & \text{if } \frac{\alpha-\beta+\alpha u^2}{2\alpha^2-2\beta^2} > \pi^0 
\end{cases} \]

Above, we let

\[ W_1 = \left(\sqrt{\frac{(2\alpha+\beta)(2\alpha(u^2+u-1)\beta(1-\beta))}{32(\alpha+\beta)}}\right)^2, \quad W_2 = \sqrt{\frac{8\alpha^2\pi^0-2(4\beta\pi^0+2\alpha-1)(1-\beta(u^2))}{2\alpha-\beta}}, \quad W_3 = \frac{32\alpha^2\pi^0+3\pi^0+3\alpha^2+2\alpha - 2\beta - \beta(u^2-1)^2}{\alpha-\beta}.\]

The manufacturer’s and retailer’s corresponding profits are

(a) If \( u \geq \frac{2\sqrt{2}\beta+\beta-2}{2\beta} \), then

\[ \Pi_{PH}^{(2)}(\pi^0) = -2f + \begin{cases} 
\frac{(2\alpha(u+1)+\beta(u-1))^2}{16\alpha(2\alpha-\beta)(\alpha+\beta)}, & \text{if } \pi^0 \leq \frac{4\alpha^2(5u^2-6u+5)+4\alpha\beta(u^2-1)-3\beta^2(u^2-1)^2}{32\alpha(2\alpha-\beta)(\alpha+\beta)} \\
\frac{S_1(\beta+\beta)+2(2\alpha(u+1)\beta(u-1))}{4\alpha(2\alpha+\beta)}, & \text{if } \frac{4\alpha^2(5u^2-6u+5)+4\alpha\beta(u^2-1)-3\beta^2(u^2-1)^2}{32\alpha(2\alpha-\beta)(\alpha+\beta)} < \pi^0 \leq \frac{\alpha+\alpha u^2}{2\alpha^2-2\beta^2} \\
0, & \text{if } \frac{\alpha+\alpha u^2}{2\alpha^2-2\beta^2} < \pi^0 \leq \frac{\alpha-\beta+\alpha u^2}{2\alpha^2-2\beta^2} \\
0, & \text{if } \frac{\alpha-\beta+\alpha u^2}{2\alpha^2-2\beta^2} > \pi^0 
\end{cases} \]

and

\[ \pi_{PH}^{(2)}(\pi^0) = \begin{cases} 
\pi^0, & \text{if } \pi^0 \leq \frac{4\alpha^2(5u^2-6u+5)+4\alpha\beta(u^2-1)-3\beta^2(u^2-1)^2}{32\alpha(2\alpha-\beta)(\alpha+\beta)} \\
\frac{4\alpha^2(5u^2-6u+5)+4\alpha\beta(u^2-1)-3\beta^2(u^2-1)^2}{32\alpha(2\alpha-\beta)(\alpha+\beta)}, & \text{if } \frac{4\alpha^2(5u^2-6u+5)+4\alpha\beta(u^2-1)-3\beta^2(u^2-1)^2}{32\alpha(2\alpha-\beta)(\alpha+\beta)} < \pi^0 \leq \frac{\alpha+\alpha u^2}{2\alpha^2-2\beta^2} \\
0, & \text{if } \frac{\alpha+\alpha u^2}{2\alpha^2-2\beta^2} < \pi^0 \leq \frac{\alpha-\beta+\alpha u^2}{2\alpha^2-2\beta^2} \\
0, & \text{if } \frac{\alpha-\beta+\alpha u^2}{2\alpha^2-2\beta^2} > \pi^0 
\end{cases} \]

(b) If \( \frac{2\sqrt{2}\beta+\beta-2}{2\beta} \leq u < \frac{2\sqrt{2}\beta+\beta-2}{2\beta} \), then
\[ \Pi^{(2)*}_{PH}(\pi^0) = -2f + \begin{cases} \frac{1}{4(\alpha+\beta)}, & \text{if } \pi^0 \leq \frac{1}{8(\alpha+\beta)} \\ \left(\sqrt{\frac{2\pi^0}{\alpha+\beta}} - 2\pi^0, \right. & \text{if } \frac{1}{8(\alpha+\beta)} < \pi^0 \leq W_1 \\ \frac{16\alpha(2\alpha-\beta)(\alpha+\beta)}{(2\alpha(u+1)+\beta(u-1))^2}, & \text{if } W_1 < \pi^0 \leq 4\alpha^2(5u^2-6u+5) + 4\alpha\beta(u^2-1) - 3\beta^2(u-1)^2 \\ S_1(\beta + S_2(\alpha - \beta + (u-1) - \beta u), & \text{if } \frac{32\alpha(2\alpha-\beta)(\alpha+\beta)}{4\alpha^2\beta} < \pi^0 \leq \frac{\alpha + \alpha u^2}{2\alpha^2 + 2\alpha \beta} \\ 0, & \text{if } \pi^0 > \frac{\alpha + \alpha u^2}{2\alpha^2 + 2\alpha \beta} \end{cases} \]

and

\[ \pi^{(2)*}_{PH}(\pi^0) = \begin{cases} \frac{1}{8(\alpha+\beta)}, & \text{if } \pi^0 \leq \frac{1}{8(\alpha+\beta)} \\ \pi^0, & \text{if } \frac{1}{8(\alpha+\beta)} < \pi^0 \leq W_1 \\ \frac{4\alpha^2(5u^2-6u+5) + 4\alpha\beta(u^2-1) - 3\beta^2(u-1)^2}{32\alpha(2\alpha-\beta)(\alpha+\beta)}, & \text{if } W_1 < \pi^0 \leq 4\alpha^2(5u^2-6u+5) + 4\alpha\beta(u^2-1) - 3\beta^2(u-1)^2 \\ 0, & \text{if } \pi^0 > \frac{\alpha + \alpha u^2}{2\alpha^2 + 2\alpha \beta} \end{cases} \]

(c) If \( u < \frac{2-\beta}{6-\beta} \), then

\[ \Pi^{(2)*}_{PH}(\pi^0) = -2f + \begin{cases} \frac{1}{4(\alpha+\beta)}, & \text{if } \pi^0 \leq \frac{1}{8(\alpha+\beta)} \\ \sqrt{\frac{2\pi^0}{\alpha+\beta}} - 2\pi^0, & \text{if } \frac{1}{8(\alpha+\beta)} < \pi^0 \leq \frac{1+u^2-2u}{2(\alpha+\beta)} \\ \frac{16\alpha(2\alpha-\beta)(\alpha+\beta)}{(2\alpha(u+1)+\beta(u-1))^2}, & \text{if } \frac{1+u^2-2u}{2(\alpha+\beta)} < \pi^0 \leq \frac{\alpha + \alpha u^2}{2\alpha^2 + 2\alpha \beta} \\ S_1(\beta + S_2(\alpha - \beta + (u-1) - \beta u), & \text{if } \frac{\alpha + \alpha u^2}{2\alpha^2 + 2\alpha \beta} < \pi^0 \leq \frac{\alpha + \alpha u^2}{2\alpha^2 + 2\alpha \beta} \\ 0, & \text{if } \pi^0 > \frac{\alpha + \alpha u^2}{2\alpha^2 + 2\alpha \beta} \end{cases} \]

\[ \pi^{(2)*}_{PH}(\pi^0) = \begin{cases} \frac{1}{8(\alpha+\beta)}, & \text{if } \pi^0 \leq \frac{1}{8(\alpha+\beta)} \\ \pi^0, & \text{if } \frac{1}{8(\alpha+\beta)} < \pi^0 \leq \frac{\alpha + \alpha u^2}{2\alpha^2 + 2\alpha \beta} \\ 0, & \text{if } \pi^0 > \frac{\alpha + \alpha u^2}{2\alpha^2 + 2\alpha \beta} \end{cases} \]

where \( S_1 = \sqrt{8\alpha^2\pi^0 - 2\alpha(-4\beta\pi^0 + \alpha - 2u + 1) - \beta(u-1)^2} \) and \( S_2 = \sqrt{8\alpha^2\pi^0 + \alpha(8\beta\pi^0 - 3u^2 + 2u - 3) - \beta(u-1)^2} \).

\( \Pi^{(2)*}_{PH}(\pi^0) \) decreases in \( \pi^0 \) for \( 0 \leq \pi^0 \leq \frac{\alpha - \beta + \alpha u^2}{2\alpha^2 + 2\alpha \beta} \). Thus there exists \( \hat{\pi}^{(2)}_{PH} \leq \frac{\alpha - \beta + \alpha u^2}{2\alpha^2 + 2\alpha \beta} \) such that \( \Pi^{(2)*}_{PH}(\pi^0) > 0 \) when \( \pi^0 < \hat{\pi}^{(2)}_{PH} \) and \( \Pi^{(2)*}_{PH}(\pi^0) < 0 \) when \( \pi^0 > \hat{\pi}^{(2)}_{PH} \). Therefore when \( \pi^0 > \hat{\pi}^{(2)}_{PH} \), PH2 is not feasible.

\section*{B.1.3 PH1 vs. PH2}

Proof. Now, we compare profits to determine when PH1 and PH2 should be expected in equilibrium. To do this, one can check that \( \Pi^{(2)*}_{PH}(\pi^0) \) is decreasing in \( \beta \) and \( \Pi^{(1)*}_{PH}(\pi^0) \) is independent in \( \beta \).

(a) If \( \Pi^{(2)*}_{PH}(\pi^0) > \max\{\Pi^{(1)*}_{PH}(\pi^0), 0\} \) for any \( \beta \in [0, \alpha \frac{1-u}{1+u}] \), then let \( \hat{\beta}_{PH}(\pi^0) = \alpha \frac{1-u}{1+u} \).

(b) If \( \Pi^{(2)*}_{PH}(\pi^0) < \max\{\Pi^{(1)*}_{PH}(\pi^0), 0\} \) for any \( \beta \in [0, \alpha \frac{1-u}{1+u}] \), then let \( \hat{\beta}_{PH}(\pi^0) = 0 \).
(c) Otherwise there exists \( \hat{\beta}_{PH}(\pi^0) \in [0, \alpha \frac{1-u}{1+u}] \) such that \( \Pi^{(2)^*}_{PH}(\pi^0) > \max\{\Pi^{(1)^*}_{PH}(\pi^0), 0\} \) whenever \( \beta < \hat{\beta}_{PH}(\pi^0) \), \( \Pi^{(2)^*}_{PH}(\pi^0) < \max\{\Pi^{(1)^*}_{PH}(\pi^0), 0\} \), and \( \beta > \hat{\beta}_{PH}(\pi^0) \).

Thus if \( \beta < \hat{\beta}_{PH}(\pi^0) \), then \( N^* = 2 \) under PH. If \( \beta > \hat{\beta}_{PH}(\pi^0) \), then \( N^* = 1 \) whenever \( \pi^0 \leq \hat{\pi}_{PH} \); and \( N^* = 0 \) whenever \( \pi^0 > \hat{\pi}_{PH} \) since \( N = 1 \) under PH is not feasible. This completes the proof of Proposition 3.

\[ \square \]

C Proof of Proposition 4

Proof. From Proposition 1, we can see that in the centralized supply chain, when \( f > \frac{(\alpha(1-u)-\beta(1+u))^2}{4\alpha(\alpha^2-\beta^2)} \), the manufacturer will not sell two product versions if the realized market potential is \( A = (1+u, 1-u) \) or \((1-u, 1+u)\).

From the proof of Proposition 3, one can check that under PH contract, \( \frac{d\Pi^{(2)^*}_{PH}(\pi^0) - \max\{\Pi^{(1)^*}_{PH}(\pi^0), 0\}}{df} < 0 \), that is the manufacturer’s profit difference under PH2 and PH1 is decreasing in \( f \). For any given \( \beta \in [0, \alpha \frac{1-u}{1+u}] \) and \( \pi^0 \in [0, \hat{\pi}_{PH}] \), we have \( \Pi^{(2)^*}_{PH}(\pi^0) - \max\{\Pi^{(1)^*}_{PH}(\pi^0), 0\} \geq 0 \) for \( f = 0 \). Therefore, there exists \( f_{PH}(\beta, \pi^0) \geq 0 \) such that \( \Pi^{(2)^*}_{PH}(\pi^0) - \max\{\Pi^{(1)^*}_{PH}(\pi^0), 0\} \geq 0 \) for \( f \leq f_{PH}(\beta, \pi^0) \), that it the manufacturer will be optimal to produce two product version under PH.

Therefore, when \( \frac{(\alpha(1-u)-\beta(1+u))^2}{4\alpha(\alpha^2-\beta^2)} < f < f_{PH}(\beta, \pi^0) \), the manufacturer will always sell two product versions under PH, but will sell less than two product versions in the centralized supply chain if the realized market potential is \( A = (1+u, 1-u) \) or \((1-u, 1+u)\).

\[ \square \]

D Proof of Corollary 1

Proof. From the proof of Proposition 3, one can check that \( \frac{\partial \Pi^{(2)^*}_{PH}(\pi^0)}{\partial \beta} < 0, \frac{\partial \Pi^{(1)^*}_{PH}(\pi^0)}{\partial \beta} < 0 \) and \( \frac{\partial \Pi^{(1)^*}_{PH}(\pi^0)}{\partial \pi^0} > 0 \). \( \hat{\pi}_{PH} \) is defined as \( \Pi^{(1)^*}_{PH}(\hat{\pi}_{PH}) = 0 \). Thus \( \frac{\partial \hat{\pi}_{PH}}{\partial f} = -\frac{\partial \Pi^{(1)^*}_{PH}(\pi^0)}{\partial f} / \frac{\partial \Pi^{(1)^*}_{PH}(\pi^0)}{\partial \pi^0} < 0 \) and \( \frac{\partial \Pi^{(2)^*}_{PH}}{\partial \beta} > 0. \) Therefore \( \hat{\pi}_{PH} \) decreases in \( f \) and increases in \( u \).

\( \hat{\beta}_{PH}(\pi^0) \) is defined as \( \Pi^{(2)^*}_{PH}(\pi^0) = \max\{\Pi^{(1)^*}_{PH}(\pi^0), 0\} \) for \( \beta = \hat{\beta}_{PH}(\pi^0) \). When \( \pi^0 > \hat{\pi}_{PH} \), PH1 is not feasible and thus \( \Pi^{(2)^*}_{PH}(\pi^0) = 0 \) for \( \beta = \hat{\beta}_{PH}(\pi^0) \). One can check that \( \Pi^{(2)^*}_{PH}(\pi^0) \) decreases in \( \beta \) and \( \pi^0 \), and increases in \( u \). Thus \( \frac{\partial \hat{\beta}_{PH}(\pi^0)}{\partial \beta} = -\frac{\partial \Pi^{(2)^*}_{PH}(\pi^0)}{\partial \beta} / \frac{\partial \Pi^{(2)^*}_{PH}(\pi^0)}{\partial \pi^0} < 0 \) and \( \frac{\partial \hat{\beta}_{PH}(\pi^0)}{\partial \pi^0} = -\frac{\partial \Pi^{(2)^*}_{PH}(\pi^0)}{\partial \pi^0} / \frac{\partial \Pi^{(2)^*}_{PH}(\pi^0)}{\partial \beta} > 0 \), that is \( \hat{\beta}_{PH}(\pi^0) \) decreases in \( \pi^0 \) and increases in \( u \).

When \( \pi^0 \leq \hat{\pi}_{PH} \), PH1 is feasible and \( \hat{\beta}_{PH}(\pi^0) \) is defined as \( \Pi^{(2)^*}_{PH}(\pi^0) = \Pi^{(1)^*}_{PH}(\pi^0) \) for \( \beta = \hat{\beta}_{PH}(\pi^0) \). We can verify that \( \frac{\partial \Pi^{(2)^*}_{PH}(\pi^0) - \Pi^{(1)^*}_{PH}(\pi^0)}{\partial \beta} < 0 \) and \( \frac{\partial \Pi^{(2)^*}_{PH}(\pi^0) - \Pi^{(1)^*}_{PH}(\pi^0)}{\partial \pi^0} > 0 \) if \( u > \sqrt{2} - 1 \). Thus if \( u > \sqrt{2} - 1 \), then \( \frac{\partial \hat{\beta}_{PH}(\pi^0)}{\partial \beta} = -\frac{\partial \Pi^{(1)^*}_{PH}(\pi^0) - \Pi^{(1)^*}_{PH}(\pi^0)}{\partial \beta} / \frac{\partial \Pi^{(2)^*}_{PH}(\pi^0) - \Pi^{(1)^*}_{PH}(\pi^0)}{\partial \beta} > 0 \), that is \( \hat{\beta}_{PH}(\pi^0) \) increases in \( \pi^0 \).

At last, one can verify that \( \Pi^{(2)^*}_{PH}(\pi^0) - \max\{\Pi^{(1)^*}_{PH}(\pi^0), 0\} \) decreases in \( \beta \) and \( f \). Thus we have \( \frac{\partial \hat{\beta}_{PH}(\pi^0)}{\partial f} = -\frac{\partial \Pi^{(2)^*}_{PH}(\pi^0) - \max\{\Pi^{(1)^*}_{PH}(\pi^0), 0\}}{\partial f} / \frac{\partial \Pi^{(2)^*}_{PH}(\pi^0) - \max\{\Pi^{(1)^*}_{PH}(\pi^0), 0\}}{\partial \beta} > 0 \), that is \( \hat{\beta}_{PH}(\pi^0) \) decreases in \( f \).

\[ \square \]
E Proof of Proposition 5

As in the proof of Proposition 3, we first derive the equilibrium outcome when $N = 1$ (we label this case using a mnemonic PL1); then we obtain the equilibrium outcome when $N = 2$ (labeled PL2). Finally, we arrive at the result in Proposition 4 by comparing profits across both cases.

E.1 PL1

Proof. The equilibrium is derived using backward induction.

Retail Pricing and Retail Assortment Decision. In Stage 4, after the state of the world, $A$, is revealed, the retailer chooses retail price, $p$, which determines her sales, $s(p, A)$. Then using her sales revenue, $p \cdot s(p, A)$, she pays $w_{PL} \cdot s(p, A)$ to the manufacturer. Thus the retailer chooses a vector of retail price, $p$, so as to maximize her profit subject to an inventory constraint

$$\begin{align*}
\max_{p \geq w_{PL}} (p - w_{PL}) \cdot s(p, A) \quad \text{s.t.} \quad s(p, A) \leq q_{PL}. 
\end{align*}$$

(E.1)

where $s(p, A) = (A - p)/\alpha$. $q_{PL}$ is large enough due to zero production cost so the constraint $s(p, A) \leq q_{PL}$ is never binding in equilibrium. Thus the optimal solution is given by

$$p^*_{PL} = \min \left\{ \frac{A + w_{PL}}{2}, A \right\} \quad \text{and} \quad s^*_{PL} = \frac{(A - w_{PL})^+}{2\alpha}.$$

Wholesale Pricing and Stocking Decisions. In Stage 2, the manufacturer chooses wholesale price, $w_{PL}$, and production quantity, $q_{PL}$, that are optimal in

$$\begin{align*}
\max_{q_{PL}, w_{PL}} \Pi^{(1)}_{P_L}(w_{PL}) = w_{PL} \cdot \mathbb{E}[s^*_{PL}] - f \\
\text{s.t.} \quad \pi^{(1)}_{P_L}(w_{PL}) = \mathbb{E}[(p^*_{PL} - w_{PL}) \cdot s^*_{PL}] \geq \pi^0,
\end{align*}$$

(E.2a)

(E.2b)

where $s^*_{PL}$ is the retailer’s equilibrium sales and (E.2b) is the retailer’s participation constraint.

Since the unit production cost is zero, the manufacturer’s profit weakly increases in $q_{PL}$ when $q_{PL} \leq \frac{1+u}{2\alpha}$ and is independent in $q_{PL}$ when $q_{PL} > \frac{1+u}{2\alpha}$. Thus we just consider that the manufacturer will produce $q_{PL} = \frac{1+u}{2\alpha}$.

Given the retailer’s ordering decision in Stage 4, the manufacturer’s and retailer’s expected profits can be simplified as

$$\Pi^{(1)}_{P_L}(w_{PL}) = -f + \begin{cases} 
\frac{w_{PL}(1-w_{PL})}{2\alpha}, & \text{if } w_{PL} \leq 1 - u \\
\frac{w_{PL}(1+u-w_{PL})}{4\alpha}, & \text{if } 1 - u < w_{PL} \leq 1 + u \\
0, & \text{if } w_{PL} > 1 + u
\end{cases}$$
and

\[
\pi_{PL}^{(1)}(w_{PL}) = \begin{cases} 
\frac{u^2+(1-w_{PL})^2}{4u}, & \text{if } w_{PL} \leq 1 - u \\
\frac{(1+u-w_{PL})^2}{8u}, & \text{if } 1 - u < w_{PL} \leq 1 + u \\
0, & \text{if } w_{PL} > 1 + u
\end{cases}
\]

For any given \(\pi^0\), let \(w_{PL}^*(\pi^0)\), \(\Pi_{PL}^{(1)*}(\pi^0)\), and \(\pi_{PL}^{(1)*}(\pi^0)\) denote the manufacturer’s optimal wholesale price, the manufacturer’s corresponding profit, and the retailer’s corresponding profit, respectively. We first solve the unconstrained problem where \(\pi^0 = 0\). We just need to optimize \(\Pi_{PL}^{(1)}(w_{PL})\) and the optimal wholesale price is

\[
w_{PL}^*(0) = \begin{cases} 
1/2, & \text{if } u \leq \sqrt{2} - 1 \\
(1 + u)/2, & \text{if } u > \sqrt{2} - 1
\end{cases}
\]

The manufacturer’s and retailer’s corresponding profits are

\[
\Pi_{PL}^{(1)*}(0) = \Pi_{PL}^{(1)}(w_{PL}^*(0)) = -f + \begin{cases} 
\frac{1}{8u^2}, & \text{if } u \leq \sqrt{2} - 1 \\
\frac{(1+u)^2}{16u}, & \text{if } u > \sqrt{2} - 1
\end{cases}
\]

and

\[
\pi_{PL}^{(1)*}(0) = \pi_{PL}^{(1)}(w_{PL}^*(0)) = \begin{cases} 
\frac{1+4u^2}{16u^2}, & \text{if } u \leq \sqrt{2} - 1 \\
\frac{(1+u)^2}{32u}, & \text{if } u > \sqrt{2} - 1
\end{cases}
\]

When \(\pi^0 \leq \pi_{PL}^{(1)*}(0)\), the manufacturer faces the unconstrained problem and the solution is the same as \(\pi^0 = 0\). When \(\pi^0 > \pi_{PL}^{(1)*}(0)\), in order to satisfy the retailer’s reservation constraint E.2b, the manufacturer has to set the wholesale price \(w_{PL} < w_{PL}^*(0)\) since the retailer’s profit decreases in \(w_{PL}\). The manufacturer’s and retailer’s expected profits are shown in Figure E.1. If \(u \geq 1/2\) or \(u < \sqrt{2} - 1\) (see Figure E.1(a) and E.1(c)), then \(\Pi_{PL}^{(1)}(w_{PL})\) increases in \(w_{PL}\) for \(w_{PL} < w_{PL}^*(0)\). Thus if \(\pi^0 > \pi_{PL}^{(1)*}(0)\), then the retailer’s reservation constraint E.2b will be active and the optimal solution is solved from \(\pi_{PL}^{(1)}(w_{PL}) = \pi^0\). If \(\sqrt{2} - 1 \leq u < 1/2\) (see Figure E.1(b)), then \(\Pi_{PL}^{(1)}(w_{PL})\) first increases, then decreases, and then increases in \(w_{PL}\) for \(w_{PL} < w_{PL}^*(0)\). We define \(\tilde{w}_{PL}^{(1)} \in [1-u, \frac{1+u}{2}]\) as \(\Pi_{PL}^{(1)}(\tilde{w}_{PL}^{(1)}) = \Pi_{PL}^{(1)}(1/2)\). When \(\pi_{PL}^{(1)}(0) \leq \pi^0 < \pi_{PL}^{(1)}(\tilde{w}_{PL}^{(1)})\), the retailer’s reservation constraint will be binding. When \(\pi_{PL}^{(1)}(\tilde{w}_{PL}^{(1)}) \leq \pi^0 \leq \pi_{PL}^{(1)}(1/2)\), the manufacturer’s optimal wholesale price will be \(w_{PL} = 1/2\). And when \(\pi^0 > \pi_{PL}^{(1)}(1/2)\), the retailer’s reservation constraint will be binding again.

Thus the optimal wholesale price is

\[
w_{PL}^*(\pi^0) = \begin{cases} 
w_{PL,hv}(\pi^0), & \text{if } u > \sqrt{2} - 1 \\
w_{PL,lv}(\pi^0), & \text{if } u \leq \sqrt{2} - 1
\end{cases}
\]
Figure E.1: Firms’ Ex-ante Profit Function under PL1

(a) \( u \geq 1/2 \) and \( w_{PL}^*(0) = \frac{1+u}{2} \)

(b) \( \sqrt{2} - 1 < u < 1/2 \) and \( w_{PL}^*(0) = \frac{1+u}{2} \)

(c) \( u \leq \sqrt{2} - 1 \) and \( w_{PL}^*(0) = 1/2 \)

where

\[
    w_{PL,lv}(\pi^0) = \begin{cases} 
      1/2, & \text{if } \pi^0 \leq \frac{1+4u^2}{16\alpha} \\
      1 - \sqrt{4\alpha \pi^0 - u^2}, & \text{if } \frac{1+4u^2}{16\alpha} < \pi^0 \leq \frac{1+u^2}{4\alpha} \\
      0, & \text{if } \pi^0 > \frac{1+u^2}{4\alpha}
    \end{cases}
\]

and

\[
    w_{PL,hv}(\pi^0) = \begin{cases} 
      \frac{1+u}{2}, & \text{if } \pi^0 \leq \frac{(1+u)^2}{32\alpha} \\
      1 + u - 2\sqrt{2\alpha\pi^0}, & \text{if } \frac{(1+u)^2}{32\alpha} < \pi^0 \leq \frac{u^2}{2\alpha} + \frac{u^2+2u}{16\alpha} + \sqrt{u^4+4u^2+4u^2-1}, \text{ if } u \geq 1/2 \\
      1/2, & \text{if } \frac{u^2+2u}{16\alpha} + \sqrt{u^4+4u^2+4u^2-1} < \pi^0 \leq \frac{1+4u^2}{16\alpha} \text{ and } \sqrt{2} - 1 \leq u < 1/2 \\
      1 - \sqrt{4\alpha \pi^0 - u^2}, & \text{if } \frac{u^2}{2\alpha} + \frac{u^2+2u}{16\alpha} + \sqrt{u^4+4u^2+4u^2-1} < \pi^0 \leq \frac{1+4u^2}{16\alpha} \text{ and } \sqrt{2} - 1 \leq u < 1/2 \\
      0, & \text{if } \pi^0 > \frac{1+u^2}{4\alpha}
    \end{cases}
\]
The manufacturer’s and retailer’s corresponding profits are as follows
(a) If \( u > \sqrt{2} - 1 \), then

\[
\Pi_{PL}^*(\pi^0) = -f + \begin{cases} 
\frac{(1+u)^2}{32\alpha}, & \text{if } \pi^0 \leq \frac{(1+u)^2}{32\alpha} \\
\frac{\sqrt{\pi^0(u+1) - 2\sqrt{2}u\pi^0}}{\sqrt{2\alpha}} & \text{if } \frac{(1+u)^2}{32\alpha} < \pi^0 \leq \begin{cases} 
\frac{u^2}{2\alpha}, & \text{if } u \geq 1/2 \\
\frac{u^2 + 2u}{16\alpha} + \frac{\sqrt{u^4 + 4u^2 - 1}}{16\alpha} & \text{if } \sqrt{2} - 1 \leq u < 1/2
\end{cases} \\
\frac{1}{8\alpha}, & \text{if } u^2 + 2u + \frac{\sqrt{u^4 + 4u^2 - 1}}{16\alpha} < \pi^0 \leq \frac{1+4u^2}{16\alpha} \text{ and } \sqrt{2} - 1 \leq u < 1/2 \\
-4\alpha\pi^0 + \sqrt{4\alpha\pi^0 - u^2 + u^2} & \text{if } \pi^0 > \frac{1}{4\alpha} \\
0, & \text{if } \pi^0 > \frac{1+u^2}{4\alpha}
\end{cases}
\]

and

\[
\pi_{PL}^*(\pi^0) = \begin{cases} 
\frac{(1+u)^2}{32\alpha}, & \text{if } \pi^0 \leq \frac{(1+u)^2}{32\alpha} \\
\pi^0, & \text{if } \frac{(1+u)^2}{32\alpha} < \pi^0 \leq \begin{cases} 
\frac{u^2}{2\alpha}, & \text{if } u \geq 1/2 \\
\frac{1+4u^2}{16\alpha} & \text{if } \sqrt{2} - 1 \leq u < 1/2
\end{cases} \\
\frac{1+4u^2}{16\alpha}, & \text{if } \frac{u^2 + 2u}{16\alpha} + \frac{\sqrt{u^4 + 4u^2 - 1}}{16\alpha} < \pi^0 \leq \frac{1+4u^2}{16\alpha} \text{ and } \sqrt{2} - 1 \leq u < 1/2 \\
\frac{u^2}{2\alpha}, & \text{if } u \geq 1/2 \\
\frac{1+4u^2}{16\alpha}, & \text{if } \sqrt{2} - 1 \leq u < 1/2 \text{ and } \pi^0 < \frac{1+u^2}{4\alpha} \\
0, & \text{if } \pi^0 > \frac{1+u^2}{4\alpha}
\end{cases}
\]

(b) If \( u \leq \sqrt{2} - 1 \), then

\[
\Pi_{PL}^*(\pi^0) = -f + \begin{cases} 
\frac{1}{8\alpha}, & \text{if } \pi^0 \leq \frac{1+4u^2}{16\alpha} \\
\frac{-4\alpha\pi^0 + \sqrt{4\alpha\pi^0 - u^2 + u^2}}{2\alpha}, & \text{if } \frac{1+4u^2}{16\alpha} < \pi^0 \leq \frac{1+u^2}{4\alpha} \\
0, & \text{if } \pi^0 > \frac{1+u^2}{4\alpha}
\end{cases}
\]

and

\[
\pi_{PL}^*(\pi^0) = \begin{cases} 
\frac{1+4u^2}{16\alpha}, & \text{if } \pi^0 \leq \frac{1+4u^2}{16\alpha} \\
\pi^0, & \text{if } \frac{1+4u^2}{16\alpha} < \pi^0 \leq \frac{1+u^2}{4\alpha}
\end{cases}
\]

\( \Pi_{PL}^*(\pi^0) \) decreases in \( \pi^0 \) for \( 0 \leq \pi^0 \leq \frac{1+u^2}{4\alpha} \). Thus there exists \( 0 \leq \hat{\pi}_{PL} \leq \frac{1+u^2}{4\alpha} \) such that \( \Pi_{PL}^*(\pi^0) > 0 \) when \( \pi^0 < \hat{\pi}_{PL} \) and \( \Pi_{PL}^*(\pi^0) < 0 \) when \( \pi^0 > \hat{\pi}_{PL} \). Thus PL1 is not feasible when \( \pi^0 > \hat{\pi}_{PL} \).

\[\square\]

**E.2 PL2**

**Proof.** As before, the equilibrium is derived using backward induction.
**Retail Pricing and Retail Assortment Decision.** In Stage 4, after the state of the world, $A$, is revealed, the retailer chooses a vector of retail prices, $p$, which determine her sales, $s(p,A)$. Then using her sales revenue, $p \cdot s(p,A)$, she pays $w_{PL} \cdot s(p,A)$ to the manufacturer. As such, the retailer chooses a vector of retail prices, $p$, so as to maximize her profit subject to an inventory constraint

$$\max_{p \geq w_{PL}} (p - w_{PL}) \cdot s(p,A) \quad \text{s.t.} \quad s(p,A) \leq q_{PL}. \quad (E.3)$$

As in PL1, $q_{PL}$ is large enough so that the quantity constraint is never binding. Thus the optimal solution is given by

$$s_{PL,i}^* = \begin{cases} \frac{\alpha(A_i - w_i) - \beta(A_j - w_j)}{2(\alpha^2 - \beta^2)}, & \text{if } w_i < A_i \text{ and } \frac{\beta}{\alpha}(A_j - w_j) < A_i - w_i < \frac{\alpha}{\beta}(A_j - w_j) \\ \frac{A_i - w_i}{2\alpha}, & \text{if } A_i - w_i \geq \frac{\alpha}{\beta}(A_j - w_j) \quad \text{and} \\ 0, & \text{otherwise}, \end{cases}$$

$$p_{PL,i}^* = A_i - \alpha s_{PL,i}^* - \beta s_{PL,j}^*, \text{ for } i, j = 1, 2, i \neq j.$$

**Wholesale Pricing and Stocking Decisions.** In Stage 2, the manufacturer chooses wholesale prices, $w_{PL}$, and production quantities, $q_{PL}$, that are optimal in

$$\max_{q_{PL},w_{PL}} \Pi^{(2)}_{PL}(w_{PL}) = w_{PL} \cdot E[s_{PL}^*] - 2f \quad (E.4a)$$

$$\text{s.t.} \quad \pi^{(2)}_{PL}(w_{PL}) = E[(p_{PL}^* - w_{PL}) \cdot s_{PL}^*] \geq \pi^0, \quad (E.4b)$$

where $s_{PL}^*$ is the retailer’s equilibrium sales and (E.4b) is the retailer’s participation constraint.

Since the unit production cost is zero, the manufacturer’s profit weakly increases in $q_{PL}$ when $q_{PL} \leq (\frac{1}{2\alpha}, \frac{1+u}{2\alpha})$ and is independent in $q_{PL}$ when $q_{PL} > (\frac{1+u}{2\alpha}, \frac{1+u}{2\alpha})$. Thus we just consider the manufacturer will produce $q_{PL} = (\frac{1}{2\alpha}, \frac{1+u}{2\alpha})$.

The following lemma shows that under PL2, we must have $w_{PL,1} = w_{PL,2}$ in equilibrium.

**Lemma E.1.** If $N = 2$, then neither $w_{PL,1} < w_{PL,2}$ nor $w_{PL,1} > w_{PL,2}$ can be supported as an equilibrium with the PL contract.

Proof of Lemma E.1. We show that given $N = 2$ under PL, any $w_{PL} = (w_{PL,1}, w_{PL,2})$ with $w_{PL,1} < w_{PL,2}$ is not optimal for the manufacturer. We just need to show that there exist $0 \leq \delta w_1, \delta w_2 < (w_{PL,2} - w_{PL,1})/2$, such that $\tilde{w}_{PL} = (w_{PL,1} + \delta w_1, w_{PL,2} - \delta w_2)$ dominates $w_{PL}$, i.e., $\Pi^{(2)}_{PL}(\tilde{w}_{PL}) > \Pi^{(2)}_{PL}(w_{PL})$ and $\pi^{(2)}_{PL}(\tilde{w}_{PL}) > \pi^{(2)}_{PL}(w_{PL})$.

The manufacturer’s and retailer’s expected profits can be written as

$$\Pi^{(2)}_{PL}(w_{PL}) = w_{PL} \cdot E[s_{PL}^*(w_{PL})] - 2f$$

and

$$\pi^{(2)}_{PL}(w_{PL}) = E[(p_{PL}(s_{PL}^*(w_{PL})) - w_{PL}) \cdot s_{PL}^*(w_{PL})].$$

For small enough $\delta w_1$ and $\delta w_2$, we have
\[ \Pi_{PL}^{(2)}(w_{PL}) = \Pi_{PL}^{(2)}(w_{PL}) + \delta w_1 \frac{\partial \Pi_{PL}^{(2)}(w_{PL})}{\partial w_{PL,1}} - \delta w_2 \frac{\partial \Pi_{PL}^{(2)}(w_{PL})}{\partial w_{PL,2}} + o(\delta w_1) + o(\delta w_2) \quad \text{and} \]
\[ \pi_{PL}^{(2)}(w_{PL}) = \pi_{PL}^{(2)}(w_{PL}) + \delta w_1 \frac{\partial \pi_{PL}^{(2)}(w_{PL})}{\partial w_{PL,1}} - \delta w_2 \frac{\partial \pi_{PL}^{(2)}(w_{PL})}{\partial w_{PL,2}} + o(\delta w_1) + o(\delta w_2). \]

To show \( \Pi_{PL}^{(2)}(w_{PL}) > \Pi_{PL}^{(2)}(w_{PL}) \) and \( \pi_{PL}^{(2)}(w_{PL}) > \pi_{PL}^{(2)}(w_{PL}) \), we just need to show that \( \delta w_1 \frac{\partial \Pi_{PL}^{(2)}(w_{PL})}{\partial w_{PL,1}} - \delta w_2 \frac{\partial \Pi_{PL}^{(2)}(w_{PL})}{\partial w_{PL,2}} > 0 \) and \( \delta w_1 \frac{\partial \pi_{PL}^{(2)}(w_{PL})}{\partial w_{PL,1}} - \delta w_2 \frac{\partial \pi_{PL}^{(2)}(w_{PL})}{\partial w_{PL,2}} > 0 \).

One can easily verify that \( \frac{\partial \pi_{PL}^{(2)}(w_{PL})}{\partial w_{PL,1}} < \frac{\partial \pi_{PL}^{(2)}(w_{PL})}{\partial w_{PL,2}} < 0 \), that is the retailer’s profit decreases in the wholesale prices and is more sensitive to the lower wholesale price \( w_{PL,1} \) than the higher wholesale price \( w_{PL,2} \).

If \( \frac{\partial \Pi_{PL}^{(2)}(w_{PL})}{\partial w_{PL,2}} < 0 \), then we just need to take \( \delta w_1 = 0 \) and small enough \( \delta w_2 > 0 \). Then we have \( \delta w_1 \frac{\partial \Pi_{PL}^{(2)}(w_{PL})}{\partial w_{PL,1}} - \delta w_2 \frac{\partial \Pi_{PL}^{(2)}(w_{PL})}{\partial w_{PL,2}} = -\delta w_2 \frac{\partial \Pi_{PL}^{(2)}(w_{PL})}{\partial w_{PL,2}} > 0 \) and \( \delta w_1 \frac{\partial \pi_{PL}^{(2)}(w_{PL})}{\partial w_{PL,1}} - \delta w_2 \frac{\partial \pi_{PL}^{(2)}(w_{PL})}{\partial w_{PL,2}} = -\delta w_2 \frac{\partial \pi_{PL}^{(2)}(w_{PL})}{\partial w_{PL,2}} > 0 \). Thus \( \Pi_{PL}^{(2)}(w_{PL}) > \Pi_{PL}^{(2)}(w_{PL}) \) and \( \pi_{PL}^{(2)}(w_{PL}) > \pi_{PL}^{(2)}(w_{PL}) \).

If \( \frac{\partial \Pi_{PL}^{(2)}(w_{PL})}{\partial w_{PL,2}} \geq 0 \), then one can verify that \( \frac{\partial \Pi_{PL}^{(2)}(w_{PL})}{\partial w_{PL,1}} \geq \frac{\partial \Pi_{PL}^{(2)}(w_{PL})}{\partial w_{PL,2}} \), that is the manufacturer’s profit increases in the wholesale prices and is more sensitive to the lower wholesale price \( w_{PL,1} \) than the higher wholesale price \( w_{PL,2} \). We can also verify that \( 0 < \frac{\partial \Pi_{PL}^{(2)}(w_{PL})}{\partial w_{PL,1}} \leq \frac{\partial \Pi_{PL}^{(2)}(w_{PL})}{\partial w_{PL,2}} \), \( \frac{\partial \pi_{PL}^{(2)}(w_{PL})}{\partial w_{PL,2}} \) and \( \frac{\partial \pi_{PL}^{(2)}(w_{PL})}{\partial w_{PL,1}} \) which leads to

\[ \delta w_1 \frac{\partial \Pi_{PL}^{(2)}(w_{PL})}{\partial w_{PL,1}} - \delta w_2 \frac{\partial \Pi_{PL}^{(2)}(w_{PL})}{\partial w_{PL,2}} > 0 \quad \text{and} \quad \delta w_1 \frac{\partial \pi_{PL}^{(2)}(w_{PL})}{\partial w_{PL,1}} - \delta w_2 \frac{\partial \pi_{PL}^{(2)}(w_{PL})}{\partial w_{PL,2}} > 0. \]

Therefore \( \Pi_{PL}^{(2)}(w_{PL}) > \Pi_{PL}^{(2)}(w_{PL}) \) and \( \pi_{PL}^{(2)}(w_{PL}) > \pi_{PL}^{(2)}(w_{PL}) \). We have shown that any \( w_{PL} = (w_{PL,1}, w_{PL,2}) \) with \( w_{PL,1} < w_{PL,2} \) can not be supported as an equilibrium. Similarly, we can show that any \( w_{PL} = (w_{PL,1}, w_{PL,2}) \) with \( w_{PL,1} > w_{PL,2} \) can not be supported as an equilibrium, either. Therefore, given \( N = 2 \) under PL, we must have \( w_{PL,1} = w_{PL,2} \) in an equilibrium.

Due to Lemma E.1, we just need to consider \( w_{PL,1} = w_{PL,2} = w_{PL}^{(2)} \). Given the retailer’s ordering decision in Stage 4, the manufacturer’s and retailer’s expected profits can therefore be simplified as

\[ \Pi_{PL}^{(2)}(w_{PL}^{(2)}) = -2f + \begin{cases} \frac{w_{PL}^{(2)} - w_{PL}^{(2)}}{\alpha + \beta}, & \text{if } w_{PL}^{(2)} \leq \frac{a(1-u) - \beta(1+u)}{a - \beta} \leq 1 - u \\ \frac{w_{PL}^{(2)} a(3+u - 3 w_{PL}^{(2)}) + \beta(1+u - w_{PL}^{(2)})}{4 a (\alpha + \beta)}, & \text{if } 1 - u < w_{PL}^{(2)} \leq 1 + u \\ \frac{w_{PL}^{(2)} (2a+\beta)(1+u - w_{PL}^{(2)})}{4 a (\alpha + \beta)}, & \text{if } w_{PL}^{(2)} > 1 + u \end{cases} \]
and

\[ \pi_{PL}^{(2)}(w_{PL}^{(2)}) = \begin{cases} 
\frac{\alpha(u^2 + (1-w_{PL}^{(2)})^2) - \beta(1-w_{PL}^{(2)})^2}{2(\alpha - \beta)^2}, & \text{if } w_{PL}^{(2)} \leq \frac{\alpha(1-u) - \beta(1+u)}{\alpha - \beta} \\
\frac{\alpha(3u^2 + 2u(1-w_{PL}^{(2)}) + 3(1-w_{PL}^{(2)})^2) + \beta(1+u-w_{PL}^{(2)})^2}{8\alpha(\alpha + \beta)}, & \text{if } \frac{\alpha(1-u) - \beta(1+u)}{\alpha - \beta} < w_{PL}^{(2)} \leq 1 - u \\
0, & \text{if } 1 - u < w_{PL}^{(2)} \leq 1 + u \\
\end{cases} \]

For any given \( \pi^0 \), let \( w_{PL}^{(2)*}(\pi^0) \), \( \Pi_{PL}^{(2)*}(\pi^0) \), and \( \pi_{PL}^{(2)*}(\pi^0) \) denote the manufacturer’s optimal wholesale price, the manufacturer’s corresponding profit, and the retailer’s corresponding profit, respectively. We first solve the unconstrained problem where \( \pi^0 = 0 \). We just need to optimize \( \Pi_{PL}^{(2)}(w_{PL}^{(2)}) \) and the optimal wholesale price is

\[ w_{PL}^{(2)*}(0) = \begin{cases} 
\frac{1+u}{2}, & \text{if } u \geq u_1(\beta) \\
\frac{\alpha(3+u) + \beta(1+u)}{6\alpha + 2\beta}, & \text{if } u_2(\beta) < u < u_1(\beta), \\
1/2, & \text{if } u \leq u_2(\beta) 
\end{cases} \]

where \( u_1(\beta) = \max\{2\sqrt{\frac{\alpha}{2\alpha + \beta}} - 1, 2\sqrt{\frac{6\alpha^2 + 5\alpha\beta - \beta^2 - 3\alpha - \beta}{5\alpha + 3\beta}}\} \), \( u_2(\beta) = \min\{2\sqrt{\frac{\alpha}{2\alpha + \beta}} - 1, 2\sqrt{\frac{\alpha(3\alpha + \beta) - 3\alpha - \beta}{\alpha + \beta}}\} \).

The manufacturer’s and retailer’s corresponding profits are

\[ \Pi_{PL}^{(2)*}(0) = \Pi_{PL}^{(2)}(w_{PL}^{(2)*}(0)) = -2f + \begin{cases} 
\frac{(2\alpha + \beta)(1+u)^2}{16\alpha(\alpha + \beta)}, & \text{if } u \geq u_1(\beta) \\
\frac{(\alpha(3+u) + \beta(1+u))^2}{16\alpha(\alpha + \beta)(3\alpha + \beta)}, & \text{if } u_2(\beta) < u < u_1(\beta) \\
\frac{1}{\alpha(\alpha + \beta)}, & \text{if } u \leq u_2(\beta) 
\end{cases} \]

and

\[ \pi_{PL}^{(2)*}(0) = \pi_{PL}^{(2)}(w_{PL}^{(2)*}(0)) = \begin{cases} 
\frac{(2\alpha + \beta)(1+u)^2}{32\alpha(\alpha + \beta)}, & \text{if } u \geq u_1(\beta) \\
\frac{(9+6u+3u^2)\alpha^2 + 2(3+4u+9u^2)\alpha\beta + (1+u)^2\beta^2}{32\alpha(\alpha + \beta)(3\alpha + \beta)}, & \text{if } u_2(\beta) < u < u_1(\beta) \\
\frac{\alpha(1+4u^2) - \beta}{8(\alpha^2 - \beta^2)}, & \text{if } u \leq u_2(\beta) 
\end{cases} \]

When \( \pi^0 \leq \pi_{PL}^{(2)*}(0) \), the manufacturer faces the unconstrained problem and the solution is the same as \( \pi^0 = 0 \). When \( \pi^0 > \pi_{PL}^{(2)*}(0) \), similar to the analysis in PL1 and PH2, we can show that the optimal wholesale prices under PL2 is

\[ w_{PL}^{(2)*}(\pi^0) = \begin{cases} 
\frac{w_{PL,h1}(\pi^0)}{u_{PL,h1}(\pi^0)}, & \text{if } u \geq u_1(\beta) \\
\frac{w_{PL,mv}(\pi^0)}{u_{PL,mv}(\pi^0)}, & \text{if } u_2(\beta) < u < u_1(\beta) \\
\frac{w_{PL,lv}(\pi^0)}{u_{PL,lv}(\pi^0)}, & \text{if } u \leq u_2(\beta) 
\end{cases} \]
where

$$\begin{align*}
u_{P_L,Lv}^{(2)}(\pi^0) &= \begin{cases} 
\frac{1+u}{2}, & \text{if } \pi^0 \leq \frac{(2\alpha+\beta)(1+u)^2}{32\alpha(\alpha+\beta)} \\
2 \left( \frac{1}{2} + \frac{\beta(1+u)}{6\alpha+2\beta} \right) - \frac{\sqrt{2} \alpha(\alpha+\beta)}{\alpha+\beta}, & \text{if } \frac{(2\alpha+\beta)(1+u)^2}{32\alpha(\alpha+\beta)} < \pi^0 \leq \pi_1^0 \\
\frac{\alpha(3+u)+\beta(1+u)}{6\alpha+2\beta} - \frac{\beta(u+1)}{3\alpha+2\beta}, & \text{if } \pi_1^0 < \pi^0 \leq \pi_2^0 \\
\frac{1}{2}, & \text{if } \pi_2^0 < \pi^0 \leq \pi_3^0 \\
1 - \sqrt{\frac{2\alpha^2\pi^0 - 2\beta^2\pi^0 - \alpha u^2}{\alpha-\beta}}, & \text{if } \pi_3^0 < \pi^0 \leq \pi_4^0 \\
0, & \text{if } \pi_4^0 < \pi^0 \leq \frac{\alpha^2+\beta^2-\beta}{2(\alpha^2-\beta^2)} \\
\frac{\alpha(3+u)+\beta(1+u)}{6\alpha+2\beta} - \frac{\beta(u+1)}{3\alpha+2\beta}, & \text{if } \frac{\alpha^2+\beta^2-\beta}{2(\alpha^2-\beta^2)} < \pi^0 \\
\frac{1}{2}, & \text{if } \pi^0 > \frac{\alpha^2+\beta^2-\beta}{2(\alpha^2-\beta^2)} \\
1 - \sqrt{\frac{2\alpha^2\pi^0 - 2\beta^2\pi^0 - \alpha u^2}{\alpha-\beta}}, & \text{if } \pi^0 > \frac{\alpha^2+\beta^2-\beta}{2(\alpha^2-\beta^2)} \\
0, & \text{if } \pi^0 > \frac{\alpha^2+\beta^2-\beta}{2(\alpha^2-\beta^2)} 
\end{cases}
\end{align*}$$

$$\begin{align*}
u_{P_L,mv}^{(2)}(\pi^0) &= \begin{cases} 
\frac{1}{2}, & \text{if } \pi^0 \leq M_1 \\
\frac{\alpha(3+u)+\beta(1+u)}{6\alpha+2\beta} - \frac{\beta(u+1)}{3\alpha+2\beta}, & \text{if } M_1 < \pi^0 \leq \pi_5^0 \\
\frac{1}{2}, & \text{if } \pi_5^0 < \pi^0 \leq \pi_4^0 \\
1 - \sqrt{\frac{2\alpha^2\pi^0 - 2\beta^2\pi^0 - \alpha u^2}{\alpha-\beta}}, & \text{if } \pi_4^0 < \pi^0 \leq \frac{\alpha^2+\beta^2-\beta}{2(\alpha^2-\beta^2)} \\
0, & \text{if } \pi^0 > \frac{\alpha^2+\beta^2-\beta}{2(\alpha^2-\beta^2)} 
\end{cases}
\end{align*}$$

and

$$\begin{align*}
u_{P_L,lv}^{(2)}(\pi^0) &= \begin{cases} 
\frac{1}{2}, & \text{if } \pi^0 \leq \frac{\alpha^2+\beta^2-\beta}{2(\alpha^2-\beta^2)} \\
\frac{1}{2}, & \text{if } \frac{\alpha^2+\beta^2-\beta}{2(\alpha^2-\beta^2)} < \pi^0 \\
1 - \sqrt{\frac{2\alpha^2\pi^0 - 2\beta^2\pi^0 - \alpha u^2}{\alpha-\beta}}, & \text{if } \frac{\alpha^2+\beta^2-\beta}{2(\alpha^2-\beta^2)} < \pi^0 \leq \frac{\alpha^2+\beta^2-\beta}{2(\alpha^2-\beta^2)} \\
0, & \text{if } \pi^0 > \frac{\alpha^2+\beta^2-\beta}{2(\alpha^2-\beta^2)}
\end{cases}
\end{align*}$$

Above, we let

$$Z_1 = \sqrt{2\alpha^0(3\alpha^2+4\alpha\beta+\beta^2)-u^2(2\alpha+\beta)}, \quad M_1 = \frac{\alpha^2(33u^2+6u+9)+2\alpha\beta(9u^2+4u+3)+\beta^2(u+1)^2}{32\alpha(\alpha+\beta)(3\alpha+\beta)}, \quad Q_1 = \sqrt{\alpha^2(u^2+6u-3)+2\alpha\beta(u^2+4u+1)+\beta^2(u+1)^2}, \quad u_4 = \max\left\{\frac{3\alpha+\beta}{\alpha+3\beta}, \frac{1}{2}(1-\frac{\beta}{2\alpha+\beta})\right\},$$

$$\begin{align*}
u_3 &= \begin{cases} 
\left(1-\frac{\beta}{2\alpha+\beta}\right)/2, & \text{if } 2\sqrt{\frac{\alpha^2(3\alpha+\beta)^2-3\alpha-\beta}{\alpha+3\beta}} > \frac{3\alpha+\beta}{\alpha+3\beta}, \\
\max\left\{\frac{2\sqrt{\alpha(3\alpha+\beta)-3\alpha-\beta}}{\alpha+3\beta}, \frac{2\sqrt{6\alpha^2+5\alpha\beta+\beta^2-3\alpha-\beta}}{5\alpha+3\beta}\right\}, & \text{otherwise}
\end{cases}
\end{align*}$$

$$\begin{align*}
u_1^0 &= \begin{cases} 
\left(\frac{\beta}{\sqrt{2(2\alpha+\beta)}}\right)^2(u^2+2u-1)+\beta(u+1)^2+\beta(2u+1)+\beta u, & \text{if } u_1 < u \leq u_3 \\
\frac{(\alpha^2(1+u)\left[\frac{(2\alpha^2+6u+3)+2\alpha(2\alpha+\beta)}{(3\alpha+\beta)}\right] + 2\alpha(\alpha+\beta) + \beta(u+1))^2}{32\alpha(\alpha+\beta)(3\alpha+\beta)}, & \text{if } u_3 < u \leq u_4 \\
\frac{\beta^2(2\alpha+\beta)}{2\alpha(\alpha+\beta)}, & \text{if } u > u_4
\end{cases}
\end{align*}$$

$$\begin{align*}
u_2^0 &= \begin{cases} 
\frac{\beta^2(2\alpha+\beta)}{2\alpha(\alpha+\beta)}, & \text{if } u_1 < u \leq u_3 \\
\frac{\beta^2(2\alpha+\beta)}{2\alpha(\alpha+\beta)}, & \text{if } u_3 < u \leq u_4 \\
\frac{\beta^2(2\alpha+\beta)}{2\alpha(\alpha+\beta)}, & \text{if } u > u_4
\end{cases}
\end{align*}$$

$$\begin{align*}
u_3^0 &= \begin{cases} 
\frac{\beta^2(2\alpha+\beta)}{2\alpha(\alpha+\beta)}, & \text{if } u_1 < u \leq u_3 \\
\frac{\beta^2(2\alpha+\beta)}{2\alpha(\alpha+\beta)}, & \text{if } u > u_3
\end{cases}
\end{align*}$$
$\pi_4^0 = \begin{cases} 
\alpha + 4u^2 \alpha - \beta, & \text{if } u \leq \frac{\alpha - \beta}{2(\alpha + \beta)} \\
u^2(2\alpha^2 + \alpha\beta + \beta^2) - 8u(\alpha^2 - \beta^2), & \text{if } u > \frac{\alpha - \beta}{2(\alpha + \beta)} 
\end{cases}$

and

$\pi_5^0 = \begin{cases} 
\alpha^2(17u^2 + 6u + 3) + 2\alpha\beta(5u^2 + 4u + 2) + \beta^2(u + 1)^2 + Q_1(\alpha(u + 3) + \beta(u + 1)), & \text{if } u \leq \frac{\alpha - \beta}{2(\alpha + \beta)} \\
u^2(2\alpha^2 + \alpha\beta + \beta^2), & \text{if } u > \frac{\alpha - \beta}{2(\alpha + \beta)} 
\end{cases}$

The manufacturer's and retailer's corresponding profits are as follows.

(a) If $u \geq u_1(\beta)$, then

$\Pi_{PL}^{(2)}(\pi^0) = -2f + \begin{cases} 
\frac{(2\alpha + \beta)(1 + u)^2}{32\alpha(\alpha + \beta)}, & \text{if } \pi^0 \leq \frac{(2\alpha + \beta)(1 + u)^2}{32\alpha(\alpha + \beta)} \\
\frac{(u + 1)(2\alpha + \beta)}{\sqrt{\alpha(\alpha + \beta)(3\alpha + \beta)}}, & \text{if } \frac{(2\alpha + \beta)(1 + u)^2}{32\alpha(\alpha + \beta)} < \pi^0 \leq \pi_1^0 \\
\frac{(\alpha + \beta)(u + 1) + \alpha(u - 2y_1 + 3)}{2(\alpha + \beta)(3\alpha + \beta)}, & \text{if } \pi_1^0 < \pi^0 \leq \pi_2^0 \\
1 - \frac{1}{\sqrt{\alpha + \beta}}, & \text{if } \pi_2^0 < \pi^0 \leq \pi_3^0 \\
0, & \text{if } \pi_3^0 < \pi^0 \leq \pi_4^0 \\
\frac{1}{2(\alpha + \beta)}, & \text{if } \pi_4^0 < \pi^0 \leq \frac{\alpha + \alpha u^2 - \beta}{2(\alpha^2 - \beta^2)} \\
\pi_4^0, & \text{if } \pi^0 > \frac{\alpha + \alpha u^2 - \beta}{2(\alpha^2 - \beta^2)} 
\end{cases}$

and

$\Pi_{PL}^{(2)}(\pi^0) = \begin{cases} 
\frac{(2\alpha + \beta)(1 + u)^2}{32\alpha(\alpha + \beta)}, & \text{if } \pi^0 \leq \frac{(2\alpha + \beta)(1 + u)^2}{32\alpha(\alpha + \beta)} \\
\frac{\alpha^2(17u^2 + 6u + 3) + 2\alpha\beta(5u^2 + 4u + 2) + \beta^2(u + 1)^2 + Q_1(\alpha(u + 3) + \beta(u + 1))}{32\alpha(\alpha + \beta)(3\alpha + \beta)}, & \text{if } \frac{(2\alpha + \beta)(1 + u)^2}{32\alpha(\alpha + \beta)} < \pi^0 \leq \pi_1^0 \\
\pi_1^0, & \text{if } \pi_1^0 < \pi^0 \leq \pi_2^0 \\
\frac{\alpha - \beta + 4u^2}{8\alpha^2 - 8\beta^2}, & \text{if } \pi_2^0 < \pi^0 \leq \pi_3^0 \\
\pi_3^0, & \text{if } \pi_3^0 < \pi^0 \leq \pi_4^0 \\
0, & \text{if } \pi_4^0 < \pi^0 \leq \frac{\alpha + \alpha u^2 - \beta}{2(\alpha^2 - \beta^2)} \\
\pi_4^0, & \text{if } \pi^0 > \frac{\alpha + \alpha u^2 - \beta}{2(\alpha^2 - \beta^2)} 
\end{cases}$

(b) If $u_2(\beta) < u < u_1(\beta)$, then

$\Pi_{PL}^{(2)}(\pi^0) = -2f + \begin{cases} 
\frac{(\alpha + \beta)(u + 1)^2}{32\alpha(\alpha + \beta)}, & \text{if } \pi^0 \leq M_1 \\
y_1(\beta(u + 1) + \alpha(u - 2y_1 + 3)), & \text{if } M_1 < \pi^0 \leq \pi_5^0 \\
1 - \frac{1}{\sqrt{\alpha + \beta}}, & \text{if } \pi_5^0 < \pi^0 \leq \pi_4^0 \\
0, & \text{if } \pi_4^0 < \pi^0 \leq \frac{\alpha + \alpha u^2 - \beta}{2(\alpha^2 - \beta^2)} \\
\pi_4^0, & \text{if } \pi^0 > \frac{\alpha + \alpha u^2 - \beta}{2(\alpha^2 - \beta^2)} 
\end{cases}$
and

\[
\pi_{PL}^{(2)}(\pi^0) = \begin{cases} 
\frac{\alpha^2 (3\alpha^2 + 6\alpha + 9) + 2\alpha \beta (9\alpha^2 + 4\alpha + 3) + \beta^2 (u+1)^2}{32\alpha (\alpha + \beta)(3\alpha + \beta)}, & \text{if } \pi^0 \leq M_1 \\
\pi^0, & \text{if } M_1 < \pi^0 \leq \pi_5^0 \\
\frac{-\beta + 4\alpha u^2}{8\alpha^2 - 8\beta}, & \text{if } \pi_5^0 < \pi^0 \leq \pi_4^0 \\
\pi^0, & \text{if } \pi_4^0 < \pi^0 \leq \frac{\alpha + \alpha u^2 - \beta}{2(\alpha^2 - \beta^2)} \\
0, & \text{if } \pi^0 > \frac{\alpha + \alpha u^2 - \beta}{2(\alpha^2 - \beta^2)}
\end{cases}
\]

(c) If \( u \leq u_2(\beta) \), then

\[
\Pi_{PL}^{(2)}(\pi^0) = -2f + \begin{cases} 
\frac{1}{4(\alpha + \beta)} \left( \frac{-2\alpha^2 \alpha^2 + \beta^2 + \alpha^2 + \alpha^2}{\alpha + \beta} \left( 1 - \frac{-2\alpha^2 \alpha^2 + \beta^2 + \alpha^2 + \alpha^2}{\alpha + \beta} \right) \right), & \text{if } \pi^0 \leq \frac{-\beta + 4\alpha u^2}{8\alpha^2 - 8\beta^2} \\
0, & \text{if } \frac{-\beta + 4\alpha u^2}{8\alpha^2 - 8\beta^2} < \pi^0 \leq \frac{\alpha + \alpha u^2 - \beta}{2(\alpha^2 - \beta^2)} \\
\end{cases}
\]

and

\[
\pi_{PL}^{(1)}(\pi^0) = \begin{cases} 
\frac{-\beta + 4\alpha u^2}{8\alpha^2 - 8\beta^2}, & \text{if } \pi^0 \leq \frac{-\beta + 4\alpha u^2}{8\alpha^2 - 8\beta^2} \\
\pi^0, & \text{if } \frac{-\beta + 4\alpha u^2}{8\alpha^2 - 8\beta^2} < \pi^0 \leq \frac{\alpha + \alpha u^2 - \beta}{2(\alpha^2 - \beta^2)} \\
0, & \text{if } \pi^0 > \frac{\alpha + \alpha u^2 - \beta}{2(\alpha^2 - \beta^2)}
\end{cases}
\]

where \( Y_1 = \sqrt{\frac{2\pi\rho(3\alpha^2 + 4\alpha \beta + \beta^2) - u^2 (2\alpha + \beta)}}{\alpha^2 - \beta^2} \).

\[ \Pi_{PL}^{(2)}(\pi^0) \] decreases in \( \pi^0 \) for \( 0 \leq \pi^0 \leq \frac{\alpha + \alpha u^2 - \beta}{2(\alpha^2 - \beta^2)} \). Thus there exists \( 0 \leq \hat{\pi}_{PL}^{(2)} \leq \frac{\alpha + \alpha u^2 - \beta}{2(\alpha^2 - \beta^2)} \) such that \( \Pi_{PL}^{(2)}(\pi^0) > 0 \) when \( \pi^0 < \hat{\pi}_{PL}^{(2)} \) and \( \Pi_{PL}^{(2)}(\pi^0) < 0 \) when \( \pi^0 > \hat{\pi}_{PL}^{(2)} \). Therefore when \( \pi^0 > \hat{\pi}_{PL}^{(2)} \), PL2 is not feasible.

\[ \square \]

### E.2.1 PL1 vs. PL2

**Proof.** Now, we compare profits to determine when PL1 and PL2 should be expected in equilibrium. To do this, one can check that \( \Pi_{PL}^{(2)}(\pi^0) \) is decreasing in \( \beta \) and \( \Pi_{PL}^{(1)}(\pi^0) \) is independent in \( \beta \). Thus,

(a) If \( \Pi_{PL}^{(1)}(\pi^0) = \max\{\Pi_{PL}^{(1)}(\pi^0), 0\} \) for any \( \beta \in [0, \alpha \frac{1-u}{1+u}] \), then let \( \hat{\beta}_{PL}(\pi^0) = \alpha \frac{1-u}{1+u} \);

(b) If \( \Pi_{PL}^{(2)}(\pi^0) = \min\{\Pi_{PL}^{(1)}(\pi^0), 0\} \) for any \( \beta \in [0, \alpha \frac{1-u}{1+u}] \), then let \( \hat{\beta}_{PL}(\pi^0) = 0 \);

(c) Otherwise there exists \( \hat{\beta}_{PL}(\pi^0) \in [0, \alpha \frac{1-u}{1+u}] \) such that \( \Pi_{PL}^{(2)}(\pi^0) > \max\{\Pi_{PL}^{(1)}(\pi^0), 0\} \) whenever \( \beta < \hat{\beta}_{PL}(\pi^0) \), and \( \Pi_{PL}^{(2)}(\pi^0) < \min\{\Pi_{PL}^{(1)}(\pi^0), 0\} \) whenever \( \beta > \hat{\beta}_{PL}(\pi^0) \).

Thus if \( \beta < \hat{\beta}_{PL}(\pi^0) \), \( N^* = 2 \) under PL. If \( \beta > \hat{\beta}_{PL}(\pi^0) \), \( N^* = 1 \) when \( \pi^0 \leq \hat{\pi}_{PL} \); and \( N^* = 0 \) when \( \pi^0 > \hat{\pi}_{PL} \) since \( N = 1 \) under PL is not feasible. This completes the proof of Proposition 5.

\[ \square \]

### F Proof of Proposition 6

**Proof.** Let \( \Pi_j^{(N)}(\pi^0) = \max\{\Pi_j^{(1)}(\pi^0), \Pi_j^{(2)}(\pi^0)\} \) denote the manufacturer’s optimal profit under contract type \( j \in \{PH, PL\} \). We compare the manufacturer’s optimal profit under each contract.
1. When IF is feasible, i.e., $\pi^0 \leq \hat{\pi}_{IF}$, we can show that $\Pi_{IF}^*(\pi^0) > \Pi_{PH}^*(\pi^0)$ and $\Pi_{IF}^*(\pi^0) > \Pi_{PL}^{(N)*}(\pi^0)$. Thus the optimal contract is IF.

2. When IF is not feasible, i.e., $\pi^0 > \hat{\pi}_{IF}$, we just need to compare PH vs. PL. Let $\Pi_{PHPL}^{(2)*}(\pi^0) = \max\{\Pi_{PH}^{(2)*}(\pi^0), \Pi_{PL}^{(2)*}(\pi^0)\}$ and $\Pi_{PHPL}^{(1)*}(\pi^0) = \max\{\Pi_{PH}^{(1)*}(\pi^0), \Pi_{PL}^{(1)*}(\pi^0)\}$ denote the manufacturer’s optimal profit with two product versions and a single product version, respectively. Since $\Pi_{PHPL}^{(2)*}(\pi^0)$ decreases in $\beta$ and $\Pi_{PHPL}^{(1)*}(\pi^0)$ is independent in $\beta$, there exists $\bar{\beta}(\pi^0)$ such that $\Pi_{PHPL}^{(2)*}(\pi^0) \geq \max\{\Pi_{PHPL}^{(1)*}(\pi^0), 0\}$ when $\beta \leq \bar{\beta}(\pi^0)$; and $\Pi_{PHPL}^{(2)*}(\pi^0) < \max\{\Pi_{PHPL}^{(1)*}(\pi^0), 0\}$ when $\beta > \bar{\beta}(\pi^0)$.

(a) Thus when $\beta \leq \bar{\beta}(\pi^0)$, offering two product versions is optimal. We just need to compare PH2 and PL2.

If $0 \leq u \leq \sqrt{2} - 1$, then $\Pi_{PH}^{(2)*}(\pi^0) - \Pi_{PL}^{(2)*}(\pi^0) \leq 0$ when $\beta = 0$ and when $\beta = \alpha \frac{1-u}{1+u}$, and $\Pi_{PH}^{(2)*}(\pi^0) - \Pi_{PL}^{(2)*}(\pi^0)$ first decreases, then increases, then decreases, and then increases in $\beta$ for $\beta \in [0, \alpha \frac{1-u}{1+u}]$. If $\Pi_{PH}^{(2)*}(\pi^0) - \Pi_{PL}^{(2)*}(\pi^0) \leq 0$ for any $\beta \in [0, \alpha \frac{1-u}{1+u}]$, then we define $\underline{\beta}_{PHPL}(\pi^0) = 0$ and $\bar{\beta}_{PHPL}(\pi^0) = 0$. Otherwise, there exist $0 \leq \underline{\beta}_{PHPL}(\pi^0) < \bar{\beta}_{PHPL}(\pi^0) \leq \alpha \frac{1-u}{1+u}$, such that $\Pi_{PH}^{(2)*}(\pi^0) - \Pi_{PL}^{(2)*}(\pi^0) \geq 0$ when $\beta \in (\underline{\beta}_{PHPL}(\pi^0), \bar{\beta}_{PHPL}(\pi^0))$ and $\Pi_{PH}^{(2)*}(\pi^0) - \Pi_{PL}^{(2)*}(\pi^0) \leq 0$ if $\beta \notin (\underline{\beta}_{PHPL}(\pi^0), \bar{\beta}_{PHPL}(\pi^0))$.

If $u > \sqrt{2} - 1$, then $\Pi_{PH}^{(2)*}(\pi^0) - \Pi_{PL}^{(2)*}(\pi^0) > 0$ when $\beta = 0$, and $\Pi_{PH}^{(2)*}(\pi^0) - \Pi_{PL}^{(2)*}(\pi^0)$ first increases and then decreases in $\beta$ for $\beta < \frac{1+u^2-2u}{2u^2+u}$, and $\Pi_{PH}^{(2)*}(\pi^0) - \Pi_{PL}^{(2)*}(\pi^0) \leq 0$ when $\beta \geq \frac{1+u^2-2u}{2u^2+u}$. Thus there exists $0 \leq \bar{\beta}_{PHPL}(\pi^0)$, such that $\Pi_{PH}^{(2)*}(\pi^0) - \Pi_{PL}^{(2)*}(\pi^0) > 0$ when $\beta < \bar{\beta}_{PHPL}(\pi^0)$ and $\Pi_{PH}^{(2)*}(\pi^0) - \Pi_{PL}^{(2)*}(\pi^0) \leq 0$ otherwise. Let $\underline{\beta}_{PHPL}(\pi^0) = 0$ when $u > \sqrt{2} - 1$.

Thus, for any given $u \in [0, 1]$, we have

- PH2 if $\underline{\beta}_{PHPL}(\pi^0) < \beta < \min\{\bar{\beta}_{PHPL}(\pi^0), \bar{\beta}(\pi^0)\}$.
- PL2 otherwise.

(b) When $\beta > \bar{\beta}(\pi^0)$, offering two product versions is not optimal. Define $\hat{\pi} = \max\{\hat{\pi}_{PH}, \hat{\pi}_{PL}\}$. When $\pi^0 \geq \hat{\pi}$, both PL1 and PH1 are not feasible and thus $N = 0$. When $\pi^0 \leq \hat{\pi}$, we just need to compare PH1 and PL1.

One can easily show that $\Pi_{PH}^{(1)*}(\pi^0) \geq \Pi_{PL}^{(1)*}(\pi^0)$ when $u > \sqrt{2} - 1$ and $\Pi_{PH}^{(1)*} \leq \Pi_{PL}^{(1)*}$ when $u < \sqrt{2} - 1$. Thus,

- PH1 if $u > \sqrt{2} - 1$.
- PL1 if $u < \sqrt{2} - 1$.

\[\square\]

G Proof of Corollary 2-4

Proof. The proofs of Corollary 2-4 follow directly from Proposition 2, 3, 5 and 6. \[\square\]