Capsules and Comments

Overfitting Bias in the Models Assessing the Predictive Power of Quarterly Reports

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In an earlier article in this Journal, Abdel-khalik and Espejo [1978] used univariate and multivariate regression models to assess the use of quarterly prediction errors by analysts in revising annual earnings predictions. They rationalized the relevance of multivariate models and, after estimating several models, concluded that analysts use the signals produced by quarterly reports to revise their forecasts of annual earnings. Their models explained about 60 percent of the variation in prediction errors of annual earnings.

In a note that followed, Brown et al. [1980] argued that multivariate models suffer from some econometric shortcomings. They favored the use of the univariate model that was suggested, but not estimated, by Abdel-khalik and Espejo. They further argued for testing the significance of the coefficients of quarterly signals against unity rather than against zero. This point was discussed in a later paper by Abdel-khalik [1981] in which the data of Brown et al. [1980] were used to show that testing against unity would not alter the findings.

Nevertheless, a technical issue concerning model specification remains unresolved and pervades both papers. The objective of this note is to

* Professor, University of Florida. I would like to express my appreciation to Bipin Ajinkya and Robert Knechel for their comments and to Art Haller and R. D. Chen for computing assistance and collecting data. [Accepted for publication July 1982.]
show that measurement and estimation problems resulted in overfits of models in both papers. While the resulting $R^2$ exaggerated the relevance of the signals produced by quarterly reports vis-à-vis other sources of information in revising annual earnings forecasts, they remained statistically significant.

**The Source of Bias**

When two random variables $X$ and $Y$ are combined to produce a third variable $Z$ such that $Z = X + Y$, it can be shown that the explanatory power of the regression $Y = \alpha + \beta X$ is different from that of the regression $Z = a + bX$, perhaps in a significant way. In particular:

\[
R^2_{zz} = b^2\frac{\text{Var}(X)}{\text{Var}(Z)}
= (1 + \beta)^2\frac{\text{Var}(X)}{\text{Var}(Z)}
= \text{Var}(X) + 2\beta\text{Var}(X) + \beta^2\text{Var}(X)/\text{Var}(Z).
\]

Since the $\text{Var}(Z) = \text{Var}(X) + \text{Var}(Y) + 2\text{Cov}(X, Y)$, by substituting and rearranging we obtain:

\[
R^2_{zz} = \left[\frac{\text{Var}(X) + 2\text{Cov}(X, Y) + \beta^2\text{Var}(X)}{\text{Var}(X) + 2\text{Cov}(X, Y) + \text{Var}(Y)}\right]
= \left[\frac{C + \beta^2\text{Var}(X)}{C + \text{Var}(Y)}\right]
\]

where:

\[C = \text{Var}(X) + 2\text{Cov}(X, Y).\]

$C$ will always be positive if $\text{Cov}(X, Y)$ is positive, or if the $\text{Var}(X)$ is greater than a negative $\text{Cov}(X, Y)$. Thus:

\[R^2_{zz} > R^2_{zx}.\]

Both the 1978 and 1980 studies mentioned above fell victim to the bias stated above. For the problem at hand, consider the following notations:

- $A_Q_{iy}$ = actual earnings for the $i$th quarter of the $y$th year.
- $FQ^*_y$ = forecasted earnings $n$ periods earlier for the $i$th quarter of the $y$th year.
- $D_{iy}$ = quarterly prediction error and is equal to $A_Q_{iy}$ minus $FQ^*_y$ defined above.
- $NR_{iy}$ = revisions of predicted annual earnings of the $y$th year conditional on the realization of quarter $i$.
- $UR_{iy}$ = the revision of forecasts in the unrealized portion of annual earnings.

Thus:

\[NR_{iy} = UR_{iy} + D_{iy}.\]

Or by analogy to the general case, $Z = Y + X$. 
Table 1

Estimating the Two Models to Evaluate the Extent of the Bias*

<table>
<thead>
<tr>
<th></th>
<th>Model 2 ( NR_{ty} = a + bD_{ty} )</th>
<th>Model 1** ( UR_{ty} = a + \beta D_{ty} )</th>
<th>Extent of Overfitting</th>
</tr>
</thead>
<tbody>
<tr>
<td>Signal of Statistics</td>
<td>Constant, ( D_{ty} ), ( R^2 )</td>
<td>Constant, ( D_{ty} ), ( R^2 ), ( \Delta R^2 )</td>
<td>( R^2 ) (M2) ( R^2 ) (M1)</td>
</tr>
<tr>
<td>Quarter 1</td>
<td>Est. Parameter (-0.079), ( 1.66 ), ( 0.55 ) ( t)-test from zero (2.55), ( 16.33 )</td>
<td>(-0.079), ( 0.66 ), ( 0.16 ) ( t)-test from one (6.49)</td>
<td>( 0.39 ), ( 3.44 )</td>
</tr>
<tr>
<td>Quarter 2</td>
<td>Est. Parameter (-0.05), ( 1.43 ), ( 0.57 ) ( t)-test from zero (2.26), ( 17.1 )</td>
<td>(-0.05), ( 0.43 ), ( 0.11 ) ( t)-test from one (5.11)</td>
<td>( 0.46 ), ( 5.18 )</td>
</tr>
<tr>
<td>Quarter 3</td>
<td>Est. Parameter (-0.07), ( 1.43 ), ( 0.74 ) ( t)-test from zero (4.46), ( 24.2 )</td>
<td>(-0.07), ( 0.43 ), ( 0.21 ) ( t)-test from one (7.33)</td>
<td>( 0.53 ), ( 3.52 )</td>
</tr>
</tbody>
</table>

NA = Not applicable.
* Number of observations are equal to or greater than 205, depending on the number of missing observations.

All coefficients of \( D_{ty} \) are statistically significant at a level below 0.01.
** The reader is invited to verify the results by using the variances and covariances and the coefficients pertaining to Model 1, which are as follows:

<table>
<thead>
<tr>
<th></th>
<th>Var ( (UR_{ty}) )</th>
<th>Var ( (D_{ty}) )</th>
<th>Cov ( (UR_{ty}, D_{ty}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quarter 1</td>
<td>0.247</td>
<td>0.092</td>
<td>0.0609</td>
</tr>
<tr>
<td>Quarter 2</td>
<td>0.119</td>
<td>0.0705</td>
<td>0.03</td>
</tr>
<tr>
<td>Quarter 3</td>
<td>0.061</td>
<td>0.0675</td>
<td>0.0294</td>
</tr>
</tbody>
</table>

Using these notations, Brown et al. [1980, pp. 282-85] regressed \( NR_{ty} \) against \( g(D_{ty}) \), rather than regressing \( UR_{ty} \) against \( D_{ty} \). Since \( NR_{ty} \) includes both \( UR_{ty} \) and \( D_{ty} \), a bias exists in the results.

**Empirical Evaluation**

A sample of 76 companies was chosen from among 11 industries from the Value Line Investment Survey (the same source used by prior studies). Forecasted and actual data for both quarterly and annual reports were collected from the period 1975-77. The two models fitted were of the form:

\[
UR_{ty} = \alpha + \beta D_{ty}, \quad \text{(Model 1)}
\]

\[
NR_{ty} = \alpha + bD_{ty}, \quad \text{(Model 2)}
\]

Several estimations of those two models were carried out. Due to the consistency of results, only the estimates of the combined (over the period 1975-77) data for each of the first three quarters are reported in table 1. As shown: (1) the coefficients of Model 1 are smaller than those of Model 2 exactly by one; (2) the intercepts are unchanged; (3) the statistical significance of the coefficients is relatively weakened as we...
move from Model 2 to Model 1, but continues to be significant at below the 0.01 level; (4) $R^2$ was about four times higher for Model 2 than for Model 1.

**Conclusion**

This note is a technical extension to the discussion that took place in this *Journal* concerning the importance of the quarterly signals in revising analysts' annual earnings forecasts. It has been shown that the models used for that purpose were misspecified and were overfitted. Although the results still show that the signals produced by quarterly reports are significantly related to revisions of the unrealized portion of annual earnings, they also indicate that the relative importance of those signals is about one-fourth of what it was purported to be. Thus, much of the revisions of annual earnings appears to be due to information obtained from other sources.

**REFERENCES**


