Do Joint Audits Improve or Impair Audit Quality?

Mingcherng Deng
University of Minnesota

Tong Lu
University of Houston

Dan A. Simunic
University of British Columbia

Minlei Ye
University of Toronto

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ABSTRACT

Joint audits involve two audit firms and may have the desired effect of “Two heads are better than one.” However, joint audits may also induce one firm to free-ride on another firm’s performance and therefore damage the total precision of audit evidence. In addition, since joint audits involve two firms, it may be more expensive for the audited company to “bribe” its auditors. However, adding another audit firm may create an “opinion shopping” opportunity for the company, thereby threatening auditor independence. So the possible consequences of joint audits are quite complex, and we incorporate these various trade-offs in our analysis.

We investigate three regimes: single audits by a big firm; joint audits by two big firms; joint audits by one big firm and one small firm. We compare the three regimes along three dimensions: total audit evidence precision; auditor independence; and total audit fee. Our analysis suggests that auditor independence is more likely to be compromised under joint audits. Moreover, joint audits involving a technologically inefficient firm (a small firm) may impair audit quality since a free-riding problem would prevail and result in lower total audit evidence precision. Finally, audit fees under joint audits are lower than under single audits when the technological difference between the two audit firms is small and/or the big firm bears a large proportion of misstatement cost.

By providing the first theoretical study of joint audits, we advance understanding of audit quality. We also generate a new set of empirically testable predictions and explain current mixed empirical findings on joint audits.

Keywords: joint audit, audit quality, precision, auditor independence, audit fee.
1 Introduction

Are two heads better than one? The answer to this question seems straightforward and it is exactly one of the reasons cited in support of joint audits, in which two audit firms simultaneously and separately audit a company and jointly sign an audit report. Proponents of joint audits argue that two pieces of audit evidence produce higher total information precision than just a single piece. For example, in auditing a company’s fair value estimate of a certain asset, auditors are weakly better off with another piece of audit evidence about the fair value, because they always have the option of ignoring that piece of evidence if it is completely uninformative.

Another benefit of joint audits may be to enhance auditor independence. The conventional wisdom suggests that it is more expensive for a company to “bribe” two audit firms in joint audits than a single firm in single audits. The reason is that, under joint audits, the audit report must be co-signed by both firms and if one of the two refuses to sign, the audit report cannot be released. This makes compromising auditor independence much more difficult because the company has to pay a sufficiently large bribe to satisfy both audit firms.

Joint audits are not uncommon. For example, France has mandated by law joint audits of public companies since 1966. The same thing is true for the financial services sector in South Africa. Various countries, such as India, Germany, Switzerland, and the U.K., have proposed voluntary joint audits. In 2010, the European Commission seriously considered mandating joint audits, and continues to debate the issue at this time (EC (2010)).

If two heads were indeed better than one, and if two firms were more expensive to buy off than one, then joint audits probably would be more prevalent in the world. However, single audits are still the norm in significant portions of the world, with the U.S. being a notable example. An interesting about-face change occurred in Denmark, which mandated joint audits in 1930 but abolished this requirement in 2005.
Besides the benefits of joint audits, we identify two economic forces against joint audits that are absent in the extant literature. The first one is free-riding. In joint audits, one of the audit firms may save its auditing costs by investing less in its audit work and taking advantage of the other audit firm’s hard work. In equilibrium, the precision of the audit evidence is lower than that in the first-best world. The second one is auditor independence. When a client intends to compromise audit evidence, joint audits provide an opportunity of internal opinion shopping, because having more audit firms on board is like having more draws in a lottery. Thus, joint audits can endanger auditor independence.

In theory, joint audits entail significant trade-offs in both audit evidence precision and auditor independence, and in practice, the audit arrangements are diverse. Thus it is unclear ex ante under what conditions joint audits dominate single audits and under what conditions the converse is true.

We investigate the interactions among audit evidence precision, auditor independence, and audit fee in three regimes: single audits by a big firm (Regime $B$); joint audits by two big firms (Regime $BB$); joint audits by one big firm and one small firm (Regime $BS$). While the first two regimes serve as benchmarks, the third regime is particularly interesting because it is the European Commission’s target regime. Specifically, the European Commission states that, Regime $BS$ “could act as a catalyst for dynamising the audit market and allowing small and medium-sized firms to participate more substantially in the segment of large audits” (EC (2010), p.17). We make two assumptions to capture the differences between a big audit firm and a small one. First, a big audit firm has an advantage in its auditing technology in the sense that it has a lower marginal cost of audit evidence precision than a small firm. Second, a big audit firm bears a larger proportion of misstatement cost (such as litigation risk, reputation loss, etc.) than a small firm.

Comparing Regime $BB$ with Regime $B$, we find the joint audit generates the same
audit evidence precision as the single audit. Though it costs more to compromise auditor independence under the joint audit, the *ex ante* likelihood of non-independence is higher under the joint audit. Audit fees are lower under the joint audit.

Regarding the differences between a single audit regime $B$ and a joint audit regime $BS$, we derive the same result regarding auditor independence as that in Regime $BB$: auditor independence is more likely to be compromised, though it costs more to compromise independence. Additionally, we find the total precision of audit evidence under joint audits is lower than that under single audits. Furthermore, the audit fee under joint audit is less than that under single audit if the big audit firm has a sufficiently small technological advantage over the small firm and/or bears a large proportion of misstatement cost.

Our research contributes to several streams of literature. First, to our knowledge, there has been no previous theoretical study of joint audits. Joint audits provide a unique setting for analyzing both audit evidence precision and auditor independence, two components of audit quality.

Second, we extend the theoretical literature on audit quality by introducing two new strategic interactions into an auditing game. One is the company’s strategic shopping of audit opinions between its two joint auditors. The other is the joint auditors’ strategic free-riding incentives between each other. Previous research analyzed different factors that may impair auditor independence (e.g., DeAngelo 1981a, Antle 1984, Simunic 1984, Magee and Tseng 1990, Kanodia and Mukherji 1994, Lu 2006) or may influence audit information precision/audit effort (Dye 1993, Pae and Yoo 2001, Schwartz 1997, Zhang 2007). These papers focus on single audits. Our paper enriches the literature by studying joint audits and thus identifying additional strategic factors.

Third, the existing empirical research provides mixed evidence on the impact of joint audits on audit quality and audit fees (e.g., Francis et al. 2009, Gonthier-Besacier and Schatt

Fourth, this study provides timely policy implications for regulators. To encourage the growth of small-sized audit practices, the European Commission is considering mandating large companies to hire at least one audit firm outside the Big-Four firms to conduct joint audits (EC 2010). Our analysis suggests mandating joint audits with small audit firms for the purpose of reducing market concentration could lead to detrimental effects on audit quality. In light of the global convergence of accounting and auditing standards, this paper can help inform regulators’ deliberations on joint audits.

The remainder of the paper proceeds as follows. Section 2 provides institutional background on how joint audits are conducted in practice. Section 3 presents the structure and ingredients of the model under the three regimes, $B$, $BB$, and $BS$. Section 4 establishes the equilibrium audit quality and audit fees in these regimes. Section 5 compares those regimes. Section 6 develops the empirical predictions. We conclude in Section 7. We relegate all proofs to the Appendix.

2 Institutional Background

This section explains the current joint audit practice in France to provide a foundation for our model assumptions. Any listed company, any bank or other financial institution, and any company that prepares consolidated financial statements is required by law in France to appoint two different audit firms, who share the audit work and jointly sign the audit report. The law has evolved into a professional standard of practice requiring a balanced

\footnote{This description is based on an interview of an audit firm senior partner conducted in Paris on 12/13/2011 by one of the authors.}
division of the work of both auditors in order to ensure an efficient dual control mechanism (Gonthier-Besacier and Schatt 2007). However, in practice, auditors cannot always balance their work allocation. For example, if both Big 4 auditors conduct joint audits for a large listed company, then the workload sharing is likely to be balanced. But if one Big 4 firm and one small audit firm conduct joint audits, it is harder to share the workload equally, because the small audit firm cannot completely cover the client’s businesses (for instance, if the small firm has no audit network abroad).

After accepting an audit engagement, the two audit firms first agree on their work allocation. Their work is usually allocated either by regions (e.g., one audits America and another audits Europe) or by divisions (business units). Then they start to audit the financial statements simultaneously; they typically work this way because auditors need to meet a deadline of finishing audit reports. After finishing their part of the audit, they review each other’s audit work and prepare relevant documentation on the joint auditor’s working paper review.

At the end of the audit, each joint auditor signs the audit report on the whole financial statements, not just on the work he has done. However, courts evaluate each audit firm’s responsibility based on auditing standards. Fault, if found, may not be at the same level for each audit firm. For example, if the audited inventory is materially misstated, then the auditor who is responsible for inventory could be held more responsible than the other auditor, who simply reviewed the work.

These basic features of the French joint audit institutional regime—independent collection of audit evidence by the two audit firms with a review of each other’s work, joint agreement on the report to be issued, and separate and proportionate liability for undetected material misstatements—are consistent with the assumptions of our model in the next section.
3 Model

This section sets up the structure and ingredients of the model under three regimes:
single audits by a big firm (Regime B); joint audits by two big firms (Regime BB); joint
audits by one big firm and one small firm (Regime BS). In what follows, we first set up
the model for Regime B and then articulate how Regime BB and Regime BS deviate from
Regime B.

Regime B

Let $\tilde{x}$ denote the fundamental value of a company,\(^2\) which is distributed normally with
mean $x_0$ and precision $h$:

$$\tilde{x} \sim N(x_0, \frac{1}{h}). \quad (1)$$

Since our focus is on both dimensions of audit quality, the precision of audit evidence and
auditor independence (DeAngelo (1981a)), we model both the audit evidence accumulation
process and the subsequent company-auditor negotiation.

The auditing technology or the audit evidence accumulation process produces audit
evidence $y_B$ about $x$:

$$\tilde{y}_B|x \sim N(x, \frac{1}{e_B}), \quad (2)$$

that is, conditional on $x$, $\tilde{y}_B$ is distributed normally with mean $x$ and precision $e_B$. In other
words, the audit evidence is an unbiased but noisy estimate of $x$.

An increase in the quantity of resources utilized by the auditor can reduce the noisiness
of or enhance the precision of audit evidence. To capture this effect, we assume that the
audit resource cost is $k_B C(e)$, where $k_B > 0$ is a parameter and the precision $e$ is a choice

\(^2\)As a general rule in this paper, a symbol with a “$\sim$” indicates a random variable and the same symbol
without a “$\sim$” indicates the realized value of that random variable. For example, $x$ is a realized value of the
random variable $\tilde{x}$. 

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variable. We make the following standard assumptions about the cost function: $C(0) = 0$, $C' > 0$ (but $C'(0) = 0$), $C'' > 0$, and $C''' = 0$. A quadratic function $C(e) = e^2$, commonly used in the literature, satisfies all of the above assumptions (Chan and Pae 1998, Laux and Newman 2010, etc.).

To model the company-auditor negotiation, we introduce a pair of $(Q, r)$ representing the give-and-take between the company and its auditor. Specifically, a company may offer or “bribe” its auditors an amount of $Q$ in return for a certain report $r$ the company prefers. Following DeAngelo (1981a), we can also call $Q$ “quasi-rent.”

An independent report, $r_I \equiv \mathbb{E}[\tilde{x}|y_B]$, is the auditor’s best estimate of $x$ conditional on the audit evidence $y_B$. Note that $r_I$ is both informed and unbiased. However, if auditor independence is compromised, the released report $r$ will exceed $r_I$. We assume that, when auditor independence is compromised (when $r > r_I$), $r$ cannot exceed the observed audit evidence. This can be justified by noting that the audit evidence documented in the audit firms’ working papers is the only admissible evidence in court (Dye and Sridhar 2004).

Following the standard assumption in the literature (e.g., Antle and Nalebuff 1991, Dye and Sridhar 2004), we assume that the audit firm will bear a loss of $(r-x)^2$ if a misstatement occurs, that is, if the certified report $r$ is different from the company’s fundamental value $x$. This misstatement cost includes possible legal liability and reputation loss resulting from an audit failure.3

The sequence of events is as follows:

- The company offers a total audit fee of $F$ and hires a big audit firm in a competitive audit market. The company proposes an unaudited report $r_0$ to its auditor.

3To focus on auditing issues, we abstract away from companies’ misstatement cost. If instead the companies’ legal liability and/or reputation loss due to misstatement is sufficiently high, the company may not propose an inflated report in the first place, thereby making auditing a moot issue.
• The auditor chooses her desired precision $e^B$ of audit evidence in her audit plan.

• The audit evidence accumulation process produces audit evidence $y_B$.

• The company-auditor negotiation determines a pair of $(Q, r)$, where a company offers or “bribes” its auditor an amount of $Q$ in return for a certain audit report $r$.

• The company’s fundamental value $x$ is realized and the auditor bears a misstatement cost of $(r - x)^2$.

The company’s payoff is its share price (which is assumed to be increasing in its audited financial report $r$) net of its audit fee $F$ and its “bribe” or quasi-rent $Q$ paid to the auditor. The auditor’s payoff is the receipt of $F$ and $Q$ net of the audit resource cost of $k_B C(e^B)$ and the misstatement cost of $(r - x)^2$.

**Regime BB and Regime BS**

Having laid out the model setup for Regime $B$, we now turn to Regimes $BB$ and $BS$. The three regimes differ in the following ways:

• Regime $B$: The big audit firm has an audit resource cost function of $k_B C(e)$ and must bear 100% of the misstatement cost.

• Regime $BB$: Each of the two big audit firms has an identical cost function of $k_B C(e)$ and each must bear 50% of the misstatement cost.\(^4\) Additionally, we denote the audit evidence accumulated by the two big firms $y_B$ and $y_{B2}$, respectively.

• Regime $BS$: The big audit firm has a cost function of $k_B C(e)$ and the small audit firm has a cost function of $k_S C(e)$, where $\frac{k_S}{k_B} \equiv m > 1$ so that the big firm is more cost

\(^4\)Alternatively, we assume one auditor shares $\alpha_1 \in (0, 1)$ proportion of the misstatement cost and the other auditor shares $\alpha_2 \in (0, 1)$, where $\alpha_1 + \alpha_2 = 1$. We derive qualitative similar results, which are available upon request.
efficient than the small firm. Furthermore, the big audit firm will bear a proportion of $\alpha_B$ and the small firm will bear a proportion of $\alpha_S = 1 - \alpha_B$ of the total misstatement cost. We assume that the big audit firm bears a larger misstatement cost than the small audit firm, i.e., $\alpha_B > \alpha_S$, consistent with DeAngelo (1981b). The assumptions implies that $\alpha_B \in (\frac{1}{2}, 1)$ and $\alpha_S \in (0, \frac{1}{2})$. Additionally, we denote the audit evidence accumulated by the big firm $y_B$ and that by the small firm $y_S$.

4 Equilibrium

Using backward induction, we analyze first the “bribe” or quasi-rent $Q$ and the certified report $r$, then the precision $e$ of the audit evidence, and finally the audit fee $F$. In this section, we lay out the equilibrium analyses for the three regimes $B$, $BB$, and $BS$. Section 5 compares the three regimes and gives a full explanation of the main results.

Before we begin, a discussion of the company’s initial unaudited report $r_0$ is in order. In our model, the company has an incentive to conduct earnings management to boost its report and therefore initially it will propose a high unaudited report $r_0$ to its auditors. Because it is in the best interest of the company to propose a $r_0$ as high as possible regardless of its private information (if any), $r_0$ is uninformative and so is ignored by the auditors. This is true in our model whether or not the company has hidden information or not. Knowing that, the auditors use their own informative evidence to estimate the company value. If instead a company with hidden information could signal it by its real actions or by its audit fee, then auditing would be moot. In a nutshell, we assume away the issue of the company’s signaling in order to focus on the issue of external auditing, consistent with the literature (e.g., Antle and Nalebuff (1991)).

\[^5\text{In practice, because of tight deadlines for the release of audited financial statements, joint audit firms must work simultaneously rather than sequentially. Therefore, we model the joint audit firms’ evidence accumulation processes as simultaneous moves.}\]
4.1 Auditor Independence

In Regime B, at the stage of company-auditor negotiation about the final report to be released, the audit evidence $y_B$ is already accumulated and documented in the auditor’s working papers. The company may offer or “bribe” its auditor an amount of $Q$ in return for a certain audit report $r$ which the company prefers. The pair of $(Q,r)$ represents the give-and-take between the company and its auditor.

The auditor weighs the amount of the quasi-rent $Q$ she expects from the company against her expectation of cost of misstatement $(r-x)^2$. Because the auditor has already accumulated audit evidence, her expectation is conditional on audit evidence, that is, $E[(r-x)^2|y_B]$. Recall that the auditor always has an option of insisting on the independent report, $r_I \equiv E[\bar{x}|y_B]$. If she chooses to certify $r_I$, her expectation of the misstatement cost will be $E[(r_I-x)^2|y_B]$. If she chooses to certify an inflated report $r > r_I$, her expectation of the misstatement cost will increase to $E[(r-x)^2|y_B]$. Naturally, to overcome the auditor’s objection to an inflated report $r > r_I$, the company must offer a $Q$ large enough to cover the increase in the misstatement cost, $E[(r-x)^2|y_B] - E[(r_I-x)^2|y_B]$. In equilibrium, the company has no incentive to overpay the auditor, so it will set $Q$ equal to $E[(r-x)^2|y_B] - E[(r_I-x)^2|y_B]$.

In general, the company wants to induce its auditor to certify a report as high as possible. Because the audit evidence documented in the audit firm’s working papers is the only admissible evidence in court, the audit report $r$ cannot exceed the observed audit evidence. Thus, if the audit evidence exceeds the independent report, (that is, $y_B > r_I$), the company prefers to induce an audit report $r = y_B$. This suggests that auditor non-independence occurs when audit evidence exceeds the independent report, that is, $y_B > r_I$. Therefore, the probability of auditor independence $\Pr(AI)$ is the probability that the

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6We are interested in the probability of auditor independence when the budget constraint is not an issue to the company. Obviously, if the company were short of money to buy off its auditors in the first place, auditor independence would be a trivial issue.
independent report exceeds the audit evidence, i.e., $\Pr(r_I > y_B)$.

In Regime $BB$, besides $y_B$, the joint audit generates one additional evidence by the second big audit firm, denoted by $y_{B2}$. Analogously, the company will set $Q$ equal to $\mathbb{E}[(r - x)^2|y_B, y_{B2}] - \mathbb{E}[(r_I - x)^2|y_B, y_{B2}]$ to induce a report $r = \max\{y_B, y_{B2}\}$ if $\max\{y_B, y_{B2}\} > r_I$. Regime $BS$ is similar to Regime $BB$ except that the second evidence is $y_S$, obtained by the small firm, rather than $y_{B2}$.

**Proposition 1. [Auditor Independence]**

(i) Under Regime $B$, the negotiated pair of report $r$ and quasi-rent $Q$ is as follows:

If $y_B > r_I$, $r = y_B$ and $Q = \left(\frac{h(y_B - x_0)}{h + e_B}\right)^2$; Otherwise, $r = r_I$ and $Q = 0$.

The probability of auditor independence is

$\Pr(AI) = \Pr(y_B < x_0)$.

(ii) Under Regime $BB$, the negotiated pair of report $r$ and quasi-rent $Q$ is as follows:

If $y_B > y_{B2}$ and $y_B > r_I$, $r = y_B$ and $Q = \left(\frac{h(y_B - x_0) + e_{B2}(y_B - y_{B2})}{h + e_B + e_{B2}}\right)^2$;

If $y_{B2} > y_B$ and $y_{B2} > r_I$, $r = y_{B2}$ and $Q = \left(\frac{h(y_{B2} - x_0) + e_B(y_{B2} - y_B)}{h + e_B + e_{B2}}\right)^2$;

Otherwise, $r = r_I$ and $Q = 0$.

The probability of auditor independence is

$\Pr(AI) = \Pr(\frac{h + e_B)y_{B2} - h x_0}{e_B} < y_B < \frac{h x_0 + e_{B2} y_{B2}}{h + e_{B2}})$.

(iii) Under Regime $BS$, the negotiated pair of report $r$ and quasi-rent $Q$ is as follows:

If $y_B > y_S$ and $y_B > r_I$, $r = y_B$ and $Q = \left(\frac{h(y_B - x_0) + e_S(y_B - y_S)}{h + e_B + e_S}\right)^2$;

If $y_S > y_B$ and $y_S > r_I$, $r = y_S$ and $Q = \left(\frac{h(y_S - x_0) + e_B(y_S - y_B)}{h + e_B + e_S}\right)^2$;

Otherwise, $r = r_I$ and $Q = 0$. 

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The probability of auditor independence is
\[ \Pr(AI) = \Pr\left( \frac{(h + e^B) y_S - hx_0}{e^B} < y_B < \frac{hx_0 + e^S y_S}{h + e^S} \right). \]

### 4.2 Audit Evidence Precision

Under Regime \( B \), when the auditor plans the audit, she chooses her desired level of audit evidence precision. The benefit of higher precision is to obtain more precise audit evidence about the fundamental value of the company and thus to reduce the possibility of costly misstatement, \( \mathbb{E}[(r - \bar{x})^2|y_B] \). Higher precision requires a larger audit resource cost, \( k C(e) \). This benefit-cost trade-off determines the auditor’s optimal precision decision. Note from the preceding subsection that the company offers \( Q \) to induce the auditor to deviate from certifying the independent report \( r_I \). In other words, \( Q = \mathbb{E}[(r - \bar{x})^2|y_B] - \mathbb{E}[(r_I - \bar{x})^2|y_B] \). Therefore, the auditor’s remaining expected cost of misstatement is only \( \mathbb{E}[(r_I - \bar{x})^2|y_B] \). Formally, the big audit firm’s optimization program is

\[
\begin{align*}
\min_{e^B} & \quad \underbrace{k_B C(e^B)}_{\text{resource cost}} + \underbrace{\mathbb{E}[\mathbb{E}[(r_I - \bar{x})^2|y_B]|e^B]}_{\text{expected misstatement cost}}.
\end{align*}
\]

Under Regime \( BB \), the second big auditor obtains \( y_{B2} \). Hence, the two auditors’ expected cost of misstatement is \( \mathbb{E}[(r_I - \bar{x})^2|y_B, y_{B2}] \). Each auditor shares this cost equally, because they are economically and technologically identical. The auditors’ optimization problem is

\[
\begin{align*}
\min_{e^B} & \quad k_B C(e^B) + 0.5 \mathbb{E}[\mathbb{E}[(r_I - \bar{x})^2|y_B, y_{B2}]|e^B, e^{B2}]. \\
\min_{e^{B2}} & \quad k_B C(e^{B2}) + 0.5 \mathbb{E}[\mathbb{E}[(r_I - \bar{x})^2|y_B, y_{B2}]|e^B, e^{B2}].
\end{align*}
\]

Under Regime \( BS \), since the small audit firm receives \( y_S \), the two audit firms’ total expected cost of misstatement is \( \mathbb{E}[(r_I - \bar{x})^2|y_B, y_S] \). The big audit firm bears a proportion \( \alpha_B \) of the
misstatement cost whereas the small audit firm bears the remaining proportion of $\alpha_S$ where $\alpha_B > \alpha_S$. The big audit firm’s optimization program is

\[
\min_{e^B} k_B C(e^B) + \alpha_B \mathbb{E}[\mathbb{E}[(r_I - \bar{x})^2|y_B, y_S]|e^B, e^S].
\] (6)

Analogously, the small audit firm’s optimization program is

\[
\min_{e^S} k_S C(e^S) + \alpha_S \mathbb{E}[\mathbb{E}[(r_I - \bar{x})^2|y_B, y_S]|e^B, e^S].
\] (7)

**Proposition 2. [Audit Evidence Precision]**

(i) Under Regime $B$, the big audit firm’s optimal choice of evidence precision $e^B_B$ is determined by the following equation:

\[
k_B C'(e^B_B) - \frac{1}{(h + e^B_B)^2} = 0.
\] (8)

(ii) Under Regime $BB$, each of the big audit firm’s optimal choice of evidence precision $e^B_BB$ is determined by the following equation:

\[
k_B C'(e^B_BB) - \frac{1/2}{(h + 2e^B_BB)^2} = 0.
\]

(iii) Under Regime $BS$, the big audit firm’s optimal choice of evidence precision $e^B_BS$ and the small audit firm’s optimal choice of evidence precision $e^S_BS$ are jointly determined by the following pair of equations:

\[
k_B C'(e^B_BS) - \frac{\alpha_B}{(h + e^B_BS + e^S_BS)^2} = 0;
\]

\[
k_S C'(e^S_BS) - \frac{\alpha_S}{(h + e^B_BS + e^S_BS)^2} = 0.
\] (9)

### 4.3 Audit Fee

In a competitive audit market, the audit fee $F$ covers both the auditors’ resource cost and their expected cost of misstatement. Therefore, summing them up yields the total audit
fees in each regime:

\[ F = k_B C(e^B) + \mathbb{E}[\mathbb{E}[(r_I - \bar{x})^2|y_B]|e^B]. \]  

(10)

\[ F = k_B C(e^B) + k_B C(e^{B2}) + \mathbb{E}[\mathbb{E}[(r_I - \bar{x})^2|y_B,y_{B2}]|e^B,e^{B2}]. \]  

(11)

\[ F = k_B C(e^B) + k_S C(e^S) + \mathbb{E}[\mathbb{E}[(r_I - \bar{x})^2|y_B,y_S]|e_B,e_S]. \]  

(12)

**Proposition 3. [Audit Fee]**

(i) Under Regime \(B\), the equilibrium total audit fee \(F_B\) is as follows:

\[ F_B = k_B C(e^B_B) + \frac{1}{h + e^B_B}. \]  

(13)

(ii) Under Regime \(BB\), the equilibrium total audit fee \(F_{BB}\) is as follows:

\[ F_{BB} = 2k_B C(e^B_{BB}) + \frac{1}{h + 2e^B_{BB}}. \]  

(14)

(iii) Under Regime \(BS\), the equilibrium total audit fee \(F_{BS}\) is as follows:

\[ F_{BS} = k_B C(e^B_{BS}) + k_S C(e^S_{BS}) + \frac{1}{h + e^B_{BS} + e^S_{BS}}. \]  

(15)

5 Comparison

Having derived the results on auditor independence, audit evidence precision, and audit fee, now we are ready to compare Regimes \(B\), \(BB\), and \(BS\) along those three dimensions.

5.1 Audit Evidence Precision

To facilitate comparisons among Regimes \(B\), \(BB\), and \(BS\), we juxtapose the optimal choices of audit evidence precision given in Proposition 2 in the following table.
Table 1

<table>
<thead>
<tr>
<th>Regime</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>$k_B C'(e_B^B) - \frac{1}{(h+e_B^B)^2} = 0$</td>
</tr>
<tr>
<td>BB</td>
<td>$k_B C'(e_{BB}^B) - \frac{1/2}{(h+2e_{BB}^B)^2} = 0$</td>
</tr>
<tr>
<td></td>
<td>$k_B C'(e_{BB}^B) - \frac{1/2}{(h+2e_{BB}^B)^2} = 0$</td>
</tr>
<tr>
<td>BS</td>
<td>$k_B C'(e_{BS}^B) - \frac{\alpha_B}{(h+e_{BS}^B+e_{BS}^S)^2} = 0$</td>
</tr>
<tr>
<td></td>
<td>$k_S C'(e_{BS}^S) - \frac{\alpha_S}{(h+e_{BS}^B+e_{BS}^S)^2} = 0$</td>
</tr>
</tbody>
</table>

On appearance, it seems that the argument of “Two heads are better than one” makes sense. If the big audit firm’s evidence precision were fixed across the three regimes (that is, if $e_B^B = e_{BB}^B = e_{BS}^B$), then indeed the total evidence precision under joint audits would exceed that under single audits, that is, $e_{BS}^B + e_{BS}^S > e_B^B$ and $2e_{BB}^B > e_B^B$. However, it turns out that the big audit firm’s optimal choice of evidence precision is not fixed across the three regimes.

**Proposition 4.** The big audit firm’s optimal choice of audit evidence precision is lower under joint audits than under single audits: $e_{BS}^B < e_B^B$ and $e_{BB}^B < e_B^B$.

Contrary to the conventional wisdom, the big audit firm’s audit evidence precision is lowered under joint audits. This result is due to two factors: (i) the misstatement cost sharing with another auditor (the numerator $1/2$ or $\{\alpha_B, \alpha_S\}$ of the second term in the joint audit equations in Table 1), and (ii) free-riding (the additional term $e_{BB}^B$ or $e_{BS}^S$ in the denominator of the second term in the joint audit equations in Table 1). An audit firm may enjoy the benefits of reduction in audit risk brought about by her joint audit firm without exerting her proper share of effort, resulting in a free-riding problem. While the free-riding problem reduces an individual auditor’s audit evidence precision, the main focus should be on whether joint audits give rise to higher total audit evidence precision than single audits. The next proposition answers this question.

**Proposition 5.** [Comparison: Audit Evidence Precision]

(i) The total evidence precision in Regimes BB and B are the same: $2e_{BB}^B = e_B^B$. 

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(ii) The total evidence precision in Regime BS is less than that in Regimes B and BB: $e^B_{BS} + e^S_{BS} < e^B_B = 2e^B_{BB}$, and the big audit firm provides more effort than its share: $e^B_{BS} > \alpha e^B_B$.

Proposition 5(i) is straightforward because in Regime BB no technological differences between two big firms exist and both big firms bear an equal proportion of the misstatement costs. Therefore, each audit firm exerts one half of the effort that would have been provided by a single big firm audit, thereby leading to the same total evidence precision as in Regime B.

Proposition 5(ii) shows a dimmer picture in Regime BS, because technological differences between big and small firms exist. On one hand, since the marginal resource cost is very high for the small audit firm, it may save costs by choosing low precision, which decreases the total evidence precision. On the other hand, the big firm needs to provide more effort than its share to reduce the audit risk. Thus, the small auditor free-rides on the big auditor, because the former’s technology efficiency is lower. However, the increase in precision by the big firm is not large enough to compensate the decrease in precision by the small firm. This force causes a lower total audit evidence precision in Regime BS than in Regime B.

In summary, while the joint audit of BB provides the same information precision, the joint audit of BS impairs information precision due to free riding.

5.2 Auditor Independence

We use two measures for auditor independence: the quasi-rent required to induce the auditors to issue an inflated report, and the ex ante probability of releasing an independent report. Auditor independence may more likely to be maintained when the firm needs to pay a higher quasi-rent to “bribe” the auditors. However, the probability of releasing an
independent report is a more direct measure of auditor independence.

To facilitate comparisons among Regimes B, BB, and BS, we juxtapose the equilibrium quasi-rent and probability of auditor independence given in Proposition 1 in the following table. For the sake of comparison, we only present in Table 2 the quasi-rent offered to the big audit firm because big firms are present in all the three regimes and the small firm is not.

**Table 2**

<table>
<thead>
<tr>
<th>Regime</th>
<th>( Q )</th>
<th>( \Pr(\text{AI}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regime B</td>
<td>( Q_B = \left( \frac{h(y_B - x_0)}{h + e_B^2} \right)^2 )</td>
<td>( \Pr(\text{AI}_B) = \Pr(y_B &lt; x_0) )</td>
</tr>
<tr>
<td>Regime BB</td>
<td>( Q_{BB} = \left( \frac{h(y_B - x_0) + e_{BB}^2 (y_B - y_{BB})}{h + 2e_{BB}^2} \right)^2 )</td>
<td>( \Pr(\text{AI}<em>{BB}) = \Pr\left( \frac{(h + e</em>{BB}^2) y_{BB} - h x_0}{e_{BB}^2} &lt; y_B &lt; \frac{h x_0 + e_{BB}^2 y_{BB}}{h + e_{BB}^2} \right) )</td>
</tr>
<tr>
<td>Regime BS</td>
<td>( Q_{BS} = \left( \frac{h(y_B - x_0) + e_{BS}^2 (y_B - y_{BS})}{h + e_{BS}^2} \right)^2 )</td>
<td>( \Pr(\text{AI}<em>{BS}) = \Pr\left( \frac{(h + e</em>{BS}^2) y_{BS} - h x_0}{e_{BS}^2} &lt; y_B &lt; \frac{h x_0 + e_{BS}^2 y_{BS}}{h + e_{BS}^2} \right) )</td>
</tr>
</tbody>
</table>

Table 2 directly show that provided the same total audit evidence across regimes (\( e_{BS}^2 + 2e_{BB}^2 = e_{B}^2 \)), the quasi-rent required to bribe the auditors is higher under joint audits, because joint audits by definition involve more auditors than single audits. However, we know from Proposition 5 that the total audit evidence precision is not necessarily the same across the three regimes. So it is ex ante unclear under what conditions the quasi-rent needed to compromise auditor independence under joint audits exceeds that under single audits and under what conditions it does not. The next proposition answers this question.

**Proposition 6.** On average, the quasi-rent needed to compromise auditor independence is larger in Regime BB than in Regime B: \( \mathbb{E}[Q_{BB}] > \mathbb{E}[Q_B] \) and is larger in Regime BS than in Regime B: \( \mathbb{E}[Q_{BS}] > \mathbb{E}[Q_B] \).

Proposition 6 shows that the company must pay a higher quasi-rent under joint audits, because it must compensate both audit firms rather than one firm for the incremental cost of misstatement brought about by an inflated report. Specifically, as Regime BS provides
lower information precision than Regime $B$, the expected misstatement cost is higher in Regime $BS$. Thus, it costs more for the firm to induce the auditors to compromise their independence in joint audits.

Next we move on to investigate the probability of auditor independence across the three regimes.

**Proposition 7.** The likelihood of auditor independence under a joint audit is lower than that under single audit: $\Pr(AI_{BS}) < \Pr(AI_B)$ and $\Pr(AI_{BB}) < \Pr(AI_B)$.

The result of Proposition 7 may sound surprising. If it is more expensive to buy off two auditors, why would the probability of auditor independence be lower under joint audits? This is because joint audits provide companies an opportunity of internal opinion shopping between auditors for a favorable audit opinion. But such an opportunity does not exist under single audits.

### 5.3 Audit Fee

To facilitate comparisons among Regimes $B$, $BB$, and $BS$, we juxtapose the equilibrium audit fees given in Proposition 3 in the following table:

<table>
<thead>
<tr>
<th>Regime</th>
<th>Audit Fee Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B$</td>
<td>$F_B = k_B C(e_B^B) + \frac{1}{h + e_B^B}$</td>
</tr>
<tr>
<td>$BB$</td>
<td>$F_{BB} = 2k_B C(e_B^{BB}) + \frac{1}{h + 2e_B^{BB}}$</td>
</tr>
<tr>
<td>$BS$</td>
<td>$F_{BS} = k_B C(e_B^{BS}) + k_S C(e_S^{BS}) + \frac{1}{h + e_{BS}^B + e_{BS}^S}$</td>
</tr>
</tbody>
</table>

The audit fee covers both the audit resource cost and the expected cost of misstatement. Higher evidence precision necessarily requires a higher audit resource cost but decreases the likelihood of misstatement. Thus, an increase in precision increases one component of the audit fee but decreases the other component. Therefore, the audit fee is not monotonic in...
audit precision and it is not ex ante straightforward to rank the audit fees across the three regimes.

**Proposition 8.** [Comparison: Audit Fee]

(i) The audit fee in Regime BB is lower than that in Regime B: \( F_{BB} < F_B \).

(ii) The audit fee in Regime BS is lower than that in Regime B if and only if the big firm and small firm have similar technology efficiency and/or the big firm bears a sufficiently large proportion of misstatement cost: there exists a \( m^\dagger > 1 \) such that \( F_{BS} < F_B \) for \( m < m^\dagger \) and there exists an \( \frac{1}{2} < \alpha^\dagger < 1 \) such that \( F_{BS} < F_B \) for \( \alpha_B > \alpha^\dagger \).

Proposition 8(i) states that the total audit fee under joint audits by two big firms is lower than that under single audits by a big firm. This result is due to the convexity of the resource cost function, a standard assumption in economics. For example, one audit firm doing all the work under a completion time constraint (Regime B) may experience a higher level of staff supervision and coordination costs than if the work was split between firms (Regime BB).

Proposition 8(ii) ranks the audit fees across the alternative regimes in the dimension of the audit firms’ technological advantage and share of misstatement costs. Recall that the small audit firm suffers technological inefficiency in the sense that its marginal resource cost is larger than the big firm’s, that is, \( \frac{k_B}{k_S} \equiv m > 1 \), where \( m \) represent the extent of the small firm’s technological inefficiency. When the small audit firm’s marginal resource cost is higher (a larger \( m \)), its audit resource cost is higher. Moreover, the small audit firm chooses lower audit evidence precision, which subsequently leads to a higher likelihood of misstatement. As a result of these two factors, the total audit fee in Regime BS will be higher than its counterpart in Regime B when \( m \) is sufficiently large. By the same reasoning, the converse is true when \( m \) is sufficiently small, that is, the audit fee in Regime BS is lower than that in Regime B if the big firm and small firm have similar technology efficiency.
Second, when $\alpha_B$ is sufficiently large, the big firm has a large incentive to boost her precision because she bears a large proportion of misstatement cost. This implies a large total audit evidence precision and thus a small misstatement cost, which is reflected in the total audit fee. In brief, the total audit fee in Regime $BS$ is lower than that in Regime $B$ when the big firm bears a sufficiently large proportion of misstatement cost. Additionally, as $\alpha_B$ continues to increases, the increase of resource cost from higher precision will dominate the reduction of misstatement cost, causing the increase of audit fees. In the extreme case when $\alpha_B = 1$, the audit fee is identical under two regimes.

6 Empirical Prediction

Our model produces two sets of empirically testable predictions about joint audits on audit quality (Propositions 4 to 7) and audit fees (Proposition 8). In this section, we explain how our predictions make additional new testable hypotheses and how our predictions shed light on the extant empirical evidence.

Our model shows that audit quality (audit evidence precision and auditor independence) can be impaired or improved under various conditions. Indeed, Lesage et al. (2011) find no stable relationship between joint audits and abnormal accruals. Using theoretical analysis, we come up with the conditions under which audit quality is likely to be impaired and the conditions under which audit quality is likely to be improved.

Audit Quality.

Audit quality is determined by both audit evidence precision and auditor independence (DeAngelo 1981a). To our knowledge, there has not been any research directly examining the impact of joint audits on auditor independence. Propositions 6 and 7 together suggest that though it is more expensive to induce non-independence under joint audits than single
audits, if companies’ budgetary constraints are not binding, the *ex ante* probability of non-independence is higher under joint audits than under single audits. This is a new testable hypothesis we produce.

Our analysis on evidence precision generates the following predictions: (1) two audit firms whose technology efficiency are comparable can provide the same audit quality as a single firm audit (Proposition 5 (i)); (2) adding a firm with lower technology efficiency to form a joint audit will reduce audit quality (Proposition 5 (ii)). We find these predictions are supported by several empirical studies. For example, Holm and Thinggaard (2010) find that there is no difference in the auditors’ ability to constrain earnings management between joint and single audits in general. Using a sample of 177 French listed companies on December 31, 2003, Marmousez (2008) provides evidence that the presence of two Big 4 audit firms is associated with lower reporting quality. If one of the Big 4 firms is an industry specialist and the other is not, we can interpret the non-specialist firm as one that has lower technology efficiency, and therefore this empirical evidence is consistent with our prediction in Proposition 5(ii).

More generally, our analysis implies that adding a more efficient auditor can improve audit quality, which is supported by Francis et al. (2009). They examine auditor choice for listed companies in France. They find that companies using one Big 4 auditor paired with a non-Big 4 auditor have smaller income-increasing abnormal accruals compared to companies that use no Big 4 auditors and this effect is even stronger for companies that use two Big 4 auditors.

However, using a disclosure score for companies composing the French SBF 120 index from 2006 to 2009, Paugam and Casta (2012) provide evidence that the combination of Big 4/non-Big 4 auditors generate higher impairment-related disclosures levels than other combinations, i.e., two Big 4 or two non-Big 4. This result is inconsistent with Proposition
Therefore, we conjecture these empirical studies mainly capture the audit evidence precision aspect of audit quality. We suggest that care must be taken to separate empirically the effect of audit evidence precision and that of auditor independence.

**Audit Fee.** Proposition 8(i) proposes that the total audit fee under joint audits by two big firms is lower than that under single audits by one big firm. The empirical evidence is consistent with the direction of our prediction. Gonthier-Besacier and Schatt (2007) find that when two Big Four firms audit company accounts, the fees charged (adjusted for company size) are significantly lower in comparison with those paid in the other cases (BS or two small audit firms). Francis et al. (2009) find French audit fees are not higher under joint audits compared to other European countries that do not require joint audits.

However, Proposition 8 (ii) predicts that when a small firm is involved in joint audits, the comparison is not clear-cut. Indeed, Lesage et al. (2011) confirm the absence of any stable relationship between joint audit and audit fees. Thinggaard and Kiertzner (2008) examines audit fees paid by all 126 non-financial companies listed on the Copenhagen Stock Exchange in 2002. They find that joint audits reduce audit fees compared with audits where one auditor is dominant, albeit only for larger companies. However, the opposite evidence is documented in a different setting by Holm and Thinggaard (2010). They use the data for the whole population of non-financial Danish companies listed on the Copenhagen Stock Exchange in the five-year period surrounding the abolishment of joint audit in 2005. They find discounts (of around 25%) in audit fees in companies that change to single audits.

Moreover, we discover that when joint auditors bear the same proportion of misstatement cost, the total audit fee under Regime BS exceeds that under Regime BB because of small auditors’ technology inefficiency (see the proof of Proposition 8). Consistent with this prediction, Audousset-Coulier (2012) shows the joint audits by two big auditors do not
require a fee premium compared to joint audits by one big firm and one small firm.

7 Conclusion

To restore trust in financial reporting, in the wake of the recent financial crisis, the European Commission, among others, is re-examining the role of auditing. One of its proposed actions is to mandate joint audits. However, it is \textit{ex-ante} unclear whether joint audits are more beneficial than single audits. Though two heads may be better than one, free-riding can reduce the information precision produced by audits. Regarding auditor independence, it may be more expensive to buy off two parties than one, but adding another audit firm allows the client the opportunity of shopping for a more favorable opinion.

We develop a theory of joint audits that incorporates these trade-offs and solve for the equilibrium solution. We compare joint audits by two big firms and joint audits by one big firm and one small firm with single audits by a big firm. Three dimensions are examined: audit evidence precision, auditor independence, and audit fees.

We find that the benefits of joint audit do not always dominate its costs. Besides the out-of-pocket audit resource costs, we need to consider the more significant indirect costs of joint audits: free-riding, which may decrease the precision of the audit evidence; internal opinion shopping, which may compromise auditor independence. Though it is more expensive to compromise auditor independence under joint audits, joint audits provide an additional opportunity for a company to shop for a better audit opinion. If companies’ budgets are not binding (i.e., they can afford the “bribe” needed to get the report they want), auditor independence is more likely to be compromised under joint audits.

Therefore, the answer to the question “Do joint audits improve or impair audit quality?” is “It depends.” In this paper, we identify the conditions under which joint audits improve
audit quality and the conditions under which they impair audit quality. Our research extends the theoretical literature on audit quality and provides timely policy implications to regulators. Our model provides a theoretical framework that can explain the seemingly inconsistent and diverse empirical findings to date, and the propositions we develop provide further predictions that can potentially be tested empirically.
Appendix

Proof of Proposition 1

(i) Regime $B$.

For later references, first recall that $r_I \equiv \mathbb{E}[\tilde{x}|y_B]$. Because both $\tilde{x}$ and $\tilde{y}$ follow the normal distribution specified in (1) and (2) respectively, applying the standard formula of the conditional mean for the multivariate normal distribution yields

$$r_I = \frac{hx_0 + e^T y_B}{h + e}.$$ 

The auditor’s expectation of the misstatement cost given a report $r$ and the audit evidence $y_B$ is

$$\mathbb{E}[(r - \tilde{x})^2|y_B] = \mathbb{E}[r^2 - 2r\tilde{x} + \tilde{x}^2|y_B]$$

$$= r^2 - 2r\mathbb{E}[\tilde{x}|y_B] + \mathbb{E}[\tilde{x}^2|y_B]$$

$$= r^2 - 2r\mathbb{E}[\tilde{x}|y_B] + (\mathbb{E}[\tilde{x}|y_B])^2 + \text{Var}[\tilde{x}|y_B]$$

$$= (r - \mathbb{E}[\tilde{x}|y_B])^2 + \text{Var}[\tilde{x}|y_B]$$

$$= (r - r_I)^2 + \text{Var}[\tilde{x}|y_B]$$

$$= (r - r_I)^2 + \frac{1}{h + e}.$$ 

If an independent report $r_I$ is issued, then the auditor’s expectation of her misstatement cost is

$$\mathbb{E}[(r_I - \tilde{x})^2|y_B] = (r_I - r_I)^2 + \frac{1}{h + e} = \frac{1}{h + e}. \quad (16)$$

Therefore, to induce the auditor to certify $r$ instead of $r_I$, the company must offer the auditor an amount of $Q$ to compensate her for the increase in the misstatement cost:

$$Q = \mathbb{E}[(r - \tilde{x})^2|y_B] - \mathbb{E}[(r_I - \tilde{x})^2|y_B]$$

$$= (r - r_I)^2. \quad (17)$$

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In the text, we reasoned that the company prefers to induce a report such that \( r = y_B \) if \( y_B > r_I \), which suggests the following:

If \( y_B > r_I \), then \( r = y_B \);

Otherwise, \( r = r_I \).

Using (17), we can derive a specific expression of \( Q \) for a particular value of \( r \):

\[
\text{If } r = y_B, \quad \text{then } Q = (y_B - r_I)^2 = \left( \frac{h(y_B - x_0)}{h + e^B} \right)^2;
\]

\[
\text{If } r = r_I, \quad \text{then } Q = (r_I - r_I)^2 = 0.
\]

As discussed in the text, the probability of auditor independence \( \Pr(AI) \) is the probability of \( r_I > y_B \). Thus, we have

\[
\Pr(AI) = \Pr(r_I > y_B) = \Pr(h(y_B - x_0) < 0) = \Pr(y_B - x_0 < 0).
\]

(ii) Regime \( BB \).

Under joint audits of two big audit firms, the independent auditor report is based on two pieces of audit evidence: \( r_I \equiv \mathbb{E}[\tilde{x} | y_B, y_{B2}] \). Because both \( \tilde{x} \) and \( \tilde{y} \) follow the normal distribution, applying the standard formula of the conditional mean for the multivariate normal distribution yields \( r_I = \frac{hx_0 + e^B y_B + e^{B2} y_{B2}}{h + e^B + e^{B2}} \). The auditors’ expectation of their misstatement
costs, given a report \( r \) and their audit evidence \( y_B \) and \( y_{B2} \), is

\[
\mathbb{E}[(r - \tilde{x})^2 | y_B, y_{B2}] = \mathbb{E}[r^2 - 2r\tilde{x} + \tilde{x}^2 | y_B, y_{B2}]
\]

\[
= r^2 - 2r\mathbb{E}[\tilde{x}|y_B, y_{B2}] + \mathbb{E}[\tilde{x}^2 | y_B, y_{B2}]
\]

\[
= r^2 - 2r\mathbb{E}[\tilde{x}|y_B, y_{B2}] + (\mathbb{E}[\tilde{x}|y_B, y_{B2}])^2 + \text{Var}[\tilde{x}|y_B, y_{B2}]
\]

\[
= (r - \mathbb{E}[\tilde{x}|y_B, y_{B2}])^2 + \text{Var}[\tilde{x}|y_B, y_{B2}]
\]

\[
= (r - r_I)^2 + \frac{1}{h + e^B + e^{B2}}.
\]

If an independent report \( r_I \) is issued, then the auditors’ expectation of their misstatement cost is

\[
\mathbb{E}[(r_I - \tilde{x})^2 | y_B, y_{B2}] = (r_I - r_I)^2 + \frac{1}{h + e^B + e^{B2}} = \frac{1}{h + e^B + e^{B2}}. \tag{18}
\]

Therefore, to induce the auditors to certify \( r \) instead of \( r_I \), the company must offer the auditors an amount of \( Q \) to compensate them for the increase in the misstatement cost:

\[
Q = \mathbb{E}[(r - \tilde{x})^2 | y_B, y_{B2}] - \mathbb{E}[(r_I - \tilde{x})^2 | y_B, y_{B2}]
\]

\[
= (r - r_I)^2. \tag{19}
\]

The company prefers to induce a report such that \( r = \max\{y_B, y_{B2}\} \) if \( \max\{y_B, y_{B2}\} > r_I \). Hence, we have the following:

If \( y_B > y_{B2} \) and \( y_B > r_I \), then \( r = y_B \);

If \( y_{B2} > y_B \) and \( y_{B2} > r_I \), then \( r = y_{B2} \);

Otherwise, \( r = r_I \).
Using (19), we derive a specific expression of $Q$ for a particular value of $r$:

- If $r = y_B$, $Q = (y_B - r_I)^2 = \left(\frac{h(y_B - x_0) + e^{B^2}(y_B - y_{B2})}{h + e^B + e^{B^2}}\right)^2$;
- If $r = y_{B2}$, $Q = (y_{B2} - r_I)^2 = \left(\frac{h(y_{B2} - x_0) + e^B(y_{B2} - y_B)}{h + e^B + e^{B^2}}\right)^2$;
- If $r = r_I$, $Q = (r_I - r_I)^2 = 0$.

Now we solve the probability of auditor independence $\Pr(AI)$ under Regime $BB$. Auditor independence occurs when the independent report exceeds the audit evidence. Hence, it is the probability of $r_I > y_B$ and $r_I > y_{B2}$. Thus, we have

$$\Pr(AI) = \Pr(r_I > y_B \text{ and } r_I > y_{B2})$$

$$= \Pr(h(y_B - x_0) + e^{B^2}(y_B - y_{B2}) < 0 \text{ and } h(y_{B2} - x_0) + e^B(y_{B2} - y_B) < 0)$$

$$= \Pr\left(\frac{h + e^B)y_{B2} - hx_0}{e^B} < y_B < \frac{hx_0 + e^{B^2}y_{B2}}{h + e^{B^2}}\right).$$

(iii) Regime $BS$.

The $Q$ and $\Pr(AI)$ in Regime $BB$ are given above. Replacing $y_{B2}$ by $y_S$ and replacing $e^{B^2}$ by $e^S$ yields the counterparts in Regime $BS$. In particular, note that

$$\mathbb{E}[(r_I - \tilde{x})^2|y_B, y_S] = (r_I - r_I)^2 + \frac{1}{h + e^B + e^S} = \frac{1}{h + e^B + e^S}. \quad (20)$$

\[\square\]

Proof of Proposition 2

In Regime $B$, using (16), we can rewrite (3) as

$$\min_{e^B} k_B C'(e^B) + \frac{1}{h + e^B}. \quad (21)$$

Differentiating it with respect to $e^B$ and setting it equal to 0 yields

$$k_B C''(e^B) - \frac{1}{(h + e^B)^2} = 0.$$
In Regime \( BB \), using (18), we can rewrite (4) and (5) as

\[
\min_{e^B} k_B C(e^B) + \frac{1/2}{h + e^B + e^{B2}} \tag{22}
\]

and

\[
\min_{e^{B2}} k_B C(e^{B2}) + \frac{1/2}{h + e^B + e^{B2}} \tag{23}
\]

Differentiating them with respect to \( e^B \) and \( e^{B2} \) respectively and setting them equal to 0 yields

\[
\begin{align*}
    k_B C'(e^B) - \frac{\alpha_B}{(h + e^B + e^{B2})^2} &= 0 \\
    k_B C'(e^{B2}) - \frac{\alpha_B}{(h + e^B + e^{B2})^2} &= 0.
\end{align*}
\]

Obviously, \( e^B = e^{B2} \equiv e_{BB}^B \). So we can rewrite the above pair of equations as two identical equations: \( k_B C'(e_{BB}^B) - \frac{1/2}{(h + 2e_{BB}^B)^2} = 0 \).

In Regime \( BS \), using (20), we can rewrite (6) and (7) as

\[
\min_{e^B} k_B C(e^B) + \frac{\alpha_B}{h + e^B + e^S} \tag{24}
\]

and

\[
\min_{e^S} k_S C(e^S) + \frac{\alpha_S}{h + e^B + e^S}. \tag{25}
\]

Differentiating them with respect to \( e^B \) and \( e^S \) respectively yields

\[
\begin{align*}
    k_B C'(e^B) - \frac{\alpha_B}{(h + e^B + e^S)^2} &= 0 \\
    k_S C'(e^S) - \frac{\alpha_S}{(h + e^B + e^S)^2} &= 0.
\end{align*}
\]
Proof of Proposition 3

In Regime $B$, evaluating (21) at the optimal audit precision $e_B^B$ yields the equilibrium fee.

Analogously, in Regime $BB$, summing up (22) and (23) and evaluating the sum at the optimal precision $e_{BB}^B$ yields the equilibrium fee.

In Regime $BS$, summing up (24) and (25) and evaluating the sum at the optimal precision $e_{BS}^B$ and $e_{BS}^S$ yields the equilibrium total fee.

Proof of Proposition 4

From Table 1, under Regime $BS$, $k_B C'(e_B^B) = \frac{\alpha_B}{(h+e^B+e^S)^2}$, and under Regime $B$, $k_B C'(e_B^B) = \frac{1}{(h+e^B)^2}$. Because $\frac{\alpha_B}{(h+e^B+e^S)^2} < \frac{1}{(h+e^B)^2}$, $C'(e_B^B) < C'(e_B^B)$, and in turn, because $C'' > 0$, $e_{BS}^B < e_{B}^B$.

Analogously, from Table 1, under Regime $BB$, $k_B C'(e_B^B) = \frac{1/2}{(h+2e^B)^2}$, and under Regime $B$, $k_B C'(e_B^B) = \frac{1}{(h+e^B)^2}$. Because $\frac{1/2}{(h+2e^B)^2} < \frac{1}{(h+e^B)^2}$, $C'(e_{BB}^B) < C'(e_{B}^B)$, and in turn, because $C'' > 0$, $e_{BB}^B < e_{B}^B$.

Proof of Proposition 5

(i) From Table 1, we have $2k_B C'(e_{BB}^B)(h+2e_{BB}^B)^2 = 1$ in Regime $BB$ and $k_B C'(e_{B}^B)(h+e_{B}^B)^2 = 1$ in Regime $B$. Thus, $2C'(e_{BB}^B)(h+2e_{BB}^B)^2 = C'(e_{B}^B)(h+e_{B}^B)^2$. Because $C''' = 0$, we can rewrite the preceding equation as $C'(2e_{BB}^B)(h+2e_{BB}^B)^2 = C'(e_{B}^B)(h+e_{B}^B)^2$, which implies that $2e_{BB}^B = e_{B}^B$.

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(ii) From Table 1, under Regime $BS$, we have

\[
k_B C''(e_{BS}^B) - \frac{\alpha_B}{(h + e_{BS}^B + e_{BS}^S)^2} = 0,
\]
\[
k_S C''(e_{BS}^S) - \frac{\alpha_S}{(h + e_{BS}^B + e_{BS}^S)^2} = 0.
\]

Total differentiation of those two equations yields

\[
\begin{pmatrix}
k_B C''(e_{BS}^B) + \frac{2\alpha_B}{(h + e_{BS}^B + e_{BS}^S)^3} & \frac{2\alpha_B}{(h + e_{BS}^B + e_{BS}^S)^3} \\
\frac{2\alpha_S}{(h + e_{BS}^B + e_{BS}^S)^3} & k_S C''(e_{BS}^S) + \frac{2\alpha_S}{(h + e_{BS}^B + e_{BS}^S)^3}
\end{pmatrix} \begin{pmatrix}
de_{BS}^B \\
de_{BS}^S
\end{pmatrix} = \begin{pmatrix}
d \alpha_B \\
d \alpha_B
\end{pmatrix}
\]  \hspace{1cm} (26)

Denote the determinant of the leftmost matrix by $\Delta$, and one can easily verify that $\Delta > 0$. From (26), we use the Cramer’s Rule to solve for $\frac{de_{BS}^B}{dm}$ and $\frac{de_{BS}^S}{dm}$ and get the following:

\[
\frac{de_{BS}^B}{dm} = \frac{1}{\Delta} k_B C'(e_{BS}^S) \frac{2\alpha_B}{(h + e_{BS}^B + e_{BS}^S)^3},
\]
\[
\frac{de_{BS}^S}{dm} = -\frac{1}{\Delta} k_B C'(e_{BS}^B) \left[ \frac{2\alpha_B}{(h + e_{BS}^B + e_{BS}^S)^3} + k_B C''(e_{BS}^B) \right].
\]

It follows obviously that

\[
\frac{de_{BS}^B}{dm} > 0, \quad \frac{de_{BS}^S}{dm} < 0, \quad \text{and} \quad \left| \frac{de_{BS}^B}{dm} \right| < \left| \frac{de_{BS}^S}{dm} \right|. \hspace{1cm} (27)
\]

Therefore, $e_{BS}^B + e_{BS}^S$ is decreasing in $m$. Recall from Table 1 that in Regimes $B$ and $BB$, the evidence precision is independent of $m$. When $m = 1$, $e_{BS}^B + e_{BS}^S$ equals $e_{B}^B$ due to $C'' = 0$. Taken together, for $m > 1$, $e_{BS}^B + e_{BS}^S$ is less than the total precision in the other two regimes.

From (26), we use the Cramer’s Rule again to solve for $\frac{de_{BS}^B}{d\alpha_B}$ and $\frac{de_{BS}^S}{d\alpha_B}$ and get the following:

\[
\frac{de_{BS}^B}{d\alpha_B} = \frac{1}{\Delta(h + e_{BS}^B + e_{BS}^S)^2} \left[ \frac{2}{(h + e_{BS}^B + e_{BS}^S)^3} + k_S C''(e_{BS}^S) \right],
\]
\[
\frac{de_{BS}^S}{d\alpha_B} = \frac{1}{\Delta(h + e_{BS}^B + e_{BS}^S)^2} \left[ \frac{2}{(h + e_{BS}^B + e_{BS}^S)^3} + k_B C''(e_{BS}^B) \right]. \hspace{1cm} (28)
\]

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It follows that \( \frac{de^B_{BS}}{da_B} > 0 \) and \( \frac{de^S_{BS}}{da_B} < 0 \). And because \( k_S > k_B \) and \( C'''' = 0 \), \( |\frac{de^B_{BS}}{da_B}| > |\frac{de^S_{BS}}{da_B}| \).

Therefore, \( e^B_{BS} + e^S_{BS} \) is increasing in \( \alpha_B \). Recall from Table 1 that in Regimes \( B \) and \( BB \), the evidence precision is independent of \( \alpha_B \). When \( \alpha_B \) equals 1, the big audit firm conducts the single audit and thus, \( e^B_{BS} + e^S_{BS} \) equals \( e^B_{B} \). Therefore, for \( \alpha_B \in \left( \frac{1}{2}, 1 \right) \), \( e^B_{BS} + e^S_{BS} \) is less than the total precision in the other two regimes.

To figure out the reason for \( e^B_{BS} + e^S_{BS} < e^B_{B} \), we further examine whether thebig audit firmexerts less effort in Regime \( BS \) than it would have in Regime \( B \) with the same share of misstatement cost. Table 1 implies that

\[
\frac{C'(e^B_{BS})}{C'(e^B_{B})} = \alpha_B \left( \frac{h + e^B_{BS}}{h + e^B_{BS} + e^S_{BS}} \right)^2.
\]

The fact that \( e^B_{BS} + e^S_{BS} < e^B_{B} \) yields \( C'(e^B_{BS}) > \alpha_B C'(e^B_{B}) \). Because \( C'''' = 0 \), the above inequality implies \( e^B_{BS} > \alpha_B e^B_{B} \).

**Proof of Proposition 6**

We can specify the audit evidence as follows: \( y_B = x + \varepsilon_B \) and \( y_S = x + \varepsilon_S \) where \( \varepsilon_B \sim \mathcal{N}(0, \frac{1}{e_B}) \) and \( \varepsilon_S \sim \mathcal{N}(0, \frac{1}{e_S}) \). Now, the expectation of \( Q_{BS} \) can be developed as follows:

\[
\mathbb{E}[Q_{BS}] = \frac{1}{(h + e^B_{BS} + e^S_{BS})^2} \mathbb{E}[\left( h(y_B - x_0) + e^S_{BS}(y_B - y_S) \right]^2]
\]

\[
= \frac{1}{(h + e^B_{BS} + e^S_{BS})^2} \mathbb{E}[\left( h(x + \varepsilon_B - x_0) + e^S_{BS}(\varepsilon_B - \varepsilon_S) \right)^2]
\]

\[
= \frac{1}{(h + e^B_{BS} + e^S_{BS})^2} \mathbb{E}[\left( h(x - x_0) + (h + e^S_{BS})\varepsilon_B - e^S_{BS}\varepsilon_S \right)^2]
\]

\[
= \frac{1}{(h + e^B_{BS} + e^S_{BS})^2} \left[ h^2 \mathbb{E}[(x - x_0)^2] + \frac{(h + e^S_{BS})^2}{e^B_{BS}} + \frac{e^S_{BS}^2}{e^S_{BS}} \right]
\]

\[
= \frac{1}{(h + e^B_{BS} + e^S_{BS})^2} \left[ h + \frac{(h + e^S_{BS})^2}{e^B_{BS}} + e^S_{BS} \right]
\]

\[
= \frac{h + e^S_{BS}}{e^B_{BS}(h + e^B_{BS} + e^S_{BS})}.
\]
Analogously, we can derive the expectation of quasi-rents in Regimes B and BB:

\[
\mathbb{E}[Q_{BB}] = \frac{h + e^B_{BB}}{e^B_{BB}(h + 2e^B_{BB})}
\]

\[
\mathbb{E}[Q_B] = \frac{h}{e^B(h + e^B_B)}.
\]

Since \(e^B_{BB} < e^B_B\) (Proposition 4) and \(2e^B_{BB} = e^B_B\) (Proposition 5), \(\mathbb{E}[Q_{BB}] > \mathbb{E}[Q_B]\). This proves part (i).

Since \(e^B_{BS} < e^B_B\) (Proposition 4) and \(e^B_{BS} + e^S_{BS} < e^B_B\) (Proposition 5), \(\mathbb{E}[Q_{BS}] > \mathbb{E}[Q_B]\).

This proves part (ii).

Proof of Proposition 7

From Table 2, \(\Pr(AI_B) = \Pr(y_B < x_0)\). Because the prior mean of \(y_B\) is \(x_0\), \(\Pr(AI_B) = \frac{1}{2}\).

Now we show that \(\Pr(AI_{BS}) < \frac{1}{2}\). Recall from Table 2 that \(\Pr(AI_{BS}) = \Pr\left(\frac{(h+e^B_{BS})y_S-hx_0}{e^B_{BS}} < y_B < \frac{hx_0+e^S_{BS}y_S}{h+e^B_{BS}}\right)\). (i) If \(y_S < x_0\), the upper limit \(\frac{hx_0+e^S_{BS}y_S}{h+e^B_{BS}}\) of \(y_B\) is less than \(x_0\) (the prior mean of \(y_B\)), and so \(\Pr(AI_{BS}) < \frac{1}{2}\). (ii) If \(y_S > x_0\), the lower limit \(\frac{(h+e^B_{BS})y_S-hx_0}{e^B_{BS}}\) of \(y_B\) is greater than \(x_0\), and so again \(\Pr(AI_{BS}) < \frac{1}{2}\).

The same reasoning applies to \(\Pr(AI_{BB})\) and one can show that \(\Pr(AI_{BB}) < \frac{1}{2}\).

Proof of Proposition 8

(i) By Table 3, \(F_{BB} - F_B = 2k_B C(e^B_{BB}) - k_B C(e^B_B)\). Recall from Proposition 5 that \(2e^B_{BB} = e^B_B\). We can write \(e^B_{BB}\) as a convex combination of 0 and \(e^B_B\): \(e^B_{BB} = \frac{1}{2} \times 0 + \frac{1}{2} \times e^B_B\).

By the convexity of the cost function, we have

\[
C(e^B_{BB}) = C\left(\frac{1}{2} \times 0 + \frac{1}{2} \times e^B_B\right) < \frac{1}{2} C(0) + \frac{1}{2} C(e^B_B).
\]

Because \(C(0) = 0\), we have \(2C(e^B_{BB}) < C(e^B_B)\) and thus \(F_{BB} < F_B\).
(ii) We can rewrite the expression for $F_{BS}$ in (15) as the sum of (a) the big firm’s resource cost and its share of the expected misstatement cost expressed in (24) and (b) the small firm’s resource cost and its share of the expected misstatement cost expressed in (25):

$$F_{BS} = \left[ k_B C(e_{BS}^B) + \frac{\alpha_B}{h + e_{BS}^B + e_{BS}^S} \right] + \left[ k_S C(e_{BS}^S) - \frac{\alpha_S}{h + e_{BS}^B + e_{BS}^S} \right].$$

Note that the expression in the first bracket is the big firm’s objective function when it chooses its precision and the expression in the second bracket is the small firm’s objective function when it chooses its precision. So we can apply the envelop theorem when we derive $\frac{dF_{BS}}{dm}$ and $\frac{dF_{BS}}{d\alpha_B}$ in the following. Applying the envelop theorem to derive $\frac{dF_{BS}}{dm}$ yields

$$\frac{dF_{BS}}{dm} = \frac{\alpha_B}{(h + e_{BS}^B + e_{BS}^S)^2} \frac{de_{BS}^S}{dm} - \frac{\alpha_S}{(h + e_{BS}^B + e_{BS}^S)^2} \frac{de_{BS}^B}{dm} + k_B C(e_{BS}^S).$$

By (27), $\frac{de_{BS}^B}{dm} > 0$, $\frac{de_{BS}^S}{dm} < 0$, and $|\frac{de_{BS}^S}{dm}| < |\frac{de_{BS}^B}{dm}|$, and therefore, $\frac{de_{BS}^S}{dm} > \frac{de_{BS}^B}{dm}$. Furthermore, by assumption, $\alpha_B > \alpha_S$. Therefore, we have $\frac{dF_{BS}}{dm} > 0$. Neither $F_B$ nor $F_{BB}$ involves $m$ and thus both are independent of $m$. Moreover, when $m = 1$, $F_{BS} < F_B$ (see the proof below). Taken altogether, these three conditions ensure there exists a $m^* > 1$ such that $F_{BS} = F_B$ for $m = m^*$. Because $\frac{dF_{BS}}{dm} > 0$, this implies that $F_{BS} < F_B$ for $m < m^*$.

Next we compare the fees at the boundary when $m = 1$.

At $m = 1$, by (9), $k_B[C'(e_{BS}^B) + C'(e_{BS}^S)] = \frac{1}{(h + e_{BS}^B + e_{BS}^S)^2}$, and by (8), $k_B C''(e_B^B) = \frac{1}{(h + e_{BS}^B)^2}$. Because $C''' = 0$, we have $e_{BS}^B = e_{BS}^B + e_{BS}^S$ when $m = 1$.

Table 3 shows $F_{BS} - F_B = k_B[C(e_{BS}^B) + C(e_{BS}^S) - C(e_B^B)] + \left[ \frac{1}{(h + e_{BS}^B + e_{BS}^S)^2} - \frac{1}{h + e_{BS}^B} \right] = k_B[C(e_{BS}^B) + C(e_{BS}^S) - C(e_B^B)]$ at $m = 1$. The assumption $C'' > 0$ yields $C(e_{BS}^B) + C(e_{BS}^S) < C(e_{BS}^B + e_{BS}^S) = C(e_B^B)$ at $m = 1$. Therefore we have $F_{BS} - F_B < 0$ when $m = 1$.

Taken altogether, these three conditions ensure there exists a $m^* > 1$ such that $F_{BS}$ exceeds $F_B$.

Analogously, using the envelop theorem, we can derive $\frac{dF_{BS}}{d\alpha_B}$:

$$\frac{dF_{BS}}{d\alpha_B} = -\frac{\alpha_B}{(h + e_{BS}^B + e_{BS}^S)^2} \frac{de_{BS}^S}{d\alpha_B} - \frac{\alpha_S}{(h + e_{BS}^B + e_{BS}^S)^2} \frac{de_{BS}^B}{d\alpha_B}$$

(29)
Substituting the expressions for \( \frac{dF_B}{d\alpha_B} \) and \( \frac{dF_S}{d\alpha_B} \) in (28) into (29) and using the fact that

\[
\frac{1}{h + e_B^B + e_S^S} = \sqrt{k_B C''(e_B^B) + k_S C''(e_S^S)}
\]

implied by (9) yields

\[
dF_{BS} = \frac{1}{\Delta(h + e_B^B + e_S^S)^4} \left\{ (4\alpha_B - 2)(\sqrt{k_B C''(e_B^B) + k_S C''(e_S^S)})^3 + \alpha_B [k_B C''(e_B^B) + k_S C''(e_S^S)] - k_S C''(e_S^S) \right\}.
\]

Thus,

\[
\frac{dF_{BS}}{d\alpha_B} > 0 \iff \alpha_B > \alpha_0 \equiv \frac{2(\sqrt{k_B C''(e_B^B) + k_S C''(e_S^S)})^3 + k_S C''(e_S^S)}{4(\sqrt{k_B C''(e_B^B) + k_S C''(e_S^S)})^3 + k_S C''(e_S^S) + k_B C''(e_B^B)}.
\]

Obviously, \( \alpha_0 < 1 \), and because \( C''''(e) = 0 \) and \( k_S > k_B, \alpha_0 > \frac{1}{2} \). Therefore, \( F_{BS} \) is decreasing in \( \alpha_B \) from \( \frac{1}{2} \) to \( \alpha_0 \) and then increasing in \( \alpha_B \) from \( \alpha_0 \) to 1. Recall from Table 3 that both \( F_{BB} \) and \( F_B \) are independent of \( \alpha_B \). Note that at \( \alpha_B = 1 \), Regime BS converges to Regime B, and so \( F_{BS} = F_B \) at \( \alpha_B = 1 \).

Now the question is whether \( F_{BS} > F_B \) at \( \alpha_B = \frac{1}{2} \). If not, then \( F_{BS} \) falls short of \( F_B \) globally over all the range of \( \alpha_B \). If yes, then \( F_{BS} > F_B \) only for sufficiently small values of \( \alpha_B \). To answer this question, recall from our previous result that \( F_{BS} \) is increasing in \( m \). Therefore, we have \( F_{BS}(\alpha_B = \frac{1}{2} \text{ and } m > 1) > F_{BS}(\alpha_B = \frac{1}{2} \text{ and } m = 1) \). Note that when \( \alpha_B = \frac{1}{2} \) and \( m = 1 \), Regime BS converges to Regime BB, and so \( F_{BS}(\alpha_B = \frac{1}{2} \text{ and } m = 1) = F_{BB} \). Therefore, \( F_{BB} \) is the lower bound of \( F_{BS} \) at \( \alpha_B = \frac{1}{2} \). Because \( F_{BB} < F_B \) (proved in part (i)), it is not always the case that \( F_{BS} > F_B \) at \( \alpha_B = \frac{1}{2} \). Since \( F_{BS} \) is increasing in \( m \), for sufficiently large values of \( m \), \( F_{BS} \) will exceed \( F_B \) at \( \alpha_B = \frac{1}{2} \).

By continuity, there exists an \( \alpha^\dagger \) such that \( F_{BS} > F_B \) if \( \frac{1}{2} < \alpha_B < \alpha^\dagger \) and \( F_{BS} < F_B \) if \( \alpha^\dagger < \alpha_B < 1 \).

Note that the difference between \( F_{BS} \) and \( F_B \) is not monotonic. This is because \( F_{BS} \) is a U-shaped function of \( \alpha_B \) whereas \( F_B \) is independent of \( \alpha_B \). □
References


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