Internal Rate of Return (IRR)

A project's expected rate of return. IRR = discount rate where project

\[
NPV = 0, \quad NPV = \sum_{t=0}^{n} \frac{C_t}{(1+IRR)^t} = 0
\]

or

\[
\sum_{t=1}^{n} \frac{C_t}{(1+IRR)^t} = -C_0
\]

Accept if IRR > r (opportunity cost) on capital
Don't use for non-normal project.

Marge's IRRs

on Fin'Calc, enter CFs into CF worksheet

<table>
<thead>
<tr>
<th>Time</th>
<th>Falafel-Full</th>
<th>How 'Bout A Pretzel?</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>(20,000)</td>
<td>(20,000)</td>
</tr>
<tr>
<td>1</td>
<td>15,000</td>
<td>2,000</td>
</tr>
<tr>
<td>2</td>
<td>15,000</td>
<td>2,500</td>
</tr>
<tr>
<td>3</td>
<td>13,000</td>
<td>3,000</td>
</tr>
<tr>
<td>4</td>
<td>3,000</td>
<td>50,000</td>
</tr>
</tbody>
</table>

IRR | CPT | IRR = 54.7% |
IRR | CPT | IRR = 33.3%
If independent, accept both.
- Both IRRs > 12%
If mutually exclusive, select
Falafel-Full; higher IRR
Profitability Index (PI)

- Amount of NPV (benefit) that a project generates per dollar of investment (cost).
- Ratio of NPV/Initial Investment
- Accept project if PI > 0

Marge’s PIs: r = 12%

<table>
<thead>
<tr>
<th>Time</th>
<th>Falafel-Full</th>
<th>PV(CF)</th>
<th>How ’Bout A Pretzel?</th>
<th>PV(CF)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>(20,000)</td>
<td>(20,000)</td>
<td>(20,000)</td>
<td>(20,000)</td>
</tr>
<tr>
<td>1</td>
<td>15,000</td>
<td>13,393</td>
<td>2,000</td>
<td>1,786</td>
</tr>
<tr>
<td>2</td>
<td>15,000</td>
<td>11,958</td>
<td>2,500</td>
<td>1,993</td>
</tr>
<tr>
<td>3</td>
<td>13,000</td>
<td>9,253</td>
<td>3,000</td>
<td>2,135</td>
</tr>
<tr>
<td>4</td>
<td>3,000</td>
<td>1,907</td>
<td>50,000</td>
<td>31,776</td>
</tr>
</tbody>
</table>

- Falafel PI = 16,510/20,000 = 0.83
- Pretzel PI = 17,690/20,000 = 0.88
- Accept both if independent, since both PIs>0
- Accept Pretzel if mutually exclusive, higher PI.
- Can rank projects differently from NPV if initial investments are different.
Comparison of NPV & IRR

- For normal independent projects, both methods give the same accept/reject decision.
  - NPV > 0 yields IRR > r in order to lower NPV to 0.
- However, these methods can rank mutually exclusive projects differently.
- What to do, then?

NPV Profiles

- A graph which shows a project’s NPV at different interest rates (opportunity cost of capital).
- Can illustrate ranking conflicts between NPV and IRR.
- Below is a table of NPVs for Marge’s projects.

<table>
<thead>
<tr>
<th>r</th>
<th>Falafel-Full</th>
<th>How ‘Bout A Pretzel?</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>26,000</td>
<td>37,500</td>
</tr>
<tr>
<td>5%</td>
<td>21,589</td>
<td>27,899</td>
</tr>
<tr>
<td>10%</td>
<td>17,849</td>
<td>20,289</td>
</tr>
<tr>
<td>12%</td>
<td>16,510</td>
<td>17,690</td>
</tr>
<tr>
<td>15%</td>
<td>14,649</td>
<td>14,190</td>
</tr>
<tr>
<td>25%</td>
<td>9,485</td>
<td>5,216</td>
</tr>
<tr>
<td>35%</td>
<td>5,529</td>
<td>(874)</td>
</tr>
<tr>
<td>55%</td>
<td>(68)</td>
<td>(8,201)</td>
</tr>
</tbody>
</table>
Determining NPV/IRR Conflict Range

- For each year, subtract one project’s cash flows from the other.
- If there is a change of signs of these cash flow differences, a ranking conflict exists.
- Find IRR of these cash flow differences to find rate where the two projects have the same NPV = crossover rate.
- At a cost of capital less than this crossover rate, a ranking conflict between NPV and IRR exists.
Determining Marge’s NPV/IRR Conflict range

<table>
<thead>
<tr>
<th>Year</th>
<th>Falafel</th>
<th>Pretzel</th>
<th>F-P</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>(20,000)</td>
<td>(20,000)</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>15,000</td>
<td>2,000</td>
<td>13,000</td>
</tr>
<tr>
<td>2</td>
<td>15,000</td>
<td>2,500</td>
<td>12,500</td>
</tr>
<tr>
<td>3</td>
<td>13,000</td>
<td>3,000</td>
<td>10,000</td>
</tr>
<tr>
<td>4</td>
<td>3,000</td>
<td>50,000</td>
<td>(47,000)</td>
</tr>
</tbody>
</table>

CPT IRR = 14.1%

Marge’s Projects

- At a cost of capital less than 14.1%, Pretzel has higher NPV but lower IRR = Ranking Conflict.
- At cost of capital greater than 14.1%, Falafel has the higher NPV and IRR.
- Why? Cash flow timing differences in this case.
- Other cause: initial cost differences, but not here.
Which project is best for Marge?

- Think back to my indecent proposal from Monday.
- Which of the following investment opportunities would you prefer?
- #1) Give me $1 now and I’ll give you $2 at the end of class. NPV = $1, PI = 1, IRR = 100%
- #2) Give me $100 now and I’ll give you $150 at the end of class. NPV = $50, PI = 0.5, IRR = 50%

Reconciling NPV/IRR Ranking Conflicts

- **Shareholder Wealth Maximization:**
  - Want to add more value to the firm than less.
- **Result:** Choose project with highest NPV when NPV/IRR ranking conflict exists for mutually exclusive projects.
- Also, IRR has the multiple IRR problem for non-normal projects like the following.
Acme, Inc. Rocket-Powered Roller Blade Project

- Acme is considering the following project which would market these roller blades to coyotes trying to catch road runners. Acme expects a cash inflow in the year 1, but an outflow in the 2\textsuperscript{nd} (last) year of the project due to liability claims from injured cartoon coyotes. Acme’s opportunity cost of capital is 13%.

<table>
<thead>
<tr>
<th>Year</th>
<th>Cash Flow</th>
<th>NPV</th>
<th>IRR</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>(5)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>30</td>
<td>-1.95</td>
<td>26.8%</td>
</tr>
<tr>
<td>2</td>
<td>(30)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

At Acme’s 13\% opportunity cost of capital, the project has a negative NPV even though the IRRs is greater than 13\%.

Because of this conflict, don’t use IRR to make decisions for non-normal projects! (or look for a first IRR that is less than cost of capital)
Comparing Projects (NPV>0) with unequal lives: Equivalent Annual Annuity

- Burns Power is considering the following mutually exclusive projects in order to increase power consumption in Springfield indefinitely. Which project should be selected if Burns Power’s opportunity cost of capital is 10%?

<table>
<thead>
<tr>
<th>Year</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sun-Blocker</td>
<td>(50)</td>
<td>60</td>
<td>60</td>
<td></td>
</tr>
<tr>
<td>Fog-Maker</td>
<td>(30)</td>
<td>40</td>
<td>40</td>
<td>40</td>
</tr>
</tbody>
</table>

Find NPV and Equivalent Annual Annuity

Step 1: Find NPV of each project as is at 10%.
- NPV of Sun-Blocker = $54.1 m (CF0 = -50, C01 = 60, F02 = 2)
- NPV of Fog-Maker = $69.5 m (CFO = -30, C01 = 40, F01 = 3)
- This result is expected since the Fog-Maker project is longer. We want to put the two projects on the same footing, assuming the projects can be repeated indefinitely, we can find the Equivalent Annual Annuity (EAA) for each project.
- The EAA tells us how much NPV is generated annually on a time value of money basis.
EAA calculation

Step 2: Find EAA, “annuitize” each project’s NPV
NPV = EAA \times (PVAF_{r,n})\) where \(r\) is the cost of capital, and \(n\) is the life of the project.
- Sun-Blocker EAA: \(-54.1=PV, 10=I/Y, 2=N, 0=FV, CPT\)
  PMT = $31.2m
- Fog-Maker EAA: \(-69.5=PV, 10=I/Y, 3=N, 0=FV, CPT\)
  PMT = $27.9m
- Burns should choose the Sun-Blocker because it would add the most value on an annual basis.

Capital Rationing

Capital Rationing - Limit set on the amount of funds available for investment.
Soft Rationing - Limits on available funds imposed by management. (a company shouldn’t do this because it can limit firm value)
Hard Rationing - Limits on available funds imposed by the unavailability of funds in the capital market.
Profitability Index (PI) can help under capital rationing.
- The ratio of the net present value of a project’s cash flows to its cost.
- \( PI = \frac{NPV}{Cost} \)
- **Decision Rule: Accept if \( PI > 0 \)**
- PI can be used to rank projects under capital rationing conditions. Accept highest PI projects under the capital constraint to maximize NPV.
- **CAUTION: PI can rank mutually exclusive projects that have different initial costs differently than NPV.**

Summary of Capital Budgeting Methods
- Want a method that uses the time value of money with all project cash flows: NPV, PI, IRR.
- IRR can give erroneous decision for non-normal projects.
- Overall, NPV is the best and preferred method.
- However, under capital rationing (budget restraint), ranking projects by PI can be useful in helping to maximize NPV under capital constraint.