Example: How easy is retirement?

- Springfield mogul Montgomery Burns, age 85, wants to retire at age 100 so he can steal candy from babies full time. Once Mr. Burns retires, he wants to withdraw $300 million at the beginning of each year for 10 years from a special off-shore account that will pay 20% annually. In order to fund his retirement, Mr. Burns will make 15 equal end-of-the-year deposits in this same special account that will pay 20% annually. How large of an annual deposit must be made to fund Mr. Burns’ retirement plans?

\[
\text{Dep} \times (FVAF_{20\%,15}) = 300m \times \frac{1}{(1.2)^{15}} (1.2)
\]

\[
\text{Dep} = \frac{300m \times (FVAF_{20\%,10})}{(1.2)^{15}}
\]

\[
\text{Calc: } \text{BGN mode: } \text{2nd} \text{PMT (＝ BGN)} \quad \text{2nd ENTER (＝SET)} \quad \text{CE}
\]

\[
300 \text{PMT} 20 \text{Y} 10 \text{N} 0 \text{FV}
\]

\[
fv = -1509.3 \text{ m}
\]
$85 \quad 86 \quad \cdots \quad 99 \quad 100 \quad r = 20\%$

Dep \quad \cdots \quad \text{Dep} \quad \text{Dep}

$\text{Dep} (FVA \times F_{20\%,15}) = 1509.3 \text{ m}$

$\text{END mode :}$

\[ 1509.3 \quad \boxed{FV} \quad \boxed{0} \quad \boxed{PV} \quad 20 \quad \boxed{114} \quad 15 \quad \boxed{N} \]

\[ \boxed{\text{CPT}} \quad \boxed{PMT} = -20.95 \text{ m} \]

**Non-annual compounding**

$\text{APR} = \text{annual percentage, annual rate before compounding}$

$\text{EAR} = \text{effective annual rate, actual rate of interest earned/paid}$

$m = \# \text{ of compounding periods per year}$
Effective Annual Rate

\[ \text{EAR} = \left(1 + \frac{\text{APR}}{m}\right)^m - 1 \]

\[ \frac{\text{APR}}{m} = \text{periodic rate} \]

Example Credit Card APR

A credit card charges 18% APR compounded monthly. What is the EAR?

\[ \text{EAR} = \left(1 + \frac{\text{APR}}{m}\right)^m - 1 \]

APR = 18% or 0.18, \( m = 12 \)

\[ \text{EAR} = \left(1 + \frac{0.18}{12}\right)^{12} - 1 \]

\[ = \left(1 + 0.015\right)^{12} - 1 = 0.1956 \]

19.56%
Non-annual TVM adjustments
- use the periodic rate \( \frac{\text{APR}}{m} \) and \( \frac{1}{m} \)
- total # of compounding periods \( (t=\#y\times m) = tm \)
\[
FV = PV \left(1 + \frac{\text{APR}}{m}\right)^{tm}
\]
\[
PV = \frac{FV}{\left(1 + \frac{\text{APR}}{m}\right)^{tm}}
\]

for annuities:
assume payment period = interest compounding period
\[
PV_{ord} = C \times \left(\text{PVAF} \frac{\text{APR}}{m} \right)^{tm}
\]
\[
PV_{due} = C \times \left(\text{PVAF} \frac{\text{APR}}{m} \right)^{tm} \left(1 + \frac{\text{APR}}{m}\right)
\]
Example: Low Rate or Rebate?

- After bringing home your new-born baby, you decide your new expanded family needs a brand-new minivan. You decide upon a 2004 Dodge Grand Caravan SE costing $26,500 plus tax, doc fee and license. You have enough cash to pay the tax and other fees (~$1800 in Illinois) and need to finance $26,500.
- Dodge offers the choice of two incentives on the Grand Caravan. Rebates will be applied toward the purchase price.
  - $2,000 rebate plus 0% APR Financing for 48 months, or
  - $4,500 rebate. If you elect to take this rebate, you can get 4.29% APR (online Capital One) financing for 48 months.
  - Question: Which incentive would give you the lowest monthly payment?

\[
\text{Amortized Loan}
\]

\[
\begin{align*}
0 & \quad 1 \quad 2 \quad \cdots \quad 6m \\
\text{Loan} & \quad \text{(Pmt)} \quad \cdots \quad \text{(Pmt)} \\
\text{Amount} & \\
\text{Pmt} & = \text{PV} \cdot \left[ \frac{FVA_{\text{APR}}}{m} \right] \\
& = \frac{1}{m} \\
& = \frac{1}{48} \\
& = 48 \\
\end{align*}
\]

\[m = 12, \quad tm = 48\]

Option 1: 2000 rebate + 0% APR

\[
\text{Loan Amount} = 26,500 - 2,000 = 24,500
\]

\[
0\% \text{ APR} \implies \text{Pmt} = \frac{\text{Loan Amount}}{tm} = \frac{24,500}{48} = \$510.42
\]
Example: Homer the Entertainer (I changed my mind, this example will be done in discussion)

- Imagine Homer Simpson actually invested the $100,000 he earned providing Mr. Burns entertainment 6 years ago at 8% APR compounded quarterly and starts investing an additional $500 at the beginning of each quarter starting today 20 years at the same 8% APR compounded quarterly. How much money will Homer have 20 years from today?
Example: Enjoying your Retirement (will do Feb. 4 in lecture)

• You go ahead and make the contributions starting at age 22 from last week’s example, giving you $1,058,030 at age 65.

• You expect to live to age 85. So, you want to make 20 annual withdrawals from your IRA paying 9% at the beginning of each year starting at age 65 that keeps pace with expected inflation of 3% annually.
  – How large can this real (in terms of age 65 dollars) annual withdrawal be?
  – What will your last withdrawal be in terms of nominal (actual) dollars?