

# Collateral Policy in a World of Round-the-Clock Payment

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## *Abstract*

This paper examines competition between private and public payments settlement systems, and examines the consequences of round-the-clock private payments arrangements on the competitiveness of public systems. Central to the issue is the role of collateral both as a requirement for participation in central bank sponsored payments arrangements and as the backing for private intermediary arrangements. The presence of private systems serves as a check on the ability of a monetary authority to tighten monetary policy. Round-the-clock systems are an example of a collateral saving innovation that further pressures central bank pre-eminence in payments settlement.

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current version:  
October, 2009

## Abstract

This paper examines competition between private and public payments settlement systems, and examines the consequences of round-the-clock private payments arrangements on the competitiveness of public systems. Central to the issue is the role of collateral both as a requirement for participation in central bank sponsored payments arrangements and as the backing for private intermediary arrangements. The presence of private systems serves as a check on the ability of a monetary authority to tighten monetary policy. Round-the-clock systems are an example of a collateral-saving innovation that further pressures central bank pre-eminence in payments settlement.

Nearly twenty-four hours a day, some major national payment system is open (New York, London, Frankfurt, Tokyo). Major financial institutions deal with payments around the world, around the clock. Increasingly, private institutions compete with public ones to handle payment services: besides the established private large value payments systems such as CHIPS, the European private systems, CLS, and the quasi-private systems in India, Hong Kong and China, we have large institutions increasingly able to handle on-us transactions without recourse to outside institutions. As a result the value of transactions settled within private institutions can be vastly larger than the net flows recorded on national systems as a result.

Ultimately these private systems depend on collateral. Participants need to demonstrate that they are good for the obligated payments, and collateral is the means to make this happen. Public systems are also increasingly dependent on collateral, primarily because of the switch over recent decades to Real Time Gross Settlement Systems. These systems require much more cash to operate than do netting systems, and that cash must be borrowed, generally, on a collateralized basis, from the Government operator of the system.

The latest twist in this environment is the increasing ease with which collateral can be transferred into and out of national payment system arrangements.

Within the trading day, banks now can increase or decrease the collateral in a system, readjusting its use for other activities. Central banks have aided the process by increasingly allowing members to use as collateral securities of foreign governments. And the rise of agreements between central banks for easy shifting of collateral from one national system to another means that the day is not far off when collateral could be shifted around the world following the trading activities of payments systems around the clock.

How can we make sense of these changes? How do financial institutions decide on the use of their collateral and their participation in these systems? What are the consequences for operation of payments systems and for the effectiveness of central bank monetary policy? In this paper we will make a start at answering these questions by developing a model of competition between public and private payments arrangements. While a monetary authority will have interest rate policies available to it, a central role in the model will be played by collateral, and the real effects of the system will be related to costs of generating collateral. Twenty four-hour systems will allow for economizing on collateral; by doing so, private systems will put pressures on public systems to reduce their costs or lose market share.<sup>1</sup>

The model we develop is an extension of Berentsen-Monnet (2007) which in turn is based on Lagos-Wright (2005)'s "day-night" models. The model is analyzed in greater detail in Kahn (2008); here we focus on the application to 24-hour payments arrangements. As this is a first attempt to address these issues, many simplifications will be included. We will focus on tradeoffs between the costs of collateralization and current consumption; the possibility of additional productive investments will be ignored. Central banks use "channel systems" to carry out monetary policy—that is, they establish nominal lending and borrowing rates for central bank funds. The role of money is solely a means of payment and the need for a means of payment arises solely from the problem of limited enforcement. Individuals face uncertainty about demand for consumption, which leads to a precautionary motive for money holding. There is no aggregate uncertainty—an extension which will be important for linking the model to more macroeconomic issues. Nonetheless, competition between private and public payments arrangements will have important consequences for policy, even in this extremely simple set-up.

## 1 The model

The model is a simplified version of Kahn (2008) which in turn is a modification of Berentsen-Monnet (2009). Consider three periods 0, 1, 2 ("morning", "afternoon", "next morning.")<sup>2</sup> Agents are risk neutral. There are three different

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<sup>1</sup>The idea of financial institutions, not just payment systems, as a means of economizing on collateral by bringing idle balances together with demanders of payments media is the key feature of the paper by Berentsen Camera and Waller (2007). I thank Randy Wright for pointing out this parallel.

<sup>2</sup>The so-called "quasi-linearity" of the utility function (Lagos-Wright, 2005) allows us to isolate the normal infinite horizon problem to these three periods

goods; one can be produced in each period.

All agents can produce morning goods (produced in periods 0 and 2) at a cost of 1 per unit; the goods give utility of 1 per unit if consumed immediately. Morning good produced in period 0 can also be stored until period 2 and consumed then. We assume that agents discount period 2 by a discount factor  $\beta$ , where  $0 < \beta < 1$ . (For notational convenience, we assume no discounting between periods 0 and 1). Thus in the absence of other considerations, agents will not wish to produce morning good for storage.

Agents face individual uncertainty about preferences with respect to afternoon good. In period 1, a fraction  $n$  of the agents will be able to produce, but not to consume, afternoon good. For them the cost of production is 1 per unit. The remaining fraction  $1 - n$  can consume but not produce afternoon good. For these agents, the utility of consumption of  $q$  units of afternoon good is  $u(q)$ , a function satisfying the normal convexity and Inada conditions. Individuals learn which group they belong to in at the beginning of the afternoon. The afternoon good is not storable.

The afternoon has only anonymous trading; thus agents will need a means of payment to make purchases in this period. In all other respects we will assume markets are perfectly competitive. In particular, it is always possible to borrow or lend between periods 0 and 2. Given linear preferences, we will see that the real interest rate on a two period loan will always be fixed by the discount factor:

$$r = (1 - \beta)/\beta.$$

Let  $h_t$  be an agent's net production of morning good at time  $t = 0, 2$  (production less consumption). Let  $y$  be his production of afternoon good if the agent is an afternoon producer and  $q$  consumption of afternoon good if the agent is an afternoon consumer. Let  $x$  be consumption in period 2 of morning good stored from the previous day. Then an agent's expected utility over the three periods is

$$-h_0 - ny + (1 - n)u(q) - \beta h_2 + \beta x$$

The quantities  $h_t$  can be positive or negative;  $y, q$ , and  $x$  must be non negative. Consumption in period 2 can depend on the period 1 realizations; we will let subscripts  $b$  and  $s$  denote period 2 choices conditional on the agent turning out to be a buyer or seller respectively in period 1.

After presenting basic results, we will make an extension to the model, assuming there are *two* countries  $A$  and  $B$ , exactly as described so far, on opposite sides of the world. Specifically, we will assume that the morning and afternoon periods in one country occur between the afternoon and next morning periods in the other country, as follows:

$$\begin{array}{l} A : \quad 0 \quad 1 \quad \quad \quad 2 \\ B : \quad \quad \quad 0' \quad 1' \quad 2' \end{array}$$

## 2 Non-Monetary Equilibria

Consider the case where agents are “trustworthy,” so that afternoon trades can be handled by uncollateralized credit. In this case the two economies run independently and we only need to consider one of them. In effect, all trade can occur in period 0; we let  $f$  represent the price of afternoon good relative to morning good.

**Proposition 1** *If agents are trustworthy, then in equilibrium, afternoon consumers consume  $q^*$  units of afternoon good, where*

$$u'(q^*) = 1. \tag{1}$$

*No storage occurs in equilibrium, and*

$$f = 1.$$

As noted above,  $r$  the interest rate in period 2 morning good for borrowing a unit of period 0 morning good, is  $(1 - \beta)/\beta$ .

We will call  $q^*$  the efficient or “full-trust” level of output. An equilibrium with trustworthy agents is equivalent to a Walrasian equilibrium. In this equilibrium, individuals are indifferent between choices of working one period or the next, or of consuming newly produced morning good one period or another. Because of the linearity of costs and of preferences for morning goods, individual consumptions and productions of morning good are indeterminate. However, when comparing this economy with the rest of our examples in which agents are not trustworthy, it is natural to focus on the allocation in which all “debts” are paid the next period. That is, afternoon consumers provide output the next morning equal in value to their previous afternoon consumption, and vice versa for producers. This means that ex-consumers provide

$$\frac{q^*}{\beta}$$

units per person of morning good, and ex-producers receive, on average,

$$\frac{1 - n q^*}{n \beta}.$$

### 2.1 Commodity payment

Given that agents are not trustworthy, it will not be possible to borrow for consumption in the afternoon market. An afternoon consumer could, nonetheless, pay by trading with stored good. We can think of each agent as producing the amount  $h_0$  for storage; if he is an afternoon consumer, he will pay for consumption with stored good; if he is an afternoon producer he will hold his stored good plus any afternoon receipts for consumption the next morning. In this case, the resultant equilibria contain the analogue of a cash-in-advance constraint:

each agent maximizes utility subject to period-by-period budget constraints, including the requirement that

$$fh_0 \geq q$$

Note that  $h_0$  is chosen in advance of information about period 1 preferences: it cannot depend on whether the agent turns out to be a consumer or producer in period 1.

Determination of equilibrium in this case is aided by the following considerations: In the afternoon, producers value the stored morning good at  $\beta$  per unit. Given constant marginal costs, sellers make zero profits in the afternoon. Since agents do not know whether they will be sellers or buyers, they choose a storage level  $h$  in the morning to solve the following problem:

$$\max_h -h + (1-n)u(\beta h) + n\beta h.$$

In other words, if buyers, they sell their storage for afternoon good; if sellers, they hold their own storage until the next period. Since  $q = \beta h$ , we have the first order condition:

$$-1 + (1-n)\beta u'(q) + n\beta = 0.$$

Armed with this information one can quickly verify

**Proposition 2** *If agents can only pay for afternoon consumption with stored morning goods then in the competitive equilibrium afternoon consumers consume  $\tilde{q}$  where*

$$\frac{\beta^{-1} - n}{1 - n} = u'(\tilde{q}). \quad (2)$$

*In equilibrium the afternoon market price of afternoon good relative to morning good is*

$$f = 1$$

While the price of the afternoon good is unchanged, (again, because of the constant returns to scale in production), the shadow value of an additional unit to a buyer is higher (left side of (2)). The difference arises because buyers stock out of storage. Ex post had they known they would be buyers, they would have preferred to bring additional units of morning production into the afternoon. They do not because of the costs imposed in doing so if they turn out to be sellers. Because of the carrying costs, sellers would have preferred to bring no units of morning good into the afternoon.

We will call  $\tilde{q}$ , the level of consumption under commodity payment (“the barter level”). Since there is no intertemporal market on which afternoon good can be sold we only have spot rates of exchange between the two goods available

for trade. Note that the left side of the equation defining  $\tilde{q}$  is greater than 1 so that

$$\tilde{q} < q^*.$$

and afternoon good becomes expensive relative to stored morning good. Note that agents anticipate a capital loss on the stored good. They are willing to store the good despite the fact that in present value terms each unit will be only worth at maturity the fraction  $\beta$  of its initial cost. The difference is the liquidity premium on the morning good. As before, the real interest rate on a two period loan is  $r$ .

## 2.2 Collateralized Borrowing

This equilibrium can be given a second interpretation: suppose rather than using the stored good as an outright payment, the agents treat it as collateral; the good is held by the seller until period 2 when it is returned to the buyer in return for new morning good of equal value. Clearly this interpretation makes no substantive change in the account. But it does allow us to extend the analysis to the case where the collateral value is greater or less than the value of the goods purchased with it. It also allows us explicitly to consider interest rates for borrowing or lending between periods 1 and 2. We will include that possibility in considering the individual maximization problem, with two different rates. Of course, in the competitive equilibrium, borrowing and lending rates will be the same, but by treating them separately we will be able to use the analysis for more general situations later.

Specifically, assume traders in the afternoon engage in a “repo” transaction: buyers borrow by making a *loan* of morning good which will then be returned the following morning when the borrowing is repaid. Now buyers rather than sellers consume the old morning good, and instead buyers produce new morning good to make their payments. With linear technologies this exchange is a wash. Now we can consider “haircuts”—transactions in which the value of the collateral exceeds the value the goods received—and “loans on margin”—in which the collateral only represents a fraction of the loan value. To the extent that there are non-pecuniary costs to default, it is not necessary to require full collateral to ensure repayment. To the extent that there may be adverse selection in the collateral posted, collateral value on average will have to exceed the value of the loan.

We will let  $\alpha$  denote the fraction of the loan value which must be collateralized; thus  $\alpha < 1$  represents an incompletely collateralized loan, and  $\alpha > 1$  represents a haircut. Thus  $\alpha = 0$  is the equivalent of trustworthy agents;  $\alpha = \infty$  is an economy where commodities cannot be used to make purchases (in other words, autarky, in the absence of government-provided money).

At a cost of 1 an individual manufactures a collateral good in the morning. He can use it to guarantee payment for purchase in the afternoon and will, in any case consume the collateral good the next day, at a present value of  $\beta$  per unit.

Thus the net cost of collateral provision is  $(1 - \beta)$ . In the afternoon suppliers produce and demanders purchase afternoon good. The collateral good gives an inferior amount of consumption in period 2, but relaxes the constraint on afternoon consumption. The agent's problem becomes

$$\max_{h, q, y \geq 0} -h + (1 - n)u(q) - ny + \beta h - \beta(1 - n)(1 + r_b)q + \beta n(1 + r_s)y$$

subject to

$$\alpha^{-1}h \geq (1 + r_b)q \tag{3}$$

Here  $1 + r_b$  and  $1 + r_s$  are the number of units of period 2 morning good that must be given in exchange for an afternoon loan to buy 1 unit of afternoon good. In other words  $r_b$  and  $r_s$  are the real interest rates (in principle adjusted for the relative price of the two goods, but this again turns out to be 1).

First order conditions for this problem are as follows, using  $\lambda$  as the Lagrange multiplier for the constraint (3):

$$\begin{aligned} 1 - \beta &= \lambda \alpha^{-1} \\ (1 - n)(u'(q) - \beta(1 + r_b)) &= \lambda(1 + r_b) \\ \beta(1 + r_s) &= 1 \end{aligned}$$

The third condition means that, given the constant returns to scale for production of afternoon good, in equilibrium the relative price of afternoon good and good the subsequent morning must be equal to the marginal rate of substitution. (And thus the afternoon interest rate offered by sellers is the same as the morning interest rate  $r$ ). Eliminating  $\lambda$ , the remaining conditions say

$$u'(q) = \left( \beta + \alpha \frac{1 - \beta}{1 - n} \right) (1 + r_b)$$

If  $r_b = r_s$ , as will occur if lending is competitive, and if  $\alpha = 1$ , this condition reduces to the condition (2) determining the level of output under commodity payment. As  $\alpha$  approaches 0, the condition approaches (1) and consumption approaches the trustworthy agents case. In general consumption decreases with increasing costs of using the system (increases in  $r_b$  or  $\alpha$ ).

### 2.3 Borrowing Collateral

So far, individuals have created their own collateral (morning good). But it might be feasible for individuals to obtain collateral in other ways—to buy it, or in our framework, more relevantly, to borrow it. In analogy with what has preceded, suppose that  $r_c$  is the interest charged for borrowing collateral: the number of units of period 2 consumption that have to be given in return for the borrowing of one unit of collateral in period 0 (in addition to the return of



the collateral itself). Note that we assume that anonymity precludes borrowing collateral in period 1.

There is a basic arbitrage: if an agent produces morning good and then consumes it the next morning, the payoff to the agent is  $-1+\beta$ . If he were simply to borrow the collateral at no interest and then return it, the payoff would be zero. Thus he is indifferent between borrowing collateral and producing his own if

$$\beta r_c = 1 - \beta$$

or, in other words, if

$$r_c = r.$$

Thus if there is an external source from which collateral can be borrowed, and the interest rate is greater than this critical level, no borrowing actually occurs. At an interest rate  $r_c$  below this critical level the agent's problem becomes

$$\max_{h, q, y \geq 0} +(1-n)u(q) - ny - \beta(1-n)(1+r_b)q + \beta n(1+r_s)y - h\beta r_c$$

subject to the same constraint as before, where  $h$  is the collateral borrowed, rather than the collateral produced. (Note that once the collateral has been obtained, the purchaser still has to establish a collateralized loan with the afternoon seller.) First order conditions become

$$\begin{aligned} \beta r_c &= \lambda \alpha^{-1} \\ (1-n)(u'(q) - \beta(1+r_b)) &= \lambda(1+r_b) \\ \beta(1+r_s) &= 1 \end{aligned}$$

and as before, eliminating  $\lambda$

$$u'(q) = \left( \beta + \alpha \frac{\beta r_c}{1-n} \right) (1+r_b)$$

so consumption increases as  $r_c$  falls below the critical level. As before, under competition,  $r_b = r_s$ , and the condition simplifies to

$$u'(q) = \left( 1 + \alpha \frac{r_c}{1-n} \right).$$

In this case, as  $r_c$  falls to 0, there is zero opportunity cost to holding collateral and demand for it becomes completely elastic. Consumption reaches the full-trust level of output.

### 3 Government Inside Money

Next we contrast a government monopoly on the provision of means of payment in the economy. Again, if a government's money is only used within the country, the two economies are essentially independent. We consider a government which issues inside money. That is, at time 0 it lends money to individuals, which must be repaid at time 2. We let  $P_t$  ( $t = 0, 2$ ) be the nominal price of morning good at time  $t$ , and we let  $F$  be the nominal price of afternoon good.

We will allow for the possibility that the government pays or charges nominal interest on its money; if this is the case, the total amount of money in period 2 could exceed or fall short of the amount that the public has promised to repay to the government. We assume that the excess or shortage is mopped up by lump-sum taxes or transfers; thus when we describe monetary policy below, it always entails an implicit transfer policy to satisfy the government budget constraint.

The level of prices is indeterminate. In other words, for arbitrary positive  $P_2$ , the government can make an announcement of a willingness to buy or sell  $P_2$  units of money in return for one unit of morning goods in period 2. While government supply of money is then completely elastic at this price, private agents' aggregate supply of and demand for money in period 2 (including the transfers indicated by the government budget constraint) are completely inelastic and equal. Thus money trades at the government's specified price.

However the real money supply is independent of the stated price: The price of afternoon goods in period 1 is  $F = P_2/\beta$ . If there is zero opportunity cost to holding money then each buyer will borrow enough to purchase  $q^*$  units. No storage of morning goods takes place, and the real per capita money supply in the economy overnight is  $(1 - n)q^*/\beta$  valued at period 1 prices, or  $(1 - n)q^*$  valued at period 2 prices. The marginal rate of substitution between morning and afternoon goods is 1, so that  $P_0 = P_2/\beta$ , that is, prices deflate in line with the discount rate.

Following Berentsen-Monnet (2009) we will consider a "channel system" for conduct of monetary policy: the government establishes (nominal) borrowing and lending rates for money. Anyone who borrows money from the government overnight—that is, anyone who borrows in period 0 for repayment in period 2—will pay interest  $i_\ell$ . Anyone holding money at the end of the afternoon can deposit it with the government overnight (period 1 to period 2) and receive a deposit rate,  $i_d$ . In the absence of alternative uses money supplied by the government will end up in overnight deposits. Equally clearly, the government is restricted to combinations of  $(i_\ell, i_d)$  such that

$$i_\ell \geq i_d;$$

otherwise there will be arbitrage opportunities. (Once we put collateral in place, the restriction becomes more complicated).

If the government sets the two rates to be equal (call it  $i$ ), then there is no real effect. Again, the government can announce an arbitrary value for money

on the following morning; given this value, the price of afternoon goods in period 1 is  $F = (1 + i)^{-1}P_2/\beta$ , and again each individual borrows enough to purchase  $q^*$ . Valued at period 2 prices and including interest, the real value of the money supply (call it the “overnight money supply”) is unchanged. Valued at period 1 prices, it is smaller by the anticipated interest payments. The interest payments are also built into the inflation rate:

$$\frac{P_2}{P_0} = \beta(1 + i)$$

and if  $1 + i = \beta^{-1}$  (i.e., if  $i = r$ ) prices remain constant, period to period.

On the other hand, a spread between the interest rates does have real effects. First note that with a spread in interest rates, the public must in aggregate pay back more money on any day than is available to it. The difference is assumed to be distributed lump-sum by the government to the population as a whole; thus each pair of interest rates entails an associated (negative) tax policy.

As the interest rate spread increases, the use of money decreases. In this case analysis similar to Berentsen and Monnet (see also Kahn (2008)) shows

$$u'(q) = \frac{1 + i_\ell}{1 + i_d}$$

and

$$\frac{P_2}{P_0} = \beta(n(1 + i_d) + (1 - n)(1 + i_\ell)).$$

In other words, inflation is determined by the average of interest rates faced by buyers and sellers, and economic activity is reduced by the spread in rates.

As long as the interest rate spread remains low, no agent would actually find it useful to attempt to use commodity money. However, as the interest spread increases beyond a critical level

$$\frac{\beta^{-1} - n}{1 - n}$$

an incentive arises to develop private alternatives to government money.

Of course a monetary authority could also require that participants provide collateral in return for borrowing. An individual who borrows one dollar from the government must repay  $1 + i_\ell$  the next morning. He must post  $\gamma$  dollars worth of collateral value per dollar owed. He will pay  $F$  per unit of good bought. Thus he must post  $\gamma F(1 + i_\ell)/P_2$ . As a result, the level of consumption of afternoon good falls further.

### 3.1 Collateralized inside money competing with private collateral

Finally, we review results for competition between collateralized inside money and private collateralized loans within a single country. As before, we assume

that the money is issued one afternoon and must be repaid the next day. Let  $c$  be the amount of collateral placed in the public facility and  $b$  is the amount of collateral used in private loans. The collateral constraint says that any shortfall in payment for afternoon good that is not met by private collateralized loans must be met by borrowed money.

Money interest rates are a policy variable of the government; terms for private arrangements are set competitively. Let  $C$  be the money price in period 2 that a private borrower agrees to pay for a unit of afternoon good purchased in period 1. The equivalent value in collateral in period 2 is  $C/P_2$ . A private borrower must post collateral  $b = \alpha C/P_2$  per unit of afternoon good purchased in a private loan. An individual who borrows one dollar from the government must repay  $1 + i_\ell$  the next morning. He must post  $\gamma$  dollars of collateral value per dollar owed. He will pay  $F_1$  per unit of good bought. Thus he must post  $c = \gamma F_1(1 + i_\ell)/P_2$ .

A seller who receives a dollar in period 1 will deposit it overnight and have  $(1 + i_d)$  dollars in period 2. Thus a seller who sells a unit for money will have  $F_1(1 + i_d)$  dollars in period 2. A seller who receives a promise to pay for a unit will have  $C$  dollars in period 2. Thus for a seller to be indifferent between methods

$$C = F_1(1 + i_d) \tag{4}$$

Now the choice of use of private or public payment simply boils down to the question of which method is more expensive. For the two methods to co-exist it must be that

$$\alpha(1 + i_d) = \gamma(1 + i_\ell)$$

otherwise put, if

$$\frac{1 + i_\ell}{1 + i_d} > \alpha/\gamma$$

only private payment arrangements are used; if the inequality is reversed, only public systems are used. If only public payment arrangements are used, then the equilibrium is as in section 3. If only private arrangements are used, then the equilibrium is as in section 2.

For example, holding second period prices fixed, an increase in the haircut on borrowing money lowers the demand for money and reduces afternoon consumption. The reduction in the afternoon consumption reduces demand for collateral and thus morning prices of goods. However, once the haircut exceeds that required for private borrowing, demand for money falls to zero, and further increases in haircuts have no effect on the economy.

As the government increases the spread between interest rates, activity in the economy falls, until the spread reaches the level  $\alpha/\gamma$ . From then on, further spreads have no effect, since the economy substitutes private payments arrangements for public ones. Similarly, increasing interest rate levels affects inflation.

However, once the critical level is exceeded, then this has no significance: since public money is not actually used, the private loans could be denominated in any real good, and inflation would be irrelevant.

Because we are focusing on linear technologies, these results are knife-edged. Unless the policy is exactly right, private and public payments arrangements do not co-exist. Kahn (2008) extends to convex technologies allowing for the two sorts of arrangements to co-exist; calculations are summarized in the appendix.

## 4 Twenty-four hour systems: Economizing on Collateral

We have noted that if an economy can borrow cheaper collateral it will be able to achieve a more desirable outcome. A potential source of cheaper collateral is the other economy: after all, the collateral in the other economy is sitting idle overnight. To illustrate the ideas most clearly, let us assume that it is possible for individuals in economy  $B$  to borrow collateral from economy  $A$  during  $B$ 's morning (overnight for  $A$ ) and return it in time for  $A$ 's morning. Recall that  $r_c$  is the interest rate for collateral. We have already seen the analysis from the side of the borrowing country; let us therefore consider the process from the side of the lenders:

$$\max_{h,q,y \geq 0} -h + (1-n)u(q) - ny + \beta(1+r_c)h - \beta(1-n)(1+r_b)q + \beta n(1+r_s)y$$

Now the conditions are:

$$\begin{aligned} 1 - \beta(1+r_c) &= \lambda\alpha^{-1} \\ (1-n)(u'(q) - \beta(1+r_b)) &= \lambda(1+r_b) \\ \beta(1+r_s) &= 1 \end{aligned}$$

And they reduce to

$$u'(q) = \left( \beta + \alpha \frac{1 - \beta(1+r_c)}{1-n} \right) (1+r_b)$$

Again assume competition in the market for collateralized lending, so  $r_b = r_s$ . Now in equilibrium, all of  $A$ 's collateral will be lent to  $B$  (there is zero opportunity cost of doing so while balances are idle overnight). This collateral will buy the same amount of goods in each country. So, provided that country  $B$  produces no collateral of its own,  $q$  is identical in each country and  $r_c$  is determined by this fact:

$$\left( \beta + \alpha \frac{1 - \beta(1+r_c)}{1-n} \right) (1+r_b) = \left( \beta + \alpha \frac{\beta r_c}{1-n} \right) (1+r_b)$$

or simplifying

$$\frac{1 - \beta}{2\beta} = r_c.$$

Note at this interest rate, country  $B$  prefers borrowing collateral to making it on its own. Furthermore, at this interest rate, the amount consumed in each country is greater than it was before trade in collateral was instituted.

#### 4.1 Implications for Monetary Authority

Instituting this private international payment arrangement reduces the cost of making payments, and draws business away from the public system. First, focus on country  $A$ 's public system, assuming that country  $B$  just uses the private system. Before the international arrangement, the decision to use public money required

$$\frac{1 + i_\ell}{1 + i_d} \gamma < \alpha.$$

Now it requires the more stringent condition:

$$\frac{1 + i_\ell}{1 + i_d} \gamma < \alpha \frac{1 - \delta(1 + r_c)}{1 - \delta}$$

the new factor stems from the fact that collateral moved into the private system earns its own interest overnight. For the example at hand this simplifies to

$$\frac{1 + i_\ell}{1 + i_d} \gamma < \frac{\alpha}{2}. \tag{5}$$

Allowing each dollar of collateral to buy twice as many units as before, in effect cuts the haircut in half.

The considerations are nearly symmetric for country  $B$ . If country  $A$  produces private collateral, and if the collateral can be used in either the private or public system, the analysis from the previous section holds. The crucial issue is whether the collateral can be returned to country  $A$  before it is needed there. If the public system, for example, retains the collateral until period  $2'$ , beyond the consumption date in country  $A$ , then this doubles its cost relative to the private system, and again the condition (5) determines whether  $B$ 's public system is used. Note, moreover, the potential interactions: suppose timing is such that neither public system allows the use of the other country's private collateral, but the private systems do allow for collateral to be shared. Start from public system costs such that consumers in each country prefer to use the public system. Then let the costs in one country rise. Eventually consumers in that country prefer to adopt the private system, sharing collateral with agents in the other country. As a result, the public system in the other country loses its own customers, despite having made no changes in its own charges.

In summary, there are several possible public responses to an internationalization of the private system (in addition, of course, to regulations prohibiting it). Attempting to extend the use of public payments internationally will only be effective if the costs of doing so are made competitive with the private system costs.<sup>3</sup> There are essentially two ways that the costs can be reduced: one is to allow agents to redeem their publicly pledged collateral for use overnight (for example, through arrangements making it easier to transfer collateral from one system to another) and the other is to reduce the time necessary for money to be deposited in order to receive the interest—in other words, to allow additional use of idle deposits.

## 5 Literature

The Berentsen Monnet model can be regarded as a formalization of the ideas of Woodford (2000 et seq.) about conducting monetary policy in a world with no outside money. The macroeconomic role of money as a medium of exchange has also been explored in numerous cash-in-advance models; in most of them, however, there is no flexibility in the use of publicly provided cash in payment for so-called cash goods. A recent partial exception is Sauer (2008), which examines the trade in which investors can sell illiquid shares or liquidate assets in order to trade by making payments on a goods market. In his model the central bank can prevent this liquidation by entering a repo market.

The issue of private competition with public payments arrangements is, of course, not new. In an important early paper Wallace (1983) argues, in the context of retail payment systems, that the only reasons that U.S. government issued interest bearing securities do not replace non-interest bearing Federal Reserve Notes as a transaction medium are their non-negotiability (in the case of savings bonds) and limitation to large denominations (in the case of treasury bills). But private intermediaries could solve the latter problem in particular, and make a profit, by establishing narrow banks which hold large value treasuries and issue small denomination, riskless private notes suitable for payment. The lack of such notes in the U.S. is clearly due to legal restriction (notably, in Scotland, such legal restrictions are still not in place, and commercial banks do issue their own circulating notes). In an intriguing footnote (p.4), Wallace asks if checking accounts might in effect play the same role. He then states that “interest ceilings, reserve requirements, zero marginal-cost check clearing by the Federal Reserve and the failure to tax income in the form of transaction services ... explain the way checking account services have been priced.” In the context of retail banking in the U.S. nowadays, it is hard to argue that any of these considerations make a significant difference. Thus the following sentences of the footnote become the relevant ones: “In the absence of these forms of government interference, most observers predict that checking accounts would pay interest

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<sup>3</sup>China has made highly publicized calls for the replacement of the dollar as the world's settlement currency. It has also made changes to extend the use of its currency and payments arrangements internationally, see Chen, Peng and Shu (2009) for an overview of the effects.

at the market rate with charges levied on a per transaction basis”—a prediction that seems largely to have come true.<sup>4</sup>

But then, in Wallace’s view, provided the public and private arrangements have the same ability to effect payments, an open market operation which reduces the available reserves of treasury bills to commercial banks and substitutes central bank money simply shifts payments services from private to public arrangements, without affecting interest rates, prices or economic activity. This is equivalent to our arrangement in which  $\alpha = \gamma$  and interest rates are nil. Wallace assumes, unlike us, that the government has the possibility of restricting the payments in the system through legal requirements. On the other hand he assumes that the government system is constrained not to incur losses. Under these circumstances, there is an upper bound on the interest rate on default free securities when they co exist with non interest bearing government currency.

Sargent and Wallace (1982) use the overlapping generations framework of Samuelson (1958) to examine the “real-bills doctrine.” In their framework, a fiat currency can compete with private credit instruments. Differences in endowments in alternating generations lead to a natural variation in relative prices of consumption good in adjacent periods. If fiat money and private lending co-exist, then the return on the two must be the same, that is, the nominal interest rate on lending must be zero. When a monetary equilibrium exists there are a continuum of equilibria in general, each consistent with a different initial value of a unit of fiat money. Monetary equilibria exist as long as the population is not “too impatient.” In all of these monetary equilibria but one, the value of money goes asymptotically to zero. In the remaining equilibrium, the value of money remains stationary, fluctuating with the periodicity of endowments; goods prices and money stock are positively correlated. (In addition there is always a nonmonetary equilibrium, in which private borrowing and lending occurs, but money does not effect intergenerational changes.) Sargent and Wallace then consider a restriction so that some households cannot engage in private lending (because of a minimum restriction on the size of privately issued securities), forcing them to hold government issued securities. If these securities have lower return than private securities in equilibrium, rich savers hold the private securities, and the difference in returns implies suboptimal equilibria, despite the fact that by constraining the poor lenders from the market, price fluctuation can be eliminated. Sargent and Wallace argue that use of government borrowing at low levels will undo the restriction on small bills.

Goodhart (2000) considers the role of central bank in a world where electronic payments have become dominant. He has two arguments in favor of the continuing importance of the central bank: the first is that currency and electronic moneys are imperfect substitutes, particularly with regard to privacy. The second, which he contrasts to “free banking” approaches of the papers described above, is that a central bank, as a bank for a government, is able to run losses financed by the government’s tax levying powers. Using the govern-

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<sup>4</sup>More questionable, however, is Wallace’s view, that this effectively puts checking accounts on the “non-cash” side in inventory models of money demand.



ment's deep pockets, the central bank can always wrest control of the money supply from the private provider by standing ready to engage in loss-making open market operations. The public's knowledge of the bank's power to do so, means that in fact these activities do not need to be carried out much of the time; instead the bank can engage in "open-mouth" operations. Goodhart has in mind the exchange of central bank notes for government debt, or possibly the purchase of private bank debt. However, as we have seen, in a world where provision of private bank debt is only constrained by the availability of collateralizable assets the crucial determinant of the power of a central bank to restrict the money supply is the elasticity of the supply of collateralizable assets.

The issue of the role of cross-border collateral has been examined in several papers by central bankers. Manning and Willison examine cross-country provision of collateral, when collateral is expensive, banks engage in activity in multiple countries, and delay in payment is costly. They show that in many circumstances permitting cross-border collateral induces banks to increase the pool of collateral available for backing payments. This becomes important in the case where there is uncertainty in the overall demand for payment.

## 6 Conclusions

Private payments systems are commonly described as "piggy-backing" on public systems: while they may engage in independent activity during the day, at the end of the day, the final settling-up is generally entrusted to a public large value payment system. However this view is misleading: in fact private and public systems are in competition, and in that competition, the cost of collateral is a major consideration.

This paper has examined the implications of private "round-the-clock" systems for the competitiveness of public payment arrangements. It has argued that the ability to use collateral on the other side of the world during down time in a home country reduces the cost of running the private system, and puts further constraint on the ability of public systems to remain competitive, further limiting the ability of a public authority to run a restrictive monetary policy.

## 7 Appendix: Variable Collateral Requirements for Private Loans

So far we have not addressed the issue of the source of the collateral requirements. While public requirements are largely a policy variable, private requirements depend on reliability, information, incentives and enforcement considerations. In practice, payments arrangements have collateral requirements which vary with the identity of the participants and the amount of their participation. Private systems place a variety of restrictions on membership and collateral requirements for participants, including differentiation between various classes of participants. As a result, only the larger and (presumably) better collateralized institutions participate in the private systems directly.

The important consequence is that changes in the collateral requirements of the public system yield continuous responses in the use of the private system. For example, increased collateral requirements in the public system induce a move to the private system by some institutions who would formerly have found the private requirements too stringent.

Suppose that in order to borrow at date 1 an amount equivalent to  $bP_2$  in nominal value at date 2, it is necessary to post an amount of collateral equal to  $\kappa\alpha(b)$ , where  $\alpha$  is an increasing convex function of  $b$ , satisfying the Inada conditions. The problem becomes

$$\max_{h,q,y,M_3,x,c} -h + (1-n)u(q) - ny + E\beta(x+c+b + \frac{M_3}{P_2})$$

subject to

$$(P_0(h-c-b) + F_1y)(1+i_d) - T \geq P_2x_s + M_{3s} \quad (6)$$

$$(P_0(h-c-b) - F_1q)(1+i_\ell) - T \geq P_2x_b + M_{3b} \quad (7)$$

$$(P_0(h-c-b) - F_1q) + ((1+i_\ell)^{-1}\gamma^{-1}c + (1+i_d)^{-1}\kappa^{-1}\alpha^{-1}(b))P_2 \geq 0 \quad (8)$$

where we have used the condition (4) for the seller to be indifferent between the two methods. (Here  $T$  denotes the lump-sum tax/subsidy from the government's budget balance condition).

The first order conditions

$$-1 + P_0(\lambda_1(1 + i_d) + \lambda_2(1 + i_\ell) + \lambda_3) = 0$$

$$(1 - n)u'(q) - (\lambda_2(1 + i_\ell) + \lambda_3)F_1 = 0$$

$$-n + \lambda_1 F_1(1 + i_d) = 0$$

$$n\beta = \lambda_1 P_2$$

$$(1 - n)\beta = \lambda_2 P_2$$

$$\frac{n\beta}{P_2} \leq \lambda_1; \quad M_{3s} \geq 0$$

$$\frac{(1 - n)\beta}{P_2} \leq \lambda_2; \quad M_{3b} \geq 0$$

$$\beta - P_0(\lambda_1(1 + i_d) + \lambda_2(1 + i_\ell) + \lambda_3) + \lambda_3(1 + i_\ell)^{-1}\gamma^{-1}P_2 \leq 0; \quad c \geq 0$$

$$\beta - P_0(\lambda_1(1 + i_d) + \lambda_2(1 + i_\ell) + \lambda_3) + \lambda_3(1 + i_d)^{-1}\kappa^{-1}[(\alpha^{-1})'(b)]P_2 = 0$$

simplify to

$$P_0(n\beta(1 + i_d) + (1 - n)\beta(1 + i_\ell) + \lambda_3 P_2) = P_2$$

$$(1 - n)(u'(q) - \Delta) = \frac{\lambda_3 P_2}{\beta(1 + i_d)}$$

$$F_1 = \frac{P_2}{\beta(1 + i_d)}$$

$$(1 + i_\ell)(1 - \beta) \geq \gamma^{-1}\lambda_3 P_2; \quad c \geq 0$$

$$(1 + i_d)(1 - \beta) = \kappa^{-1}[(\alpha^{-1})'(b)]\lambda_3 P_2$$

$$\frac{\Delta q}{R\beta} = \gamma^{-1}c + \Delta\kappa^{-1}\alpha^{-1}(b).$$

There are two cases to consider:  $c > 0$ :

$$P_2/P_0 = n\beta(1 + i_d) + (1 - n)\beta(1 + i_\ell) + \gamma(1 + i_\ell)(1 - \beta)$$

$$u'(q) = \Delta\left(1 + \frac{\gamma(1 - \beta)}{\beta(1 - n)}\right)$$

$$F_1 = \frac{P_2}{\beta(1 + i_d)}$$

$$\kappa^{-1}[(\alpha^{-1})'(b)]\gamma\Delta = 1$$

$$c = \frac{\alpha^{-1}(b)}{[(\alpha^{-1})'(b)]} - \frac{\gamma\Delta q}{R\beta}$$

and  $c = 0$  :

$$\begin{aligned}
P_0(n\beta(1+i_d) + (1-n)\beta(1+i_\ell) + \lambda_3 P_2) &= P_2 \\
(1-n)(u'(q) - \Delta) &= \frac{\lambda_3 P_2}{\beta(1+i_d)} \\
F_1 &= \frac{P_2}{\beta(1+i_d)} \\
(1+i_\ell)(R^{-1} - \beta) &\geq \gamma^{-1} \lambda_3 P_2; \quad c \geq 0 \\
(1+i_d)(R^{-1} - \beta) &= \kappa^{-1} [(\alpha^{-1})'(b)] \lambda_3 P_2 \\
\frac{\Delta q}{R\beta} &= \gamma^{-1} c + \Delta \kappa^{-1} \alpha^{-1}(b)
\end{aligned}$$

In other words, if  $\gamma$  exceeds a critical level, public means of payment are not used. As  $\gamma$  falls below that level, use of private means of payment shrinks and use of public means increases.

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