Channel Structure, Cross Sales, and Vertical Integration In a Multi–Channel Distribution System

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Abstract

Consider two manufacturers, each producing a single substitutable product. In any geographical area, there are two retail outlets through which the products are sold to the end consumers. Each of the two retail outlets could be privately owned (i.e. a franchised outlet) or owned by the manufacturer (i.e. a vertically integrated company store). Each manufacturer makes vertical integration decision in its best interest. However, a manufacturer incurs a fixed cost to establish an integrated channel. A manufacturer is not restricted to using only one retail outlet. We define cross sales to be the situation where at least one retail outlet sells both products. The objective of our paper is to analyze the cross sales phenomenon. We propose a game theoretic model to accomplish this objective. Our model provides answers to the following questions. When will cross sales occur in the multi–channel distribution system described above? How does the nature of competition influence cross sales? What are the equilibrium channel configurations under the scenario described above? We show that cross sale will happen in quantity competition or in a capacity constrained price competition. However, cross sales may never happen in a pure Bertrand price competition. We provide interesting and intuitive explanation for the cross sales phenomena, which to the best of our knowledge, have not been studied in the context of channel design before. We also identify various interesting economics of cross sales.

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Keywords: Channels of Distribution, competition, Channel Design, Cross Sale, Vertical Integration, Supply Chains
1. INTRODUCTION

Selecting an appropriate structure for distribution channel is a major strategic concern for any firm with a product to sell. As competition in the market place intensifies and new technologies evolve, the firms are taking fresh looks at their distribution channels to squeeze out inefficiencies. As a result, many types of distribution channels have come into being ranging from the traditional retail channel to the direct sales model. The contracting arrangement between a manufacturer and a retailer also varies from channel to channel. Some manufacturers use an exclusive arrangement with a retailer while others use multiple retail channels for distribution under both exclusive and non-exclusive channel arrangement.

The objective of this paper is to study cross sales and vertical integration issues simultaneously from a channel design perspective. Following McGuire and Staelin (1983), we consider two manufacturers, each producing a single substitutable product. In any geographical area, there are two retail outlets through which the products are sold to the end consumers. Each of these two retail outlets could be privately owned (i.e. a franchised outlet) or owned by the manufacturer (i.e. a vertically integrated company store). However, unlike McGuire and Staelin (1983), we do not necessarily restrict a retail outlet to selling product from a single manufacturer. Therefore, besides making the vertical integration decision, a manufacturer must decide whether to sell her product exclusively. We define cross sales to be the situation where at least one retailer sells both products. We assume that a manufacturer incurs a fixed cost to establish an integrated channel. Using a game theoretic model we seek answers to the following questions. When will cross sales occur in the multi-channel distribution system described above? How does the nature of competition affect cross sales? What are the equilibrium channel structures? While answering these questions, we alongside develop and identify interesting economics about cross sales.

We observe many different types of channel arrangements in practice. The most commonly observed channel arrangement is the cross sales defined above. For example, department stores, grocery stores, and retailers like Wal-Mart, K-Mart, and Target sell multiple brands of the same product. The direct distribution channel of Dell Computer is the typical example of a manufacturer-owned and vertically integrated distribution channel. Interestingly, Dell computer has recently started selling competing brand of printers in the US market alongside the Dell
printers through its website, which fits our definition of cross sales. A recent search of the Dell website for professional black and white laser printers by the authors found Brother, Lexmark, Samsung, and Xerox brands of laser printers along with Dell laser printers. Similarly, a search for the color inkjet printers yielded Canon, Epson, and Lexmark printers alongside Dell inkjet printers. As pointed out by McGuire and Staelin (1983), the exclusive arrangement between a manufacturer and a retailer is most commonly observed in industries such as gasoline, soft drinks, fast food chains etc. In addition, exclusive arrangement is also observed in manufacturer-distributor relationships in various industries ranging from chemicals and paper to appliances and Beer. For example, Hercules Inc. is the exclusive distributor for GE Specialty Materials’ water treatment solutions to the paper industry (Challener, 2003); Diego Suarez is the exclusive distributor of Coors brand of beers in Puerto Rico (Allio & Allio, 2002). Estee Lauder created a new company division, called BeautyBank, to develop cosmetic products exclusively for Kohl’s department stores (Merrick & Beatty, 2003).

Using stylized models, we examine the issues of cross sales and vertical integration simultaneously in presence of fixed costs. Our model allows wholesale price discrimination by the manufacturers. We use quantity competition as well as a Bertrand-like price competition to model the strategic interactions between the manufacturers and the retailers and show the existence of a unique equilibrium outcome. We provide an intuitive explanation for cross sale and for the existence of each of the channel structures described above. We show that cross sale might happen in a price competition even in absence of retail differentiation. We characterize the conditions under which cross sales will happen in the integrated as well as in the decentralized channels. When a decentralized manufacturer uses a common wholesale price for both the retailers, we show that cross sale is more likely to happen in each channel configuration when the two products are less substitutable. On the other hand, when a decentralized manufacture is allowed wholesale price discrimination, cross sale will happen even when the products are highly substitutable. We provide an intuitive explanation for these results. In addition, in a mixed channel configuration, the integrated manufacturer will not like to sell her product through the decentralized retail outlet when the decentralized manufacturer uses a common wholesale price. However, the integrated manufacturer will sell her competitor’s product along with her own product through her integrated channel under certain conditions. This might explain the presence of the competitors’ products on Dell’s website and the absence of Dell products in a competitor’s
distribution channel in the US.

Our work relates to channel design and channel competition literatures in marketing. The classic work of McGuire and Staelin (1983) looks at channel design issues under exclusive arrangement between a manufacturer and a retailer. Choi (1996), on the other hand studies channel competition where the channel structures are assumed exogenous. Our work combines these two streams of literature by observing cross sale to be an equilibrium outcome of a game between two manufacturers and two retailers. The main contribution of our work is two-fold. First, we provide an explanation of the cross sales phenomenon by combining the vertical integration decision with the cross selling decision. To the best of our knowledge, this has not been studied before. Second, and on the theoretical side, we establish that cross sales may not happen in a pure Bertrand price competition. However, cross sale will happen in quantity competition or in a capacity constrained price competition. Our model also underscores the fact that cross sale is an equilibrium outcome even in the absence of retail differentiation.

The remainder of this paper is organized as follows. The next section provides a review of the related literature. We describe our model in Section 3. The analyses and results are presented in Section 4. Section 5 summarizes and concludes the paper.

2. LITERATURE REVIEW

Channel structure, competition, and coordination have been studied in various forms both in marketing and in operations literature beginning with the classic work of McGuire and Staelin (1983). McGuire and Staelin (1983) study the effect of product substitutability on Nash equilibrium channel configurations involving two manufacturers and two retail outlets. Each manufacturer produces one product, uses an exclusive arrangement with a retail outlet to distribute the product, and is allowed to vertically integrate with the downstream retailer. They show that for low degrees of substitutability, each manufacturer will distribute her product through an integrated channel; while for high degree of substitutability, each manufacturer will use either a decentralized or an integrated distribution channel. Other researchers studying similar problems are McGuire and Staelin (1986), Moorthy (1988), and Coughlan and Wernerfelt (1989). Gupta & Loulou (1998) model manufacturers’ investments in cost reduction efforts and their impact on vertical integration decisions in an oligopolistic market with two manufacturers.
producing substitutable products. Choi (1991) studies the channel competition problem where two manufacturers producing substitutable products distribute their products through a single common retailer. Choi (1996) considers price competition under product and store differentiations for various channel structures. One of his channel structures is similar to our channel configuration for cross sales. However, the focus of our work is fundamentally different from that of Choi. Choi assumes the channel structure to be given and studies price competition between retailers. Channel design issues, thus, are not considered by him. In contrast, channel configurations and cross sales in our model arise as an equilibrium outcome. Our model, thus, explains when and why cross sale arises. In addition, Choi’s analysis is restricted to the decentralized channel only. Moner-Colonques et al. (2004) also study channel competition under product substitutability. However, unlike our work, they do not consider channel design issues or the possibility of an integrated channel competing with a decentralized channel. Neither do they allow the possibility of wholesale price discrimination by a manufacturer.

A parallel stream of literature (Jeuland and Shugan, 1983; Shugan, 1985; Moorthy, 1987) focuses on single manufacturer-retailer dyads and analyzes channel coordination incentives in relation to various contractual arrangements as well as manufacturer and retailer efforts in improving quality or increasing demand. Channel coordination has also received considerable attention in Operations Management literature. A vast majority of this literature assumes a single manufacturer and a single retailer. The typical analytical approach is to study the source of inefficiency (often related to double marginalization), and mechanisms for achieving coordination. Cachon (2004) provides a review of this literature. Tsay and Agrawal (2004) provide a review of recent research in modeling channel conflict and coordination. However, none of these studies consider the issue of channel design. The distribution channel structures are treated as given in these models.

3. MODEL

We consider a multi-channel distribution system involving two manufacturers, denoted by $M_1$ and $M_2$, each producing a single substitutable product. In any given geographical area, there are two retail outlets $R_1$ and $R_2$. Each retail outlet could be privately owned (a decentralized franchised outlet, for example) or owned by a manufacturer (i.e. a vertically integrated company store). Each manufacturer gets to make the vertical integration decision about her channel, taking
into account her competitor’s action. Our primary goal in this paper is to develop insights on the
cross sale phenomenon commonly observed in practice. We define cross sale to be the situation
where at least one retailer sells both products. Thus, in addition to making vertical integration
decisions, each manufacturer gets to decide which retailer(s) to use. In addition, we assume that a
manufacturer incurs a fixed cost $F$ to establish a vertically integrated channel. This fixed cost
might represent the cost of establishing a physical store, investment in information technology,
and the cost of coordination. No such cost is incurred by a manufacturer in a decentralized
channel. We also assume that $F$ is common to both channels. Having different fixed costs for the
two channels does not alter our conclusions. In any decentralized channel, each party maximizes
its own profit; while a manufacturer maximizes the total channel profit in an integrated channel.
To simplify the exposition of our model, the unit production cost at the manufacturer-level, and
the unit handling cost at the retailer-level are normalized to zero.

We model the scenario described above as a four-stage extensive game. At stage 1 of the
game, each manufacturer decides whether to vertically integrate her downstream retail store.
Consequently, there are four subgames at stage 2, each of which is associated with a channel
configuration after integration/decentralization decisions are made. We will denote these
subgames and the associated channel configurations by (D, D), (I, D), (D, I), and (I, I), where (D,
D) indicates that both channels are decentralized, (D, I) indicates that channel 1 is decentralized
while channel 2 is integrated, etc.

After the first stage, each manufacturer learns whether the competing manufacturer has
chosen integration or decentralization. At stage 2, each manufacturer, integrated or not,
determines the retail outlet(s) she will use. A manufacturer can use either of the two retail outlets
or both. However, we assume that if an integrated manufacturer decides to use an exclusive
arrangement with a retail outlet, then she must use her own integrated outlet. This assumption
makes sense as the manufacturer incurs a cost to establish an integrated channel in our model.
For example, in the (D, D) channel, each manufacturer might use $R_1$, or $R_2$, or both $R_1$ and $R_2$.
The strategy space for each manufacturer is denoted by $\{1, 2, 12\}$ where 1 represents that a
manufacturer chooses $R_i$, etc. On the other hand, the strategy space of the integrated
manufacturer is $\{2, 12\}$ in the (D, I) channel and $\{1, 12\}$ in the (I, D) channel. The strategy space
for the decentralized manufacturer in the mixed structures is $\{1, 2, 12\}$. In (I, I) structure, the
respective strategy spaces for the two manufacturers are $\{1, 12\}$ and $\{2, 12\}$. Consider the (D, D)
configuration described earlier. Since each manufacturer has 3 different strategies to choose from, there are 9 subgames at stage 3, denoted by (D1, D1), (D1, D2), (D1, D12), (D2, D1), (D2, D2), (D2, D12), (D12, D1), (D12, D2), and (D12, D12). Similarly, the 6 subgames for (D, I) arrangement are (D1, I2), (D1, I12), (D2, I2), (D2, I12), (D12, I2), and (D12, I12). Lastly, (I, I) has four subgames: (I1, I2), (I1, I12), (I12, I2), and (I12, I12).

At the end of stage 2, each manufacturer knows the channel structure of the competing distribution channel. At stage 3, each manufacturer sets her wholesale price where applicable. We allow each manufacturer to set different wholesale prices for the two retail outlets as well as to set a common wholesale price for both the retail outlets. No wholesale price is involved in an integrated channel.

At stage 4, having known the channel structure and the wholesale price(s), each retail store, integrated or not, competes over quantity. In other words, the retail prices are determined by quantity competition. Let \( Q_{ij} \) be the order quantity of product \( i \) by retail store \( j, i, j = 1,2 \) and let \( Q_i = Q_{i1} + Q_{i2} \) be the total quantity of product \( i \) ordered. We assume that both manufacturers produce the exact quantities which meet the orders placed by both retail stores. At each stage of the game, the decisions are assumed to be made simultaneously and non-cooperatively. Each firm seeks to maximize its profit. Furthermore, we assume no information asymmetry exists among the players.

### 3.1. The Demand Functions and Solution Procedures

Recall that \( Q_i = Q_{i1} + Q_{i2} \) is the total quantity of product \( i \) ordered. The demand function we use is as follows:

\[
p_i = 1 - Q_i - \theta Q_j, \quad i, j = 1,2, \quad j \neq i,
\]

where \( p_i \) is the retail price of product \( i \), and \( \theta \in [0,1] \) is the substitutability between the two products. A value of \( \theta = 0 \) implies that the two products are completely different, while the products become maximally substitutable as \( \theta \) approaches 1.

We let \( w_{ij} \) denote the wholesale price paid by the retailer \( R_j \) pays for product \( i \). We will simply use the notation \( w_i \) for \( M_i \)'s wholesale price when she chooses to have a common
wholesale price for both the retailers. When channel \( i \) is decentralized, \( M_i \) chooses \( w_{ij} \) or \( w_i \), while \( R_i \) chooses \( Q_{si} \) and \( Q_{2i} \). Let \( \Pi_i^s \) denote the profit of party \( l \) under channel configuration (or strategy profile) \( s \), where, \( s = (D1, D2), (I1, D12), (D1, I12), (I12, I2), \) etc.; and \( l = M_i, R_i, i = 1,2 \). In addition, we use the notation \( \pi_i^s \) to denote the gross profit of party \( l \) under channel arrangement \( s \) before accounting for the fixed cost \( F \). Throughout our paper, the notation \((D1, I12), (I, D), \) etc. are interchangeable with the notations “D1I12”, “ID”, etc. Finally, we denote the optimal value of any quantity with superscript of “*”.

We are now ready to describe the solution procedure for stages 1-3 of our model. Consider the case where a manufacturer sets a common wholesale price for both retail outlets first. For the reasons of brevity, we will only describe the solution procedure for the \((D12, I2)\) channel. The solutions for other configurations will follow a similar procedure. We solve for the game by working backwards. Note that no wholesale price is involved in the integrated channel. Thus, given the common wholesale price \( w_1 \), the two retail stores maximize their profits independently. Since \( M_2 \) decides to sell her product exclusively, we have \( Q_{21} = 0 \). The problem of the integrated manufacturer and the decentralized retailer are as follows:

\[
\begin{align*}
Max_{Q_{11}} & (p_1 - w_1)Q_{11}, \\
Max_{Q_{12}, Q_{22}} & (p_1 - w_1)Q_{12} + p_2 Q_{22} - F,
\end{align*}
\]

where, \( p_1 = 1 - (Q_{11} + Q_{12}) - \theta Q_{22} \) and \( p_2 = 1 - Q_{22} - \theta (Q_{11} + Q_{12}) \). By solving the two first-order conditions, we derive the best response functions:

\[
Q_{ik} = Q_{ik}(w_1), i, k = 1,2.
\]

Therefore, we can write the manufacturers’ profit functions as follows:

\[
\begin{align*}
\Pi_{M_i}^{D12/2} &= w_i [Q_{11}(w_1) + Q_{12}(w_1)] \\
\Pi_{M_i}^{D12/2} &= (p_1 - w_1)Q_{12}(w_1) + p_2 Q_{22}(w_1) - F
\end{align*}
\]

By optimizing the manufacturer \( i \)'s profit (equations 5 and 6) with respect to \( w_i \), we can find the optimal wholesale prices and calculate the optimal retail prices, optimal quantities, and optimal profits rather easily. This sequential solution guarantees that the optimal solution we derived is Nash equilibrium.
Next consider the case where a manufacturer sets different wholesale prices for the two retail outlets. We will again use the (D12, I2) channel to describe our solution procedure. In this scenario, \( M_1 \) sets \( w_{11} \) for \( R_1 \) and \( w_{12} \) for \( R_2 \). The solution procedure is then adjusted as follows:

\[
\begin{align*}
\max_{Q_{11}} (p_1 - w_{11})Q_{11},
\end{align*}
\]

\[
\begin{align*}
\max_{Q_{12}, Q_{22}} (p_1 - w_{12})Q_{12} + p_2 Q_{22} - F,
\end{align*}
\]

\[
\begin{align*}
\Pi_{M_1}^{D12I2} &= w_{11}Q_{11}(w_{11}, w_{12}) + w_{12}Q_{12}(w_{11}, w_{12}),
\end{align*}
\]

\[
\begin{align*}
\Pi_{M_2}^{D12I2} &= (p_1 - w_{12})Q_{12}(w_{11}, w_{12}) + p_2 Q_{22}(w_{11}, w_{12}) - F.
\end{align*}
\]

Table 1 and 2 summarize the optimal solutions along with the optimal values of all relevant variables for each channel arrangement. Note that we have specified the optimal profits in Table 1 before accounting for the fixed cost \( F \).

4. RESULTS AND ANALYSES

The analyses of our model are presented in this section. Throughout this section, we restrict our analyses to pure strategy equilibriums only. We study the equilibrium channel structure under cross selling and provide an interesting approach to illustrate cross sales. We seek subgame perfect equilibrium of our 4-stage game. Consequently, we need to identify the equilibrium(s) of each stage-2 subgame first. Sections 4.1 through 4.3 accomplish this. In addition, these three sub-sections identify interesting economics about cross sales. We discuss the subgame perfect equilibrium of the entire game in Section 4.4.

4.1 Analysis of the Equilibrium and Cross Sales in the (D, D) Channel

Table 3 describes the payoff matrix for all the possible scenarios associated with the (D, D) subgame.

<table>
<thead>
<tr>
<th></th>
<th>D1</th>
<th>D2</th>
<th>D12</th>
</tr>
</thead>
<tbody>
<tr>
<td>D1</td>
<td>( \pi_{M_1}^{D1D1} ), ( \pi_{M_2}^{D1D1} )</td>
<td>( \pi_{M_1}^{D1D2} ), ( \pi_{M_2}^{D1D2} )</td>
<td>( \pi_{M_1}^{D1D12} ), ( \pi_{M_2}^{D1D12} )</td>
</tr>
<tr>
<td>D2</td>
<td>( \pi_{M_1}^{D2D1} ), ( \pi_{M_2}^{D2D1} )</td>
<td>( \pi_{M_1}^{D2D2} ), ( \pi_{M_2}^{D2D2} )</td>
<td>( \pi_{M_1}^{D2D12} ), ( \pi_{M_2}^{D2D12} )</td>
</tr>
<tr>
<td>D12</td>
<td>( \pi_{M_1}^{D12D1} ), ( \pi_{M_2}^{D12D1} )</td>
<td>( \pi_{M_1}^{D12D2} ), ( \pi_{M_2}^{D12D2} )</td>
<td>( \pi_{M_1}^{D12D12} ), ( \pi_{M_2}^{D12D12} )</td>
</tr>
</tbody>
</table>
By symmetry of the (D, D) subgame, we have \( \pi_M^{D_1D_1^*} = \pi_M^{D_1D_2^*} = \pi_M^{D_2D_1^*} = \pi_M^{D_2D_2^*} \),
\( \pi_M^{D_1D_2^*} = \pi_M^{D_2D_2^*} = \pi_M^{D_2D_1^*} = \pi_M^{D_2D_2^*} \), and
\( \pi_M^{D_1D_1^*} = \pi_M^{D_1D_2^*} = \pi_M^{D_1D_1^*} = \pi_M^{D_1D_2^*} \).

Recall that we allow each manufacturer to set a common wholesale price for both retailers as well as to set different wholesale prices for the two retailers. Propositions 1 and 2 below describe the equilibrium results under these two scenarios.

**Proposition 1:** Consider the (D, D) subgame where each manufacturer sets a common wholesale price for both retailers. In this subgame, (D12, D12) is the only equilibrium if and only if 0 \(\leq\) \(\theta\) \(<\) 0.682; both (D12, D12), (D1, D2), and (D2, D1) are equilibriums if and only if 0.682 \(\leq\) \(\theta\) \(\leq\) 0.909; and (D1, D2) and (D2, D1) are the equilibriums if and only if 0.909 \(<\) \(\theta\) \(\leq\) 1. Furthermore, using a single common retailer is not an equilibrium outcome.

Proof: Since \( \pi_M^{D_1D_1^*} = \pi_M^{D_2D_2^*} \leq \pi_M^{D_1D_2^*} = \pi_M^{D_1D_2^*} \), both (D1, D1) and (D2, D2) are dominated, and thus, cannot be equilibrium. When 0 \(\leq\) \(\theta\) \(<\) 0.682, we have \( \pi_i^{D_1D_2^*} - \pi_i^{D_1D_2^*} < 0 \) and \( \pi_i^{D_1D_1^*} - \pi_i^{D_1D_2^*} < 0 \). Therefore, selling through both retailers is the dominant strategy for both manufacturers, and thus (D12, D12) is the only equilibrium if 0 \(\leq\) \(\theta\) \(<\) 0.682. Now suppose 0.682 \(\leq\) \(\theta\) \(\leq\) 0.909. It’s easy to see that \( \pi_i^{D_1D_1^*} - \pi_i^{D_1D_2^*} \geq 0 \) and \( \pi_i^{D_1D_1^*} - \pi_i^{D_1D_2^*} \leq 0 \). Given \( M_1 \) sells through one retailer, \( M_2 \)’s optimal strategy is to sell through the other retailer, vice versa. Hence (D1, D2) and (D2, D1) are equilibrium. Given \( M_1 \) uses both channels, \( M_2 \)’s optimal strategy is to use both retailers as well, vice versa. Hence (D12, D12) is an equilibrium. Finally, consider 0.909 \(<\) \(\theta\) \(\leq\) 1. If \( M_1 \) uses one retailer only, then using the other retailer only is the dominant strategy for \( M_2 \), vice versa. Hence (D1, D2) and (D2, D1) are equilibrium if 0.909 \(<\) \(\theta\) \(\leq\) 1.

Proposition 1 yields interesting insights. Cross sale will happen in the (D, D) channel when the substitutability is relatively low (0 \(\leq\) \(\theta\) \(<\) 0.682). Cross sale is the equilibrium outcome over a rather wide range of substitutability. When the two products are not very similar, inter-brand competition is weaker and the double marginalization problem is stronger in a (D, D) channel.
Therefore, a manufacturer can benefit from creating intra-brand competition. This gives rise to the decision of cross sales. On the other hand, when the two products are close substitutes, the manufacturers prefer exclusive selling because that can dampen inter-brand competition. Therefore, each manufacturer is better off selling through different retailer exclusively when the two products are highly competitive. Lastly, Proposition 1 states that the mixed arrangements (D12, D1), (D2, D12), etc. cannot be equilibriums. This implies that unidirectional cross sale will not happen in (D, D). Cross sale will either happen to both products or will not happen at all. Proposition 1 further states that using a single common retailer is never an equilibrium outcome.

Using the expressions from Table 1, it is easy to establish that \( w_i^{D1D2*} > w_i^{D12D12*} \) and \( p_i^{D1D2*} > p_i^{D12D12*} \) for all \( \theta \), indicating that both the wholesale and the retail prices are higher when both manufacturers select the exclusive arrangement. It can also be shown that the equilibrium wholesale and retail prices decrease as the substitutability between the two products increases. The results are intuitive. The manufacturers (retailers) are competing with one another directly under cross sale, resulting in a lower wholesale (retail) price. Similarly, as the products become more substitutable, the prices fall because of the increased horizontal competition between the products. The following corollary compares the retailers’ profit under exclusive and cross selling.

**Corollary 1:** The following statements hold in the (D, D) channel where a manufacturer sets a common wholesale price for the retailers, for any fixed value of \( \theta \).

(a) An exclusive arrangement gives rise to higher retailer profit compared to cross selling (i.e. \( \pi_{Ri}^{D1D2*} > \pi_{Ri}^{D12D12*} \)) for \( 0 \leq \theta < 0.202 \). However, for \( 0.202 \leq \theta < 1 \), cross sales gives rise to higher retailer profit than an exclusive arrangement (i.e. \( \pi_{Ri}^{D12D12*} > \pi_{Ri}^{D1D2*} \)).

(b) An exclusive arrangement gives rise to lower manufacturer profit compared to cross selling (i.e. \( \pi_{Mi}^{D1D2*} < \pi_{Mi}^{D12D12*} \)) for \( 0 \leq \theta < 0.510 \). However, for \( 0.510 \leq \theta < 1 \), cross sales gives rise to lower manufacturer profit than an exclusive arrangement (i.e. \( \pi_{Mi}^{D12D12*} < \pi_{Mi}^{D1D2*} \)).

The proof of Corollary 1 follows directly from the expressions in Table 1. Corollary 1 identifies interesting economics of cross sale when both channels are decentralized. When the products under consideration are more differentiated (\( 0 \leq \theta < 0.202 \)), a decentralized retailer prefers
exclusive arrangement over cross selling, while a manufacturer prefers cross sales. For $0.202 \leq \theta < 0.510$, the retailers as well as the manufacturers prefer cross sales over an exclusive arrangement, while for $0.510 \leq \theta < 1$, the retailers prefer cross selling over the exclusive arrangement and the manufacturers prefer the opposite. These results are intuitive. A differentiated product under an exclusive arrangement leads to a monopoly-like situation for the retailers, resulting in higher retailer profit compared to cross sales. The products being differentiated, the manufacturers do not face direct horizontal competition and prefers cross sales. The exact opposite scenario happens when the products are nearly identical. It is interesting to note that for the intermediate values of $\theta$ ($0.202 \leq \theta < 0.51$), both the manufacturers and the retailers prefer cross sale. However, note from Proposition 1 that $(D12, D12)$ is the only equilibrium for $0 \leq \theta < 0.682$. Therefore, for $0.202 \leq \theta < 0.51$, the equilibrium outcome is also the preferred outcome for both the manufacturers and the retailers. However no such claim can be made for $0 \leq \theta < 0.202$ or for $0.51 \leq \theta < 0.682$. By Proposition 1, $(D12, D12)$, $(D1, D2)$, and $(D2, D1)$ are all equilibrium outcome for $0.682 \leq \theta < 0.909$, and $(D1, D2)$ and $(D2, D1)$ are the only equilibrium outcomes for $0.51 \leq \theta < 1$. In each of these two regions, the retailers prefer cross sale and the manufacturers prefer exclusive arrangement to avoid direct horizontal competition. However, the equilibrium outcome will match the interest of only one party.

It can also be shown that the retailer profits decrease with $\theta$ in the $(D1, D2)$ and $(D2, D1)$ configuration while those increase with $\theta$ in the $(D12, D12)$ configuration. The later part of the above result is similar to that of Choi (1996) who concludes that horizontal product differentiation hurts a retailer under cross sale. However, unlike Choi’ model which exogenously assumes the existence of cross sale, the $(D12, D12)$ channel structure in our model is an equilibrium outcome, indicating that the existence of this channel is indeed consistent with the economic theory.

**Proposition 2:** Consider the $(D, D)$ subgame where each manufacturer sets a different wholesale price for both retailers. In this subgame, $(D12, D12)$ is equilibrium for all $\theta \in [0,1]$; $(D1, D2)$ and $(D2, D1)$ are equilibriums if and only if $0.765 \leq \theta \leq 1$. Furthermore, using a single common retailer is not an equilibrium outcome.
The proof of Proposition 2 is similar to that of Proposition 1, and thus is omitted. It is important to highlight the differences between propositions 1 and 2. When a manufacturer is allowed to set different wholesale prices for different retailers, unlike Proposition 1, bi-directional cross sales is an equilibrium for any given value of $\theta \in [0,1)$. The result is intuitive. The wholesale price discrimination allows a manufacturer to dampen the inter-brand competition when the substitutability between the two products is high. As a result, cross sales continues to be the equilibrium outcome even when the two products are highly substitutable. Moreover, when all of the channels, (D12, D12), (D1, D2), (D2, D1), are equilibriums, the exclusive arrangements (D1, D2) and (D2, D1) channels yield higher equilibrium profits for both manufacturers than the (D12, D12) channel.

It is also worth discussing how wholesale price discrimination might affect the profits of the manufacturers and retailers. Note that unidirectional cross sale is not an equilibrium outcome in the (D, D) channel. As a result, the only equilibrium channel configuration where a manufacturer might engage in wholesale price discrimination is the (D12, D12) channel (note that (D12, D2) is not an equilibrium outcome). Using Tables 1 and 2 it is easy to establish that engaging in wholesale price discrimination does not change the optimal profit of any party (either a manufacturer or a retailer) in the (D12, D12) channel. The result is intuitive. When both retailers sell both products, any threat of wholesale price discrimination by a manufacturer will result in retaliation by a retailer through his order quantity. As a result the optimal wholesale prices are same as those when no wholesale price discrimination is allowed.

4.2 Analysis of the Equilibrium and Cross Sales in the (I, D) Channel

The subgames (D, I) and (I, D) are symmetric. Hence, it is sufficient for us to study subgame (I, D) only. By our assumption, $M_1$ will always use its integrated channel. In addition, she will name a wholesale price should she desire to engage in cross sale. The decentralized manufacturer, $M_2$, on the other hand, might name a common wholesale price for the two retailers or two different wholesale prices for the two retailers. Our calculations indicate that when the decentralized manufacturer sets a common wholesale price, the optimal order quantities from the retail outlets under cross sales are not necessarily positive for all values of $\theta$. Fortunately, we are able to establish that this fact does not affect our equilibrium results later on. Lemma 1 below summarizes the situations where optimal quantities turn negative in subgame (I, D).
Lemma 1: Consider the \((I, D)\) subgame where the decentralized manufacturer sets a common wholesale price for both the retail outlets. \(Q_{12}^{12D2*} > 0\) for \(\theta < 0.641\); \(Q_{21}^{11D12*} > 0\) for \(\theta < 0.564\); \(Q_{21}^{11D12*} > 0\) for \(\theta < 0.591\), and \(Q_{12}^{12D12*} = 0\) for all \(\theta\).

The proof of Lemma 1 follows directly from Table 1. Lemma 1 states that \(Q_{12}^{12D12*} = 0\), i.e., The optimal order quantity of the integrated manufacturer’s product from the decentralized retailer is always zero. It implies that at equilibrium no actual trade will happen between the integrated manufacturer and the decentralized retailer because the named wholesale price is so high that the optimal order quantity from the decentralized retailer is zero. Knowing that she will never receive any order at her optimal wholesale price, why does the integrated manufacturer still commit to cross selling at stage 2 of the game? A possible explanation is that by doing so, the integrated manufacturer can influence the optimal behaviors of the decentralized retailer and the decentralized manufacturer.

From Lemma 1, we know that none of the optimal quantities will be negative until \(\theta > 0.564\). On the other hand, when \(\theta > 0.50\), the dominant strategy for \(M_1\) is to sell the product through her own integrated channel only. Knowing that \(M_1\) will use her own integrated channel only, \(M_2\)'s optimal strategy is to sell through \(R_2\). In other words, \((I1, D2)\) is the only equilibrium if \(\theta > 0.5\). Therefore, the equilibrium of the subgame \((I, D)\) will not be affected by negative optimal quantities described in Lemma 1. Table 4 shows the payoff matrix for \((I, D)\) subgame.

**Table 4: Payoff Matrix for \((I, D)\) subgame**

<table>
<thead>
<tr>
<th></th>
<th>D1</th>
<th>D2</th>
<th>D12</th>
</tr>
</thead>
<tbody>
<tr>
<td>I1</td>
<td>(\pi_{M1}^{11D1*} - F, \pi_{M2}^{11D1*})</td>
<td>(\pi_{M1}^{11D2*} - F, \pi_{M2}^{11D2*})</td>
<td>(\pi_{M1}^{11D12*} - F, \pi_{M2}^{11D12*})</td>
</tr>
<tr>
<td>I12</td>
<td>(\pi_{M1}^{12D1*} - F, \pi_{M2}^{12D1*})</td>
<td>(\pi_{M1}^{12D2*} - F, \pi_{M2}^{12D2*})</td>
<td>(\pi_{M1}^{12D12*} - F, \pi_{M2}^{12D12*})</td>
</tr>
</tbody>
</table>

The equilibrium results of ID subgame are summarized in Proposition 3 and 4, respectively according to whether the decentralized manufacturer is allowed wholesale price discrimination.

**Proposition 3:** Consider the \((I, D)\) subgame where the decentralized manufacturer sets a common wholesale price for both the retail outlets. In this subgame, \((I12, D12)\) is the only equilibrium if and only if \(0 \leq \theta < 0.309\); \((I1, D12)\) is the only equilibrium if and only if
$0.309 < \theta < 0.5$; and $(I_1, D_2)$ is the only equilibrium if and only if $0.5 < \theta \leq 1$. Furthermore, the decentralized manufacturer never uses the integrated retailer exclusively at equilibrium.

Proof: We can easily show that $\pi_{M_2}^{I_1D_1^*} \leq \pi_{M_2}^{I_1D_12^*}$ for all $\theta$. Therefore, $D_1$ is dominated. Consequently, $(I_1, D_1)$ and $(I_12, D_1)$ cannot be equilibrium. Consider $0 \leq \theta < 0.309$. It is easy to check that $\pi_2^{I_1D_2^*} - \pi_2^{I_1D_12^*} < \pi_2^{I_1D_2^*} - \pi_2^{I_12D_12^*} < 0$. Thus using both channels is $M_2$’s dominant strategy. In addition, $\pi_1^{I_1D_12^*} - \pi_1^{I_12D_12^*} < 0$ implies that given $M_2$ sells through both retailers, $M_1$’s optimal strategy is to use both retailers as well. Therefore, $(I_12, D_12)$ is the only equilibrium when $0 \leq \theta < 0.309$. Consider $0.309 < \theta < 0.5$. $\pi_1^{I_1D_2^*} - \pi_1^{I_12D_2^*} > \pi_1^{I_1D_12^*} - \pi_1^{I_12D_12^*} > 0$ shows that using self-owned channel only is $M_1$’s dominant strategy. Additionally, given $M_1$ decides to only use the factory outlet, $M_2$’s optimal strategy is to use both channels since $\pi_2^{I_1D_2^*} - \pi_2^{I_1D_12^*} < 0$. Thus, $(I_1, D_12)$ is the only equilibrium if $0.309 < \theta < 0.5$. Finally, consider $0.5 \leq \theta \leq 1$. It is still true that $\pi_1^{I_1D_2^*} - \pi_1^{I_12D_2^*} > \pi_1^{I_1D_12^*} - \pi_1^{I_12D_12^*} > 0$ and that selling through her own channel exclusively is $M_1$’s dominant strategy. But now $M_2$’s optimal strategy is selling through $R_2$ exclusively rather than using both retailers since $\pi_2^{I_1D_2^*} - \pi_2^{I_1D_12^*} > 0$. Consequently, $(I_1, D_2)$ is the only equilibrium when $0.5 < \theta \leq 1$.

Proposition 3 specifies the condition for cross sale to happen in the mixed channel configurations. By Proposition 3, we do not expect to see cross sale at a high level of product substitutability ( $\theta \geq 0.5$). When the substitutability is low, the cross sale will happen between the decentralized manufacturer and the integrated retail store. The integrated channel will sell its product alongside a competitor’s product through its own channel. The decentralized retailer, however, will sell the product of the decentralized manufacturer only. Even though $(I_12, D_12)$ is the unique equilibrium under low product substitutability, the optimal wholesale price named by the integrated manufacturer is too high to attract any order from the decentralized retailer. No actual trade will take place between the decentralized retailer and the integrated manufacturer. Hence, when one channel is decentralized while the other channel is integrated, the cross sale is unidirectional; i.e., cross sale only happens between the integrated retail outlet and the decentralized manufacturer.
The indirect effect comes from the lower wholesale price set by the decentralized manufacturer. The two obvious effects of this are a lower retail price of the decentralized manufacturer’s product \( p_2^{\text{IDI22}} < p_2^{\text{IDI12}} \) for \( 0 < \theta < 1 \) and less quantities sold of the integrated manufacturer’s product \( Q_1^{\text{IDI22}} < Q_1^{\text{IDI12}} \) for \( 0 < \theta < 1 \). Selling less of her own product, however, does not necessarily reduce the integrated manufacturer’s profit earned from her own product because of a higher retail price \( p_1^{\text{IDI22}} > p_1^{\text{IDI12}} \) for \( 0 < \theta < 1 \). To sum up, the integrated manufacturer always benefits from the direct effect of her carefully chosen wholesale price, while she may or may not suffer from the indirect effect. Moreover, the direct effect seems more significant than the indirect effect described above. Hence, by committing to cross sale while selecting a sufficiently high wholesale price, the integrated manufacturer can induce the decentralized manufacture and the decentralized retailer to behave in her favor. To make this happen, the integrated manufacturer’s scheme must benefit the decentralized retailer as well. This is, indeed, the case since \( \pi_{R2}^{\text{IDI22}} > \pi_{R2}^{\text{IDI12}} \) for \( 0 < \theta < 1 \). It should be noted that such a scheme is of benefit to the integrated manufacturer only if the two products are less substitutable.

We summarize the above discussion about the economics of cross sale in the following corollary.

**Corollary 2:** When the decentralized manufacturer sets a common wholesale price for both the retail outlets, cross sale actually takes place in the \((I, D)\) channel configuration if and only if \( 0 \leq \theta < 0.5 \) and that cross sale is always unidirectional. In addition, the wholesale price named by the integrated manufacturer is always higher that the wholesale price named by the decentralized manufacturer.

As mentioned in Section 1, Dell computer has recently started selling competing brand of printers alongside the Dell printers in the US market through its website. A recent search of the Dell website for professional black and white laser printers by the authors found Brother, Lexmark, Samsung, and Xerox brands of laser printers along with Dell laser printers. A search for the color inkjet printers yielded Canon, Epson, and Lexmark printers alongside Dell inkjet printers. On the other hand, Dell products are never available for sale at a competitor’s channel in the US. These observations are consistent with Proposition 5. While purely a conjecture, each brand of printer is somewhat differentiated from a competing brand not only in terms of
technical specification (resolution in dpi, printing speed, RAM etc.) but also in terms of cost of ink, toner cartridges, etc. This might have resulted in a value of \( \theta \) such that the sale of competing brands is justified through Dell’s direct channel. Dell website, however, does not sell a competing brand of desktop or notebook computer. Standardization of CPU, RAM, or hard disk space might have resulted in high substitutability (\( \theta \geq 0.5 \), in our model) between the competing brands of desktop or notebook computers. This might explain the absence of such items at Dell website. Proposition 3 also suggests that cross sale will occur in (D, I) and (I, D) configurations when the two products are completely different (\( \theta = 0 \)). This might explain the presence of digital cameras at Dell Website, which do not compete with any of the Dell products. The following corollary further explores the economics of cross sale under the (I, D) channel configuration.

**Corollary 3:** The following statements hold in the (I, D) channel for any fixed value of \( \theta \) when the decentralized manufacturer sets a common wholesale price for both the retail outlets.

(a) The decentralized retailer prefers cross sales over an exclusive arrangement (i.e. \( \pi_{R2}^{I\mid D12*} > \pi_{R2}^{I\mid D2*} \)) if and only if \( \theta > 0.354 \).

(b) The decentralized manufacturer prefers cross sales over an exclusive arrangement (i.e., \( \pi_{M2}^{I\mid D12*} > \pi_{M2}^{I\mid D2*} \)) if and only if \( 0 \leq \theta < 0.5 \).

In addition, for any fixed value of \( \theta \), the wholesales price of the decentralized manufacturer and the retail price of the decentralized retailer is always lower under cross sale compared to an exclusive arrangement (i.e., \( w_{2}^{I\mid D2*} > w_{2}^{I\mid D12*} \) and \( p_{2}^{I\mid D2*} > p_{2}^{I\mid D12*} \)).

The proof of Corollary 3 follows directly from the expressions of Table 1. The interpretation of Corollary 3 is similar to that of Corollary 1. For \( 0.354 \leq \theta < 0.5 \), both the decentralized retailer and the decentralized manufacturer prefer cross sale over an exclusive arrangement. By Proposition 3, \((I1, D12)\) is the only equilibrium outcome for any \( \theta \) within this range, indicating that the equilibrium outcome is also the preferred outcome for the decentralized manufacture and the decentralized retailer. However, the interests of these two parties do not converge for any other range of values of \( \theta \). The \((I1, D2)\) channel is the only equilibrium for \( 0.5 \leq \theta < 1 \), which is the preferred outcome of the decentralized manufacturer but not the decentralized retailer.
Corollary 3 also states that cross sales result in lower wholesale and retail prices for the product of the decentralized manufacture compared to an exclusive arrangement. The result is intuitive.

**Proposition 4:** Consider the (I, D) subgame where the decentralized manufacturer sets different wholesale prices for the two retail outlets. In this subgame, (I12, D12) is the only equilibrium if and only if $0 \leq \theta < 0.622$; there is no pure strategy equilibrium for $0.622 < \theta \leq 1$. Furthermore, the decentralized manufacturer never uses the integrated retailer exclusively at equilibrium.

The proof of Proposition 4 is similar to that of Proposition 3, and thus is omitted. Proposition 4 shows that when the decentralized manufacturer can name different wholesale price for different retailer, (I12, D12) will be the only possible pure strategy equilibrium for sufficiently low substitutability. If the product substitutability is high enough, then neither pure strategy profile will be equilibrium.

It is also worth discussing how wholesale price discrimination by $M_2$ in the (I, D) channel might affect the profits of the manufacturers and retailers. When wholesale price discrimination is not allowed, by Proposition 3, the possible equilibrium outcomes are (I12, D12), (I1, D12), and (I1, D2). On the other hand, when wholesale price discrimination is allowed, by Proposition 4, the only possible equilibrium outcome is (I12, D12). Thus, it is sufficient for us to study the of wholesale price discrimination by $M_2$ in the (I12, D12) channel.

**Corollary 4:** When $M_2$ engages in wholesale price discrimination in the (I12, D12) channel the optimal profits of both $M_1$ and $M_2$ increase than when she is not. The profit of $R_2$ follows an opposite relationship.

The proof of Corollary 4 follows directly from Tables 1 and 2. No wholesale price in involved in the integrated channel of $M_1$. Thus, $M_1$ sets only one wholesale price, $w_{12}$, in the (I12, D12) channel. When $M_2$ engages in price discrimination her optimal wholesale price for $R_2$, $w_{22}$, is higher than when she is not. This results in a higher optimal profit for $M_2$. Moreover, as $w_{22}$ increases, the best response function of $M_1$, $w_{12}(w_{22})$, also increases, resulting in a higher profit for $M_1$ as well. It is interesting to note how wholesale price discrimination affects the equilibrium outcomes in the (D, D) and (I, I) channels. Unlike the result in Corollary 4, we mentioned in Section 4.1 that wholesale price discrimination by the manufacturers does not alter the optimal profit of any party in the (D, D) channel.
4.3 Analysis of the Equilibrium and Cross Sales in the (I, I) Channel

Note that each manufacturer has self-owned sales channel. A manufacturer names one wholesale price at most when she decides to sell though the other integrated retailer as well. Consequently, it does not affect the equilibrium result at all whether a manufacturer is allowed to name different wholesale price for different retailer. The following proposition presents the equilibrium outcome for the (I, I) subgame.

**Proposition 5:** $(I12, I12)$ is the unique equilibrium of Subgame $(I, I)$.

It is easy to see from Table 1 that cross selling is the dominant strategy for both manufacturers for all $\theta \in (0,1)$. Therefore, $(I12, I12)$ is the unique equilibrium of the subgame. Proposition 5 has significant implications. In a channel where both manufacturers are integrated, each of them will be better off selling her competitor’s product besides the product of her own. One possible explanation of this is as follows. In subgame $(I, I)$, the exclusive arrangement is one special case of cross sale. The equilibrium described in Proposition 6, thus, does not preclude the possibility of an exclusive arrangement. Consider a manufacturer chooses cross selling at stage 2 of the game. Suppose it turns out that exclusive arrangement is the better choice. The manufacturer can easily switch from cross selling to exclusive arrangement by setting a sufficiently high wholesale price at stage 3 or by ordering zero quantity from her competing manufacturer at stage 4. Therefore, exclusive arrangement can still be realized even if a manufacturer has chosen cross selling at stage 2. Our calculations shows that at optimum cross selling quantities are positive. That is to say, cross selling dominates exclusive selling. This is not possible if a manufacturer has chosen to decentralize her channel. Moreover, even if a manufacturer vertically integrates her channel, the decision of the other manufacturer to decentralize her channel may prevent the integrated manufacturer from committing to cross selling.

Another interesting observation is that the optimal cross selling quantity is always positive unless the substitutability is zero, i.e., two products are completely different. When substitutability is zero, no manufacturer will sell the other product even though the other manufacturer sets wholesale price to be zero.
We will conclude this sub-section by summarizing how horizontal product differentiation affects the manufacturers and the retailers for each of the three channel configurations presented in Sections 4.1-4.3. The following corollary states our result.

**Corollary 5:** In all three types of channel configurations where a decentralized manufacturer sets a common wholesale price for the retailer(s), horizontal product differentiation (low substitutability) always helps the manufacturers, integrated or decentralized. On the other hand, horizontal product differentiation hurts the decentralized retailer(s) as long as cross-sales occur while helps the decentralized retailer(s) if there is only exclusive arrangement.

Proof: It is easy to see from Table 1 that $\pi_{M_1}^{D1D2*}$, $\pi_{M_1}^{D12D12*}$, $\pi_{M_1}^{I12I12*}$, $\pi_{M_1}^{I1D2*}$, $\pi_{M_2}^{I1D2*}$, $\pi_{M_1}^{I1D12*}$, $\pi_{M_2}^{I1D12*}$, $\pi_{R_2}^{I1D2*}$, $\pi_{R_2}^{I1D12*}$ all decrease in $\theta$. On the other hand, $\pi_{R_1}^{D12D12*}$, $\pi_{R_2}^{I1D12*}$, and $\pi_{R_2}^{I12D12*}$ increase in $\theta$.

Corollary 5 has important implications. Choi (1996) described a similar result for a (D, D) channel. Our work generalizes Choi’s finding for (I, D) and (I, I) configurations. In addition, Corollary 5 has another interesting implication. Note from Corollary 5 that $\pi_{M_1}^{I12I12*}$ decreases with $\theta$. An integrated channel, in our model, performs the functions of both the manufacturer and the retailer. However, as far as the impact of product differentiation on the profit is concerned, the integrated channel behaves more like a manufacturer rather than like a retailer indicating that product differentiation helps the integrated channel. When the wholesale price discrimination by a manufacturer is allowed, it turns out that Corollary 5 continues to hold with once exception. The profit of $R_2$ in the (I1, D12) channel, when the wholesale price discrimination by a manufacturer is allowed, is constant and is independent of $\theta$. Corollary 5 continues to hold for other configurations.

We have discussed cross sales under various channel configurations in Sections 4.1-4.3. We now show that each of these channel configurations is, indeed, an equilibrium outcome.

**4.4 Subgame Perfect Equilibrium of the Entire Game**
If a strategy profile is a subgame perfect equilibrium, then it must give an equilibrium at each stage of the game. Being an equilibrium in each subgame is a necessary condition to identify a SPE. Hence, we restrict our discussion to the equilibriums characterized by Proposition 4-6.

Recall from Proposition 4 that in subgame (D, D), we have 3 equilibriums (D12, D12), (D1, D2), and (D2, D1). We present our analyses for the case when (D1, D2) is the equilibrium of the (D, D) subgame. It can be shown by a similar analysis that all our results continue to hold when (D12, D12) is the equilibrium of the (D, D) subgame. We summarize the equilibrium results at stage-1 of the game in Proposition 6 below:

**Proposition 6:** Given any specific value of \( \theta \in [0,1] \), the equilibrium channel configuration at stage 1 is as follows. For \( F < \pi_i^{II*} - \pi_i^{DI*} \), (I, I) is the only equilibrium. For \( \pi_i^{II*} - \pi_i^{DI*} < F < \pi_i^{ID*} - \pi_i^{DD*} \), both (I, D) and (D, I) are equilibriums. For \( \pi_i^{ID*} - \pi_i^{DD*} < F \), (D, D) is the only equilibrium.

Proof: For convenience, we summarize the equilibrium results of each stage-2 subgame in Table 5 below, which is the payoff matrix at stage 1 of the game:

<table>
<thead>
<tr>
<th></th>
<th>D</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>(( \pi_{1i}^{D12112}, \pi_{2i}^{D12112} )) if 0 ≤ ( \theta ) &lt; 0.682; (( \pi_{1i}^{D1D12}, \pi_{2i}^{D1D12} )) if ( \theta ) &gt; 0.682.</td>
<td>(( \pi_{1i}^{D12112}, \pi_{2i}^{D12112} - F )) if 0 ≤ ( \theta ) &lt; 0.309; (( \pi_{1i}^{D12112}, \pi_{2i}^{D12112} - F )) if 0.309 &lt; ( \theta ) &lt; 0.5; (( \pi_{1i}^{D12112}, \pi_{2i}^{D12112} - F )) if 0.5 &lt; ( \theta ) ≤ 1.</td>
</tr>
<tr>
<td>I</td>
<td>(( \pi_{1i}^{D11112}, \pi_{2i}^{D11112} - F, \pi_{2i}^{D11112} )) if 0 ≤ ( \theta ) &lt; 0.309; (( \pi_{1i}^{D11112}, \pi_{2i}^{D11112} - F, \pi_{2i}^{D11112} )) if 0.309 &lt; ( \theta ) &lt; 0.5; (( \pi_{1i}^{D11112}, \pi_{2i}^{D11112} - F, \pi_{2i}^{D11112} )) if 0.5 &lt; ( \theta ) ≤ 1.</td>
<td>(( \pi_{1i}^{D11112}, \pi_{2i}^{D11112} - F ))</td>
</tr>
</tbody>
</table>

Consider \( F < \pi_i^{II*} - \pi_i^{DI*} \). Note that \( \pi_i^{II*} - \pi_i^{DI*} < \pi_i^{ID*} - \pi_i^{DD*} \). Thus, integration is the dominant strategy for both manufacturers. Thus, (I, I) is the unique equilibrium when \( F < \pi_i^{II*} - \pi_i^{DI*} \).

Consider \( \pi_i^{II*} - \pi_i^{DI*} < F < \pi_i^{ID*} - \pi_i^{DD*} \). Since \( \pi_i^{ID*} - \pi_i^{DD*} > F \) and \( F > \pi_i^{II*} - \pi_i^{DI*} \), if \( M_1 \) integrates her channel, \( M_2 \) will decentralize her channel; given \( M_2 \) chooses decentralization, integration is \( M_1 \)'s optimal strategy. Therefore, (I, D) is an equilibrium. Similarly, (D, I) is also an equilibrium when \( \pi_i^{II*} - \pi_i^{DI*} < F < \pi_i^{ID*} - \pi_i^{DD*} \). Consider \( F > \pi_i^{ID*} - \pi_i^{DD*} \). Decentralization
is the dominant strategy for both manufacturers. Hence, \((D, D)\) is the unique equilibrium when
\[ F > \pi_i^{ID} - \pi_i^{DD}. \]

Proposition 6 describes the possible equilibrium at stage 1. It is interesting to note that a mixed structure is an equilibrium for any possible value of \(\theta\) if the fixed cost of integration is within a certain range of values. Proposition 6, together with Proposition 1 to 5, characterizes the subgame perfect equilibrium of the entire game. When \(F\) is sufficiently low, the SPE is that both manufacturers integrate their channels at stage 1 and then at stage 2 both commit to cross selling, i.e., \((I12, I12)\). In fact, \((I12, I12)\) is also the most profitable channel arrangement for both manufacturers under \((I, I)\), where cross sale happens between the two channels.

When \(F\) is sufficiently high, the SPE is that both manufacturers choose to decentralize and then follow the strategies described in Proposition 4. In the \((D, D)\) configuration, cross sale will happen when the substitutability \(\theta < 0.682\). In other words, \((D12, D12)\) is SPE for \(0 \leq \theta < 0.682\) and high fixed cost. On the other hand, both manufacturers would commit to exclusive arrangement when the two products are highly competitive. That is, \((D1, D2)\) and \((D2, D1)\) are SPE under high substitutability and high \(F\).

When \(F\) is in between the two critical values described above, either \((I, D)\) or \((D, I)\) is an equilibrium channel configuration. The \((I12, D12)\) or the \((D12, I12)\) configuration is the SPE for lower value of \(\theta\), where cross selling will not actually happen between the integrated manufacturer and the decentralized retailer. The integrated manufacturer will always set a sufficiently high wholesale price such that the optimal order quantity from the decentralized retailer is always zero. In addition, \((I12, D2)\) will never be an equilibrium under \((I, D)\). As a result, only unidirectional cross sale will happen where the integrated manufacturer will sell both manufacturers’ products while the decentralized retailer will sell the product of the decentralized manufacturer only. \((I1, D2)\) or \((D1, I2)\) is SPE under high substitutability where both manufacturers commit to exclusive arrangement.

### 4.5 Price Competition, Quantity Competition, and Cross Sales

Our paper considers quantity competition. What will happen if we consider price competition and simultaneously allow cross sales in our model? Under such a scenario, each of the two retail stores will sell identical products, resulting in the so called Bertrand Paradox. Each retailer, then,
will set the retail price at the marginal cost of acquiring the product and will make zero profit. This allows us to make the following important observation: cross sale may never happen in a pure Bertrand price competition. However, as described in Section 1, cross sales is quite common in practice, and it is unlikely to that a retailer makes zero profit under cross sales. To explain this anomaly, the marketing researchers have proposed retail differentiation to be the driver of cross sales (Choi, 1996). In this sub-section, we show that cross sale will happen under a Bertrand-like price competition even in absence of retail differentiation and that the retailers make positive profits under this modified price competition.

One key assumption in pure Bertrand price competition is that pricing and production decisions are made simultaneously so that the demand is satisfied perfectly. We will relax this assumption to explain the existence of cross sales. We consider a price competition where production decisions precede any pricing decision. Thus, the actual quantity sold by any retail store is constrained by the quantity it ordered at an earlier stage of the game. This assumption gives rise to a capacity-constrained Bertrand-like price competition. While the assumption of Bertrand competition is more common in literature, our assumption seems to be more realistic. In most industries, production usually occurs long before the demand realization because of the manufacturing leadtime. Similarly, because of the leadtime associated with the supply process, retail stores often place orders well before the sales season, while the retail prices are determined during the actual sales season. We consider a 5-stage price competition in this subsection. The stages 1-3 of this game are identical to those of our original game described in Section 3. At stage 4, each retail store, integrated or not, chooses its order quantities of each product. We, once again, let \( Q_{ij} \) be the order quantity of product \( i \) by retail store \( j, i, j = 1, 2 \) and let \( Q_i = Q_{i1} + Q_{i2} \) be the total quantity of product \( i \) ordered. We assume that both manufacturers produce the exact quantities which meet the orders placed by both retail stores. Moreover, we assume that this is the only chance for both retail stores to procure the products from the manufacturers, i.e., no second opportunity to replenish exists. Finally, at stage 5 of the game, each retail store determines its retail price through a price competition. We once again assume that at each stage of the game, the decisions are made simultaneously and non-cooperatively and that each firm seeks to maximize its profit. We will show that the equilibrium outcome of this capacity-constrained price competition game coincides with that of a Cournot-like quantity competition game.
Kreps and Scheinkman (1983) shows that if each manufacturer in a duopoly chooses a production capacity level first and then competes over price, then this capacity constrained price competition game yields the same equilibrium results as Cournot-like quantity competition under efficient rationing rule of excess demand. Our model differs from that of Kreps and Scheinkman (1983) in that they consider a one-dimensional competition in which several stores sell a single identical product while our model studies allows each retailer to carry more than one substitutable products. We show that even if a retailer carries both substitutable products, the quantity competition characterized by demand function in (1) generates the same equilibrium as the price competition where each retailer makes ordering decision before setting retail price.

Consider the demand function where one retailer carries only one product first:
\[
q_i = \frac{1}{1+\theta} \left( 1 - p_i - \theta p_j \right).
\]  
(11)

This demand function is equivalent to those of McGuire and Staelin (1983) and is the inverse demand function of 
\[
p_i = 1 - q_i - \theta q_j. 
\]
Note that we use the notation upper case \(Q\) to denote the order quantities of the retailers, and the notation lower case \(q\) to denote the demand. Consider cross sale next. Suppose both retail stores carry product \(i\) and that the order quantities of the two store are \(Q_i\) and \(Q_{i2}\), and the retail prices are \(p_{i1}\) and \(p_{i2}\), respectively. \(Q_j\) is often referred to as the capacity of product \(i\) at the retail store \(j\). To develop the demand functions under cross sale, assume, without loss of generality, that \(p_{i1} \leq p_{i2}\). We will consider the two cases \(p_{i1} < p_{i2}\) and \(p_{i1} = p_{i2}\) separately. When \(p_{i1} < p_{i2}\), all the customers will go to retail outlet 1 first. Therefore, the demand faced by retail outlet 1 is given by the demand function in equation (11), i.e.,
\[
q_{i1} = q_i(p_{i1}, p_r).
\]  
(12)

where \(i' = 1,2\) and \(i' \neq i\). Note that it is possible that the demand at retail outlet 1 is greater than its capacity. In that case, retail outlet 2 will have a portion of the excess demand only if the demand generated by \(p_{i2}\), which is \(q_i(p_{i2}, p_r)\), has not been completely absorbed by store 1, i.e., when \(q_i(p_{i2}, p_r) > Q_{i1}\). Therefore, the demand faced by retail outlet 2 is given by:
\[
q_{i2} = Max(0, q_i(p_{i2}, p_r) - Q_{i1}).
\]  
(13)
The demand functions in (12) and (13) are called efficient rationing rule in economics. Next consider the case \( p_{i1} = p_{i2} = p_i \). In this scenario, the two stores share the demand equally unless one does not have enough capacity, in which case, the other will have all the remaining demand. The demand for product \( i \) at store \( j, j = 1, 2 \), therefore, is

\[
q_j = \text{Max} \left( \frac{q_i(p_i, p_j)}{2}, q_i(p_i, p_j) - Q_{ij} \right).
\]

(14)

Equations (12)-(14) completely specify our demand functions under cross sale.

The key here is that the actual quantity sold by any retail store is constrained by the quantity it ordered at an earlier stage of the game. Unlike the Bertrand competition, where price and quantity decisions are made simultaneously, this assumption gives rise to a capacity-constrained Bertrand-like price competition. With capacity-constrained price competition, we can show that retail differentiation is not a necessary condition for cross-sales to happen.

After the retailers’ orders \( Q_{ij} \) become common knowledge, each retailer names his retail price. We wish to establish the fact that for each pair of \((Q_1, Q_2)\), the total quantity of the each of the two products ordered, the associated subgame has a unique pure strategy equilibrium outcome which is identical to that of quantity competition.

**Lemma 2:** Suppose cross selling happens for product \( i \) and that the two retail prices are \( p_{i1} \) and \( p_{i2} \), respectively. Then, \( p_{i1} = p_{i2} \) at equilibrium.

Proof: If possible, assume that there exists an equilibrium where \( p_{i1} \neq p_{i2} \). Without loss of generality, we assume that \( p_{i1} < p_{i2} \). Retail store 1 always wants to raise its prices as long as it is capacity constrained; or else \( p_{i1} \) is such that store 1 supplies the entire demand at this price. Therefore, retail store 2 has no sales. As a result, store 2 could make a strictly positive profit by undercutting its price to \( p_{i1} \). Therefore, there cannot be an equilibrium where \( p_{i1} \neq p_{i2} \).

**Lemma 3:** Consider subgames \((Q_{11}, Q_{12}, Q_{21}, Q_{22})\) and \((Q'_{11}, Q'_{12}, Q'_{21}, Q'_{22})\) where \( Q_{ij} \neq Q'_{ij}, \ i, j = 1, 2 \). If there exists a pure strategy equilibrium in each subgame, then the two subgames have the same equilibrium prices as long as \( Q_{i1} + Q_{i2} = Q'_{i1} + Q'_{i2} \equiv Q_i \) for all \( i \). Moreover, the
equilibrium prices are unique and equal to those under quantity competition, denoted by
\[ P_i = p_i(Q_i, Q_j) = 1 - Q_i - \theta Q_j. \]

Proof: From Lemma 2 it is easy to see that \((p_{i1} = p_{i2} = P_i)\) is an equilibrium of the game. Now it
suffices to show that \((p_{i1} = p_{i2} = P_i)\) is the unique equilibrium. Suppose there exists another
equilibrium \((P'_i, P'_j)\) such that \(P'_i \neq P_i\). It’s impossible to have \(P'_i > P_i\). Otherwise, either retail
outlet has the incentive to undercut the price and grab the whole demand for product \(i\). When
\(P'_i < P_i\), either store can raise its price to \(P_i\) without losing any sales. Thus \((p_{i1} = p_{i2} = P_i)\) is the
unique equilibrium. ■

Lemma 3 has important implications. It indicates that the equilibrium in price-setting subgame is
influenced only by the total quantity (or capacity, using our terminology) of each of the two
products. Therefore, to establish the uniqueness of the equilibrium in our game in involving cross
sales, we can simply look at a \((Q_i, Q_j)\)-subgame in a channel where only exclusive arrangement
is allowed. As a result, for the remainder part, we will consider an exclusive channel
arrangement to establish the uniqueness of our equilibrium. By Lemma 2, this uniqueness result
will apply to cross sales as well. For simplicity, we ignore the wholesale prices in the following
analysis. Now consider retail outlet \(j\)'s optimization problem:

\[
\begin{align*}
\max_{p_j} & \quad (p_j - w_j) \min (q_j(p_i, p_j), Q_j).
\end{align*}
\]

Based on the definition of Nash equilibrium and the best response functions, Proposition 1 below
establishes the desired uniqueness result for our capacity-constrained price competition.

**Proposition 7:** Given any pair \((Q_i, Q_j)\) as the quantity available at the two retail outlets,
\(i, j = 1, 2, \) and \(i \neq j\), there is a unique pure strategy equilibrium in price competition. This
equilibrium can be characterized as:

\[
\begin{align*}
\text{If} & \quad Q_i \geq \frac{(1-\theta)(2+\theta) + \theta w_j - (2-\theta^2)w_i}{(1-\theta^2)(4-\theta^2)} \quad \text{and} \quad Q_j \geq \frac{(1-\theta)(2+\theta) + \theta w_i - (2-\theta^2)w_j}{(1-\theta^2)(4-\theta^2)}, \\
p_i^* & = \frac{(1-\theta)(2+\theta) + 2w_i + \theta w_j}{4-\theta^2} \quad \text{and} \quad p_j^* = \frac{(1-\theta)(2+\theta) + 2w_j + \theta w_i}{4-\theta^2}; \quad \text{(Type 1)}
\end{align*}
\]
If \[ Q_i \leq \frac{(1-\theta)(2+\theta) + \theta w_i - (2-\theta^2)w_j}{(1-\theta^2)(4-\theta^2)} \] and \[ Q_j \leq \frac{(1-\theta)(2+\theta) + \theta w_i - (2-\theta^2)w_j}{(1-\theta^2)(4-\theta^2)} \], then

\[ p_i^* = 1 - Q_i - \theta Q_j; \quad \text{(Type 2)} \]

and if \[ Q_i \geq \frac{(1-\theta)(2+\theta) + \theta w_i - (2-\theta^2)w_j}{(1-\theta^2)(4-\theta^2)} \] while \[ Q_j \leq \frac{(1-\theta)(2+\theta) + \theta w_i - (2-\theta^2)w_j}{(1-\theta^2)(4-\theta^2)} \], then

\[ p_i^* = \frac{(1-\theta^2) + w_i - 2(1-\theta^2)Q_j}{2 - \theta^2} \quad \text{and} \quad p_j^* = \frac{(1-\theta)(2+\theta) + \theta w_i - 2(1-\theta^2)Q_j}{2 - \theta^2}. \quad \text{(Type 3)} \]

Moreover, there is a unique subgame perfect equilibrium of this capacity-constrained price competition game; and that the equilibrium outcome coincides with that of a Cournot-like quantity competition.

Proof: We can represent the two best response functions in a x-y plot in the first quadrant with \( p_i \) on the x-axis and \( p_j \) on the y-axis. It, then, suffices to show that the two best response functions have only one intersection. Note that the vertical intercept of \( p_j^*(p_i) \), which is \( (1-\theta)[1-(1+\theta)Q_i] \) if \( Q_i \leq \frac{1-\theta - w_j}{2(1-\theta^2)} \) or \( \frac{1-\theta + w_j}{2} \) otherwise, is always positive since either manufacturer would choose a wholesale price so high that her downstream retailer’s optimal retail price is negative, given the competing retailer has zero retail price. Similarly, the horizontal intercept of \( p_i^*(p_j) \) is also positive. Moreover, the slope of \( p_j^*(p_i) \) is either \( \theta \) or \( \theta/2 \), both of which are less than 1. On the other hand, the slope of \( p_i^*(p_j) \) is either \( 1/\theta \) or \( 2/\theta \), both of which are greater than 1. Hence, the two best response functions have exactly one intersection in first quadrant. Solving \( p_i = p_i^*(p_j) \) and \( p_j = p_j^*(p_i) \) yields the equilibrium. It suffices to establish that in a subgame perfect equilibrium, Type 2 equilibrium is the only possible equilibrium at the last stage of the game. Note that in either Type 1 or Type 3 equilibrium, there is at least one retail store with excess quantity at equilibrium, which is impossible to occur in the subgame perfect equilibrium. This is because the retailer(s) with extra quantity can reduce its order quantity without sacrificing any sales. Solving the first-order condition of the retailer’s optimization problem under \( p_i^* = 1 - Q_i - \theta Q_j \) for \( Q_i \) and \( Q_j \) generates Cournot-like solution.
Proposition 7 guarantees that we have a unique equilibrium outcome, which is identical to that of quantity competition. All of our equilibrium results described in Sections 4.1-4.4 will, thus, continue to hold under the capacity constrained price competition described in the current subsection. This might provide another explanation for the cross sales phenomenon. In addition, Proposition 7 establishes that retail differentiation is not the sole factor contributing to cross sales under price competition. The results of our current subsection, of course, depend on the demand rationing rules described in equations 12-14. Following Davidson and Deneckere (1986), one possible limitation of the current subsection is that the equivalence of capacity constrained price competition and quantity competition might not hold for a different set of demand rationing rules. Nevertheless, that does not undermine the key contribution of the current subsection that the existence of cross sales can be explained even without retail differentiation under price competition.

5. SUMMARY AND CONCLUSIONS

Multi-channel distribution systems with differing channel configurations are widely observed in practice. In this study, we considered vertical integration and cross selling issues in channel design under product substitutability. Using a game theoretic model, we studied different channel structures involving two manufacturers and two retail outlets committing to cross sale or not and having different degrees of vertical integration. A manufacturer incurs a fixed cost whenever she establishes a vertically integrated channel. We characterize the pure strategy Nash equilibrium channel configurations. We considered cross sales and vertical integration simultaneously in our model. To the best of our knowledge, cross sales has not been studied in the context of channel design either in marketing literature before. Our work makes important contribution to the literature by providing theoretical and intuitive explanation for this phenomenon. We show the following results.

First, our analyses establish cross sales as an equilibrium outcome of the gaming behavior between the manufacturers and the retailers. A majority of the literature, on the other hand, exogenously assume the existence of cross selling. Our model, thus, suggests that a manufacturer should strategically use both channel design and cross sale decisions to maximize profit. It is interesting to note that no cross sale will happen in Choi’s (1996) model in absence of retail
differentiation. Our analyses, on the contrary, establish that cross sale is an equilibrium outcome even in absence of any retail differentiation. We have shown that, in addition to quantity competition, cross sale is also the equilibrium outcome in a capacity-constrained price competition. We have established the uniqueness of the equilibrium. We have also shown that the outcome of this game is similar to that of a quantity competition game. This means that cross sale can be explained by any of these three concepts: quantity competition, capacity constrained price competition, and price competition under retail differentiation. Interestingly, and on the contrary, cross sales may never occur in a pure Bertrand price competition. Our analysis also indicates that the equilibrium channel configuration depends not only on the level of substitutability between the two products, but also on the fixed cost of establishing a vertically integrated channel.

Second, we have examined the conditions under which cross selling will occur in each channel configuration. In (D, D), (I, D), and (D, I), configurations, when a decentralized manufacturer sets a common wholesale price for both retailers, cross sale happens when the substitutability between the two products is relatively low. On the other hand, an exclusive arrangement is preferred when the substitutability between the two products is high. When the two products are not very similar, inter-brand competition is weaker and the double marginalization problem is stronger. Therefore, a manufacturer can benefit from creating intra-brand competition. This gives rise to the decision of cross sales. On the other hand, when the two products are close substitutes, the manufacturers prefer exclusive selling because that can dampen inter-brand competition. When a decentralized manufacturer is allowed to set different wholesale prices for the two retailers in the (D, D) channel, cross sale is an equilibrium outcome for all possible values of the substitutability between the two products. The result is again intuitive. The wholesale price discrimination allows a manufacturer to dampen the inter-brand competition when the substitutability between the two products is high. As a result cross sales continues to be the equilibrium outcome even when the two products are highly substitutable. When the two products are highly substitutable, we have shown that no pure strategy equilibrium exists in the (I, D) channel, if the decentralized manufacturer is allowed to set different wholesale prices for the two retail outlets. Moreover, we have shown that using a single common retailer is never an equilibrium outcome in the (D, D) channel.
Third, cross sale is bi-directional (i.e. each of the two retail outlet stocks products from both manufacturers) in the two symmetric channel configuration, (D, D) and (I, I). Cross sale is also bidirectional in the (I, D) and (D, I) channels if the decentralized manufacturer is allowed wholesale price discrimination. However, the cross sale unidirectional in the (I, D) and (D, I) configurations when the decentralized manufacturer uses a common wholesale price, where only the integrated retail outlet sells the products of both the manufacturers. It is interesting to note that wholesale price discrimination by a manufacturer does not affect the equilibrium profit of any party in the (D, D) channel. On the other hand, wholesale price discrimination by the decentralized manufacturer results in a higher equilibrium profit for both the manufacturers and a lower equilibrium profit for the decentralized retailer in the (I, D) channel.

Fourth, under common wholesale price from a decentralized manufacturer, a decentralized retailer will never sell the product of an integrated manufacturer, even if the manufacturer chooses cross selling and sets a wholesale price. The optimal wholesale price is too high to attract any order from the decentralized retailer. The integrated manufacturer uses such a scheme rather than chooses exclusive arrangement in the first place in that she can, to some extent, influence the behaviors of the decentralized manufacturer and the decentralized retailer in her favor. This scheme is possible when the substitutability between the two products is relatively low.

Finally, cross selling is the dominant strategy for both manufacturers when both channels are integrated. Both manufacturers committing to cross selling is equilibrium for any level of substitutability between the two products. Cross sales will not actually happen between the two manufacturers only if the two products are completely different. We have also established various interesting economics of cross sale. In particular we have established when cross sales in a preferred channel choice for the manufacturers and the retailers and when these preferred choices are also the equilibrium outcomes.

Our paper has important implications for academicians as well as for practitioners. First, our analyses identify when cross sale will happen in channel. The insights provided can be useful qualitative guiding tools for marketing managers and consultants. We describe several examples involving Dell Computers that seem to provide anecdotal validation of our theoretical results. Second, we show that under some conditions, the mixed channel configurations can be the
industry equilibrium in a symmetric game. This implies stability of such mixed structures. These mixed structures were believed to be of transitional or temporary nature previously (McGuire and Staelin, 1983).

Like any other model in marketing literature, our model is not free from assumptions. We assume a deterministic demand. This allows us to get analytically tractable results and to derive interesting insights. Similar assumption has often been made in literature (McGuire and Staelin, 1983; Gupta and Loulou, 1998). Our assumption about the structure of the demand function is also standard in economics, marketing, and operations literature. As pointed out by Shubik & Levitan (1980), this kind of demand function is also consistent with individual utility maximization behavior. We consider only a dual-channel distribution system involving two substitutable products. In reality, almost any market involves multiple products, manufacturers, and retail outlets. A natural extension of our research is, therefore, to study a distribution system involving \( n \) products, \( m \) manufacturers and \( l \) retail outlets. Another possible direction of future research is to introduce demand uncertainty in our current model. We assume complete information in our model and that our model is symmetric. Relaxing these assumptions can also be potentially interesting extensions. Finally, it is important to recognize that there might be other factors beyond the scope of our model which might affect decisions regarding cross selling.

For example, consumers might value assortment and variety in a grocery store. Given these consumer characteristics, competition for store traffic will force a retailer to carry a range of products in a given category and products in several categories. Incorporating these demand side issues can also be a useful extension of our model.

REFERENCES


Table 1: Optimal Values of the Variables for Different Channel Configurations Where Decentralized Manufacturer Sets a Common Wholesale Price for the Two Retailers

<table>
<thead>
<tr>
<th>Var.</th>
<th>(D1, D1)</th>
<th>(D1, D2)</th>
<th>(D12, D2)*</th>
<th>(D12, D12)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_i^*$</td>
<td>$\frac{1-\theta}{2-\theta}$</td>
<td>$2-\theta$</td>
<td>$w_{12}^{D12D2*}$</td>
<td>$1-\theta$</td>
</tr>
<tr>
<td>$p_i^*$</td>
<td>$\frac{3-2\theta}{2(2-\theta)}$</td>
<td>$6-\theta^2$</td>
<td>$p_{12}^{D12D2*}$</td>
<td>$\frac{32-6\theta-17\theta^2}{3(16-7\theta^2)}$</td>
</tr>
<tr>
<td>$q_i^*$</td>
<td>$\frac{1}{2(1+\theta)(2-\theta)}$</td>
<td>$\frac{2}{8+2\theta-\theta^2}$</td>
<td>$q_{12}^{D12D2*}$</td>
<td>$\frac{8+3\theta-2\theta^2}{3(16-7\theta^2)}$</td>
</tr>
<tr>
<td>$\pi_{M_i}^*$</td>
<td>$\frac{1-\theta}{2(1+\theta)(2-\theta)^2}$</td>
<td>$\frac{2(2-\theta)}{(2+\theta)(4-\theta)^2}$</td>
<td>$\pi_{M_1}^{D12D2*}$</td>
<td>$\frac{(1-\theta)(4-\theta^2)(8+5\theta)^2}{6(1+\theta)(16-7\theta^2)^2}$</td>
</tr>
<tr>
<td>$\pi_{R_i}^*$</td>
<td>$\frac{1}{2(4-3\theta+\theta^2)}$</td>
<td>$\frac{4}{(2+\theta)^3(4-\theta)^2}$</td>
<td>$\pi_{R_1}^{D12D2*}$</td>
<td>$\frac{(8+3\theta-2\theta^2)^2}{9(16-7\theta^2)^2}$</td>
</tr>
<tr>
<td>$\pi_{R_2}^{D12D2*}$</td>
<td>$\frac{52+28\theta-7\theta^2-\theta^3}{36(1+\theta)(16-7\theta^2)}$</td>
<td>$\frac{2}{9(1+\theta)(2-\theta)^2}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. By symmetry, (D2, D2) are the same as (D1, D1) except that $\pi_{R_2}^*$ = 0 in (D1, D1) while $\pi_{R_1}^*$ = 0 in (D2, D2).
2. (D2, D1) and (D1, D2) are identical.
3. $\pi_{M_2}^{D1D1D2*} = \pi_{M_2}^{D2D1D2*} = \pi_{M_1}^{D1D1D2*} = \pi_{M_1}^{D1D2D2*}$ and $\pi_{M_1}^{D1D1D2*} = \pi_{M_1}^{D2D1D2*} = \pi_{M_2}^{D1D1D2*} = \pi_{M_2}^{D1D2D2*}$. The optimal values of other variables can be obtained in a similar way under the (D12, D1), (D1, D12), and (D2, D12) configurations.
### Table 1 (continued): Optimal Values of the Variables for Different Channel Configurations Where Decentralized Manufacturer Sets a Common Wholesale Price for the Two Retailers

<table>
<thead>
<tr>
<th>Var.</th>
<th>(I1, D1)</th>
<th>(I1, D2)</th>
<th>(I1, D12)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w_i^* )</td>
<td>( \frac{1 - \theta}{2} )</td>
<td>( \frac{2 - \theta}{4} )</td>
<td>( \frac{(1 - \theta)(4 + \theta)}{2(4 - \theta^2)} )</td>
</tr>
<tr>
<td>( p_i^* )</td>
<td>( p_{iD1}^* = \frac{1}{2} )</td>
<td>( p_{iD1}^* = \frac{4 + \theta}{4(2 + \theta)} )</td>
<td>( p_{iD12}^* = \frac{24 - 4\theta - 9\theta^2 + \theta^3}{12(4 - \theta^2)} )</td>
</tr>
<tr>
<td>( q_i^* )</td>
<td>( q_{iD1}^* = \frac{2 + \theta}{4(1 + \theta)} )</td>
<td>( q_{iD1}^* = \frac{4 + \theta}{4(2 + \theta)} )</td>
<td>( q_{iD12}^* = \frac{8 + 4\theta - \theta^2}{4(1 + \theta)(2 - \theta)(2 + \theta)} )</td>
</tr>
<tr>
<td>( \pi_{M1}^* )</td>
<td>( \pi_{M1}^* = \frac{5 + 3\theta}{16(1 + \theta)} )</td>
<td>( \pi_{M1}^* = \frac{(4 + \theta)^2}{16(2 + \theta)^2} )</td>
<td>( \pi_{M12}^* = \frac{640 + 160\theta - 476\theta^2 - 92\theta^3 + 61\theta^4 - 5\theta^5}{144(1 + \theta)(2 - \theta)^2(2 + \theta)^2} )</td>
</tr>
<tr>
<td>( \pi_{M2}^* )</td>
<td>( \pi_{M2}^* = \frac{1 - \theta}{8(1 + \theta)} )</td>
<td>( \pi_{M2}^* = \frac{2 - \theta}{8(2 + \theta)} )</td>
<td>( \pi_{M12}^* = \frac{(1 - \theta)(4 + \theta)^2}{24(1 + \theta)(4 - \theta^2)} )</td>
</tr>
<tr>
<td>( \pi_{R1}^* )</td>
<td>N/A</td>
<td>( \pi_{R1}^* = \frac{1}{4(2 + \theta)^2} )</td>
<td>( \pi_{R1}^* = \frac{(1 + \theta)^2(4 - \theta)^2}{36(2 - \theta)^2(2 + \theta)^2} )</td>
</tr>
</tbody>
</table>
Table 1 (cont’d): Optimal Values of the Variables for Different Channel Configurations Where Decentralized Manufacturer Sets a Common Wholesale Price for the Two Retailers

<table>
<thead>
<tr>
<th>Var.</th>
<th>(I12, D1)</th>
<th>(I12, D2)</th>
<th>(I12, D12)</th>
</tr>
</thead>
</table>
| \(w_{11}^{I12D1}\) | \(
\frac{1}{2}
\) | \((1 - \theta)(20 + 11\theta)\) \(40 - 13\theta^2\) | \((1 - \theta)(2 + \theta)\) \(8 - \theta^2\) |
| \(w_{21}^{I12D1}\) | \(1 - \theta\) \(2\) | \((1 - \theta)(20 + 10\theta - \theta^2)\) \(40 - 13\theta^2\) | \((1 - \theta)(4 + \theta)\) \(8 - \theta^2\) |
| \(p_1^{I12D1}\) | \(
\frac{1}{2}
\) | \(20 - 3\theta - 8\theta^2\) \(40 - 13\theta^2\) | \(12 - 2\theta - 3\theta^2\) \(3(8 - \theta^2)\) |
| \(p_2^{I12D1}\) | \(1 - \theta\) \(4\) | \(3(20 - 10\theta - 7\theta^2 + 3\theta^3)\) \(2(40 - 13\theta^2)\) | \(16 - 6\theta - 3\theta^2\) \(3(8 - \theta^2)\) |
| \(q_1^{I12D1}\) | \(2 + \theta\) \(4(1 + \theta)\) | \(20 - 3\theta - 8\theta^2\) \(40 - 13\theta^2\) | \(2(2 + \theta)\) \((1 + \theta)(8 - \theta^2)\) |
| \(q_2^{I12D1}\) | \(0\) | \(\theta(8 - 8\theta - 7\theta^2)\) \(2(1 + \theta)(40 - 13\theta^2)\) | \(4 - 5\theta - 3\theta^2\) \(3(1 + \theta)(8 - \theta^2)\) |
| \(q_1^{I12D2}\) | \(\frac{1}{4(1 + \theta)}\) | \(20 + 10\theta - \theta^2\) \(2(1 + \theta)(40 - 13\theta^2)\) | \(4 + 3\theta\) \(3(8 - \theta^2)\) |
| \(q_2^{I12D2}\) | \(\frac{1}{4(1 + \theta)}\) | \(\theta(8 - 8\theta - 7\theta^2)\) \(2(1 + \theta)(40 - 13\theta^2)\) | \(4 - 5\theta - 3\theta^2\) \(3(1 + \theta)(8 - \theta^2)\) |
| \(\pi_{M1}^{I12D1}\) | \(\frac{5 + 3\theta}{16(1 + \theta)}\) | \(\pi_{M1}^{I12D2}\) | \(\pi_{M1}^{I12D2}\) |
| \(\pi_{M2}^{I12D1}\) | \(\frac{1 - \theta}{8(1 + \theta)}\) | \(\pi_{M2}^{I12D2}\) | \(\pi_{M2}^{I12D2}\) |
| \(\pi_{R1}^{I12D1}\) | \(0\) | \(\pi_{R2}^{I12D2}\) | \(\pi_{R2}^{I12D2}\) |
Table 1 (Cont’d): Optimal Values of the Variables for Different Channel Configurations Where Decentralized Manufacturer Sets a Common Wholesale Price for the Two Retailers

<table>
<thead>
<tr>
<th>Var.</th>
<th>(I1, I2)</th>
<th>(I12, I2)*</th>
<th>(I12, I12)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_i*$</td>
<td>N/A</td>
<td>$w_i^{I12*} = \frac{(1-\theta)(10+\theta)}{2(10-\theta^2)}$</td>
<td>$5(1-\theta)$</td>
</tr>
<tr>
<td>$p_i*$</td>
<td>$\frac{1}{2+\theta}$</td>
<td>$p_i^{I12*} = \frac{(2-\theta)(5+\theta)}{2(10-\theta^2)}$</td>
<td>$5 - 2\theta$</td>
</tr>
<tr>
<td>$q_i*$</td>
<td>$\frac{1}{2+\theta}$</td>
<td>$q_i^{I12*} = \frac{(2-\theta)(5+\theta)}{2(10-\theta^2)}$</td>
<td>$q_i^{I12*} = \frac{(5+3\theta)}{(1+\theta)(10-\theta)}$</td>
</tr>
<tr>
<td>$\pi_M$</td>
<td>$\frac{1}{(2+\theta)^2}$</td>
<td>$\pi_M^{I12*} = \frac{(2-\theta)(10+5\theta+\theta^2)}{8(1+\theta)(10-\theta^2)}$</td>
<td>$\pi_M^{I12*} = \frac{(5+\theta)(5-2\theta)}{(1+\theta)(10-\theta)^2}$</td>
</tr>
<tr>
<td>$\pi_r$</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
</tbody>
</table>

★ By symmetry, $\pi_{M2}^{I112*} = \pi_{M1}^{I12*}$ and $\pi_{M1}^{I112*} = \pi_{M2}^{I12*}$. The optimal values of other variables can be obtained in a similar way under the (I1, I12) configuration.
Table 2: Optimal Values of the Variables for Different Channel Configurations where Decentralized Manufacturer Sets Different Wholesale Prices for the two Retailers

<table>
<thead>
<tr>
<th>Var</th>
<th>(D12, D2)</th>
<th>(D12, D12)</th>
<th>(I1, D12)*</th>
<th>(I12, D12)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_{i1}^{D12D2*}$</td>
<td>$\frac{4 - 3\theta}{4(2 - \theta)}$</td>
<td>$\frac{1 - \theta}{2 - \theta}$</td>
<td>$\frac{1 - \theta}{2 - \theta}$</td>
<td>$\frac{5(1 - \theta)(2 + \theta)}{4(5 - \theta^2)}$</td>
</tr>
<tr>
<td>$w_{i2}^{D12D2*}$</td>
<td>$\frac{1 - \theta}{2 - \theta}$</td>
<td>$\frac{1 - \theta}{2 - \theta}$</td>
<td>$\frac{1 - \theta}{2 - \theta}$</td>
<td>$\frac{1 - \theta}{2}$</td>
</tr>
<tr>
<td>$w_{ij}^{D12D12*}$</td>
<td>$\frac{1 - \theta}{2 - \theta}$</td>
<td>$\frac{1 - \theta}{2 - \theta}$</td>
<td>$\frac{1 - \theta}{2 - \theta}$</td>
<td>$(1 - \theta)(20 + 10\theta + \theta^2)$</td>
</tr>
<tr>
<td>$p_{i1}^{D12D2*}$</td>
<td>$\frac{16 - 11\theta}{12(2 - \theta)}$</td>
<td>$\frac{4 - 3\theta}{3(2 - \theta)}$</td>
<td>$\frac{6 - \theta}{12}$</td>
<td>$\frac{30 - 5\theta - 9\theta^2}{12(5 - \theta^2)}$</td>
</tr>
<tr>
<td>$p_{12}^{D12D2*}$</td>
<td>$\frac{18 - 14\theta + \theta^2}{12(2 - \theta)}$</td>
<td>$\frac{8 - 3\theta}{12}$</td>
<td>$\frac{80 - 30\theta - 21\theta^2 + 3\theta^3}{24(5 - \theta^2)}$</td>
<td></td>
</tr>
<tr>
<td>$q_{i1}^{D12D2*}$</td>
<td>$\frac{1}{6}$</td>
<td>$\frac{1}{3(1 + \theta)(2 - \theta)}$</td>
<td>$\frac{2 + \theta}{4(1 + \theta)}$</td>
<td>$\frac{20 + 10\theta - 6\theta^2 - 3\theta^3}{8(1 + \theta)(5 - \theta^2)}$</td>
</tr>
<tr>
<td>$q_{12}^{D12D2*}$</td>
<td>$\frac{4 - \theta + \theta^2}{12(1 + \theta)(2 - \theta)}$</td>
<td>$\frac{2 - \theta}{12(1 + \theta)}$</td>
<td>$\frac{(2 + \theta)\theta^2}{4(1 + \theta)(5 - \theta^2)}$</td>
<td></td>
</tr>
<tr>
<td>$q_{21}^{D12D2*}$</td>
<td>$\frac{1}{2(1 + \theta)(2 - \theta)}$</td>
<td>$\frac{1}{6}$</td>
<td>$\frac{20 - 10\theta - 9\theta^2}{24(1 + \theta)(5 - \theta^2)}$</td>
<td></td>
</tr>
<tr>
<td>$\pi_{M1}^{*}$</td>
<td>$\frac{16 - 12\theta - 3\theta^2 + \theta^3}{24(1 + \theta)(2 - \theta)^2}$</td>
<td>$\frac{2(1 - \theta)}{3(1 + \theta)(2 - \theta)^2}$</td>
<td>$\frac{20 + 8\theta - 3\theta^2}{72(1 + \theta)}$</td>
<td>$\frac{4000 + 1600\theta - 2400\theta^2 - 1260\theta^3 + 45\theta^4 + 63\theta^5}{576(1 + \theta)(5 - \theta^2)^2}$</td>
</tr>
</tbody>
</table>
\[
\begin{array}{|c|c|c|c|}
\hline
\pi_{M_2}^* &= \frac{1-\theta}{2(1+\theta)(2-\theta)^2} & \pi_{M_2}^* &= \frac{2-\theta}{12(1+\theta)} & \pi_{M_2}^* &= \frac{400-200\theta-160\theta^2+20\theta^3-39\theta^4-21\theta^5}{96(1+\theta)(5-\theta^2)^2} \\
\pi_{R_1}^* &= \frac{1}{36} & \pi_{R_2}^* &= \frac{52-12\theta+9\theta^2+\theta^3}{9(1+\theta)(2-\theta)^2} & \pi_{R_2}^* &= \frac{100+100\theta+60\theta^2+120\theta^3+105\theta^4+27\theta^5}{144(1+\theta)(5-\theta^2)^2} \\
\pi_{R_2}^* &= \frac{2}{9(1+\theta)(2-\theta)^2} & \pi_{R_2}^* &= \frac{1}{36} & \pi_{R_2}^* &= \frac{100+100\theta+60\theta^2+120\theta^3+105\theta^4+27\theta^5}{144(1+\theta)(5-\theta^2)^2} \\
\hline
\end{array}
\]

1. (D1, D1) and (D1, D2) under wholesale price discrimination are identical to (D1, D1) and (D1, D2) under a common \( w \) is applicable to both retailers, respectively.
2. By symmetry, the respective results for (D2, D1) and (D2, D2) can be derived from (D1, D2) and (D1, D1). Results for (D1, D12), (D2, D12), and (D12, D1) can be similarly derived from (D12, D2), respectively.
3. (I1, D1), (I1, D2), (I12, D1), and (I12, D2) under wholesale price discrimination are identical to (I1, D1), (I1, D2), (I12, D1), and (I12, D2) under a common \( w \) is applicable to both retailers, respectively.