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1. Introduction

The choice between money and interest rate as the monetary instrument in the presence of different structural disturbances has long been discussed, even before Poole (1970)'s formalization. In his survey article, Friedman (1989) explains the monetary instrument problem as follows.

> The "instrument problem" of monetary policy arises because of the need to specify how the central bank will conduct its open market operations. In particular, the instrument problem is the choice of a variable to be set directly by the central bank via buying and selling securities, and hence the value of which is to serve as the principal guide in carrying out that buying and selling function....

In this explanation, it is clear that not only effects of the changes in money or interest rate but also effects of the changes in government securities on the economy should be considered as a result of open market operation.

However, past analyses following Poole (1970), consider only one side of open market operation, changes in money and interest rate. This probably is due to traditional analysis that simplifies the government bond market equilibrium by simplifying the consumer's choice between government bond and money or by disregarding the implication of the government budget constraint.

There have been recent theoretical developments following Aiyagari and Gertler (1985), Sims (1988, 1994), Leeper (1991), and Woodford (1995), which emphasize careful examinations on the consumer's government bond holdings and the government budget constraint. These studies assert that there are two equilibrium relations determining the price level; the one that is consistent with conventional models, and the other that depends on fiscal policy. These studies incorporate the other side of open market operations. Based on these studies, this paper analyzes the role of fiscal policy on the choice between money and interest rate as the monetary instrument.

Following Leeper (1991), monetary and fiscal policies are categorized as "active" or "passive." The "active" policy authority sets policy variables without paying attention to the government budget constraint while the "passive" policy authority has the burden of satisfying the government budget constraint. Only when one of monetary and fiscal policies is active (and the other is passive), a unique equilibrium is obtained. In particular, a constant money growth rate policy is categorized as an active monetary policy so that a passive fiscal policy is necessary for a unique equilibrium while a constant interest rate policy is categorized as a passive monetary policy so that an active fiscal policy is necessary.

Past research that did not explicitly specify fiscal policy and did not examine the implication of the government budget constraint proceeds in two ways. First, the interest rate pegging is claimed to lead to an indeterminacy in the rational expectation model, for example, Sargent and Wallace (1975) and Smith (1988). Second, some special solution methods are used for the interest rate pegging, for example, Parkin (1978), McCallum (1981, 1983), Canzoneri et al (1983). In contrast, by specifying different fiscal policies, this paper compares the constant money growth rate and the constant interest rate policies in the model where a unique equilibrium is obtained.[1] That is, the constant money growth rate-passive fiscal regime and the constant interest rate-active fiscal regime are compared. In general, this paper compares the active monetary-passive fiscal regime where monetary policy determines the price level as in conventional models and the passive monetary-active fiscal regime where fiscal policy determines the price level as in the theory of fiscal determination of the price level.

To obtain analytic solutions, a simple structure of the model (an endowment economy model with the money-in-utility function framework) is assumed. Five kinds

[1] Appendix 2 provides some connections between the present analysis and past research.

of structural disturbances are examined – money demand shocks, aggregate demand shocks (discount rate shocks), endowment shocks, monetary policy shocks, and fiscal policy shocks. Since the production side is simplified, policy regimes are compared in terms of the variance of inflation, in contrast to Poole (1970) that assumes a fixed price level and compares policies in terms of the variance of output.[2]

When a fiscal policy is explicitly specified to guarantee a unique equilibrium for each monetary policy, Poole's results and reasoning are not useful. The difference of the regimes results from the role of monetary and fiscal policy in financing government deficit and satisfying the government budget constraint. Poole's results and reasoning are restricted to the active monetary-passive fiscal regime; for example, a combination policy (using both interest rate and money growth rate) is not better than the constant interest rate policy in the passive monetary-active fiscal regime.

In addition, some interesting results are found. First, in the passive monetary-active fiscal regime, an increase in the steady state real value of nominal government debt (bonds) reduces the variance of inflation in the presence of money demand and endowment shocks. In the regime, any shocks generating changes in the present value of net government deficits are financed by inflation tax, which depends on the steady state real value of nominal government liabilities (money and government bonds). An increase in the steady state real value of nominal government bonds decreases the size of inflation that is required to collect the given amount of inflation tax.

[2] The utility function defined in the model may be explicitly used to evaluate the policy regimes. However, since an endowment economy is assumed, there are not many interesting welfare questions and the welfare changes are minor. So, the variance of inflation, which often shows up in the policy literature as one objective, is used.

In addition, the results in the present model are similar to those in Poole (1970) with the policy objective as minimizing the variance of inflation, when we use a conventional solution method, which does not take care of the government budget constraint. Therefore, the results in this paper can be directly compared to those in Poole (1970). Refer to Appendix 2.

On the other hand, following the tradition of monetary regime comparison within general equilibrium models (for example, Sargent and Wallace (1982)), we compare a constant money growth rate rule and a constant interest rate rule, in contrast to studies following Poole (1970) that choose between money and interest rate. A similar result is obtained when a constant money and a constant interest rate rules are compared.

Second, using the result that the effects of passive policy on the contemporaneous inflation rate is almost null, we infer the following. 1) It is better for the authority with superior information on current disturbances and structural parameters and with less implementation errors to be active. 2) Only changes in the parameters of the active policy can generate the tradeoff in reducing the variance of inflation in the presence of different structural disturbances.

Beside the main objective of the paper, as a by-product, this paper provides careful examinations on dynamics of the passive monetary-active fiscal regime in the presence of different structural disturbances. It may be separate interests to see how monetary and fiscal policies interacts in response to different structural disturbances in order to satisfy the government budget constraint and what are the effects of different structural disturbances in the passive monetary-active fiscal regime (or in the model of fiscal determination of the price level).

Section 2 constructs the model and reviews the meaning of active and passive policies. Section 3 compares the constant money growth rate-passive fiscal and the constant interest rate-constant tax regimes. Section 4 extends the analysis to general active monetary-passive fiscal and passive monetary-active fiscal regimes. Section 5 concludes the paper with future research subjects.

2. The Model

2.1 The Structure of the Model

Each individual is endowed with exogenous income each period. There is fiat money and real money balance provides utility. For analytical tractability, we assume a log utility function in which consumption and real money balance are separable. Each individual maximizes his lifetime utility subject to his intertemporal budget constraint. Income is consists of endowments (Y_t) and gross interest income receipts from one-period nominal government bond holdings ($R_{t-1}B_{t-1}/P_t$, where B_{t-1} is one-period nominal government bond holdings, P_t is the price level, and R_{t-1} is the gross interest rate of the bonds). He allocates his income to consumption (C_t), changes in money holdings ($(M_t - M_{t-1})/P_t$), and nominal government bond holdings after paying net lump-sum tax (transfer if negative) to the government (τ_t). Each individual chooses C_t, M_t, and B_t, given P_t, Y_t, τ_t, V_t, K_t, and R_t.

$$\max_{\{C_t, M_t, B_t\}} E_t\left[\sum_{t=0}^{\infty} \beta^t K_t\left(\log C_t + V_t \log\frac{M_t}{P_t}\right)\right] \qquad \text{s.t.}$$

$$C_t + \frac{M_t - M_{t-1}}{P_t} + \frac{B_t - R_{t-1}B_{t-1}}{P_t} + \tau_t = Y_t \tag{1}$$

where $M_t \geq 0$, $B_t \geq 0$, and Y_t, V_t, and K_t are i.i.d. processes.

First order conditions of the consumer optimization problem are:

$$\frac{V_t C_t P_t}{M_t} = 1 - R_t^{-1} \qquad \text{and} \tag{2}$$

$$R_t^{-1} = \beta E_t\left[\frac{C_t P_t K_{t+1}}{C_{t+1} P_{t+1} K_t}\right]. \tag{3}$$

Equation (2) is the money demand relation and equation (3) is an intertemporal version of the aggregate demand relation. Therefore, we can interpret V_t as money demand shocks and K_t as a kind of aggregate demand shocks (or discount rate shocks). In response to a positive shock in K, each individual becomes less patient and the demand for current consumption increases.

The government should satisfy the budget constraint. It issues debt and money and collects lump-sum (net) tax (or transfer if negative).

$$\frac{M_t - M_{t-1}}{P_t} + \frac{B_t - R_{t-1}B_{t-1}}{P_t} + \tau_t = 0. \tag{4}$$

From the private budget constraint and the government budget constraint, the social resource constraint is:

$$Y_t = C_t. \tag{5}$$

The monetary authority is assumed to control a combination of growth rate of money and interest rate:[3]

$$\alpha_m \frac{M_t}{M_{t-1}} + \alpha_r R_t = \rho + \eta_t . \tag{6}$$

$\alpha_m = 1$ and $\alpha_r = 0$ imply that the monetary authority fixes the money growth rate at ρ. On the other hand, $\alpha_m = 0$ and $\alpha_r = 1$ imply that the monetary authority fixes the interest rate at ρ.

The fiscal authority sets the net tax level as a feedback rule from outstanding government bonds in real terms:[4]

$$\tau_t = \gamma_0 + \gamma \frac{B_{t-1}}{P_{t-1}} + \mu_t . \tag{7}$$

The disturbance terms in the monetary and fiscal policy rules (η_t and μ_t, respectively) are interpreted in two ways. First, they are interpreted as implementation errors. The effect of implementation errors on the economy is also a standard for

[3]Monetary policy is described as choosing a combination of the interest rate and the money growth rate, in order to examine the choice between them. This monetary policy can specify the complete region of active and passive monetary policy. Sims (1988) and Leeper (1991) describe monetary policy as interest rate setting rule in response to current inflation. Any combination of two variables among money growth rate, interest rate, and inflation can specify the complete region.

comparing different policy rules. Second, they are interpreted as discretionary policy. Under this interpretation, the effectiveness of discretionary policy in the presence of non-policy disturbances can be analyzed.

2.2. Steady State

By defining $h_t = M_t/M_{t-1}$, $b_t = B_t/P_t$, $\pi_t = P_t/P_{t-1}$, and $m_t = M_t/P_t$, the steady state relations follow.

$$\frac{\pi}{R} = \beta \tag{8}$$

$$h = \pi \tag{9}$$

$$m = Y/V \;, \quad m = \frac{YV}{1 - R^{-1}} \tag{10}$$

$$b = \frac{\gamma_0 - m\left(1 - \frac{1}{\pi}\right)}{\left(1 - \frac{1}{\beta} + \gamma\right)} \tag{11}$$

$$\alpha_m h + \alpha_r R = \rho \tag{12}$$

$$\tau = \gamma_0 + \gamma b \tag{13}$$

By choosing different values of policy parameters (α_m, α_r, and ρ), the monetary authority determines the steady state inflation rate. Under specific values of α_m and α_r, by choosing an appropriate steady state value of ρ, any inflation rate can be achieved.[5]

[4]This fiscal policy rule, used by Sims (1988) and Leeper (1991) (similar rules are used in Aiyagari and Gertler (1985) and Sims (1994)), can describe the complete region of active and passive fiscal policy rule. Other fiscal policy rules such as the AR-1 rule of net tax level also can describe the complete region.

[5] When the monetary authority fixes the money growth rate ($\alpha_m=1$, $\alpha_r=0$, and $h=\rho$), the steady state inflation rate is ρ from equation (9), R is determined from equation (8), m and V are determined from

2.3. Specification of Fiscal Policies for the Unique Equilibrium

The model is linearized around the steady state. Using the steady state relations, the system of equations is summarized as:

$$h_t - \pi_t + \frac{1}{(R-1)} R_t = \frac{1}{(R-1)} R_{t-1} + Y_t - Y_{t-1} + V_t - V_{t-1} \tag{14}$$

$$E_t \pi_{t+1} - R_t = Y_t - K_t \tag{15}$$

$$\begin{aligned} b_t + \left(\frac{1}{\beta} + \frac{m}{b\pi} \right) \pi_t - \frac{m}{b(R-1)} R_t = \\ \frac{1}{\beta} b_{t-1} + \left(\frac{1}{\beta} - \frac{m}{b\pi(R-1)} \right) R_{t-1} - \frac{m}{b} (Y_t + V_t) + \frac{m}{b\pi} (Y_{t-1} + V_{t-1}) - \tau_t \end{aligned} \tag{16}$$

$$h\alpha_m h_t + R\alpha_r R_t = \eta_t \tag{17}$$

$$\tau\tau_t = \gamma b b_{t-1} + \mu_t \tag{18}$$

where each variable without subscript is the steady state values and each variable with subscript is the deviation from the steady state divided by its steady state value (for both policy shocks, they are just deviations from the steady state values, not divided by their

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equations (10) given Y. In this case, fiscal policy determines only b and τ by choosing γ_0 and γ. On the other hand, in the case of the constant interest rate rule ($\alpha_m=0$, $\alpha_r=1$, and $R=\rho$), equation (8) determines the steady state inflation rate ($\beta\rho$). Note that even in the case of the constant interest rate rule, the fiscal authority cannot determine the steady state inflation rate. The fiscal authority only determines the steady state direct taxation and bonds.

Note that the monetary authority determines the steady state inflation tax on money holdings and seignorage. In contrast, from equations (11) and (13), the fiscal authority determines the size of direct taxation. Inflation tax on bond holdings is determined from the steady state inflation rate set by the monetary authority and bond holdings which can be controlled by the fiscal authority.

steady state values).[6] When the policy authorities set the policy parameters (α_m, α_r, ρ, γ_0, and γ) and the exogenous processes (K_t, Y_t, V_t, η_t, and μ_t)are realized, h_t, b_t, π_t , C_t ($=Y_t$), m_t, and R_t are determined. The steady state nominal interest rate (R) is assumed to be greater than 1, which implies the steady state inflation rate (π) and the steady state money growth rate (h) are greater than β from the steady state relations.[7]

In this system, there are two roots that can lie outside the unit circle, $1/\beta-\gamma$ and $hR\alpha_m/(h\alpha_m+R\alpha_r-R^2\alpha_r)$. Since this system includes only one expectational term ($E_{t-1}\pi_t$), only one root should lie outside the unit circle to have a unique equilibrium.[8] While the first root depends on the fiscal parameter γ, the second root depends on monetary parameters α_m and α_r. When the first root lies outside the unit circle and the second root lies inside, the fiscal authority has the burden of keeping the government budget constraint satisfied by changing the net tax level in response to government deficit changes (active monetary-passive fiscal policy).[9] On the other hand, when the first root lies inside the unit circle and the second lies outside, the monetary authority satisfies the government budget constraint by changing the growth rate of money and by collecting seignorage and inflation tax (passive monetary-active fiscal policy). In the first case, Quantity Theory of Money and Ricardian Equivalence hold as in conventional models

[6]Equation (14), (15), and (17) can be expressed with m_t instead of h_t. I use h_t to simplify the monetary policy equation (17).

[7] First, it makes the steady state government bond and money holdings positive (equation (10)). Second, when R is less than 1, the behavior of the system is unusual in the sense that the demand for (the growth rate of) real money balance is an increasing function of the interest rate (from equation (14)). Third, we can interpret a money growth rate rule as an active monetary policy without reference to the steady state value (Refer to the next section).

[8]This is the condition from Blanchard and Kahn (1980). In general, we need additional conditions found by Sims (1995). In the present model, the condition from Blanchard and Kahn (1980) is a necessary and sufficient condition for a unique equilibrium.

[9]More precisely, the first root needs to be less than $1/\beta$. When the first root is equal or greater than 1 and less than $1/\beta$, real debt (and tax) explodes but this equilibrium violates neither the transversality condition nor the feasibility condition. Refer to McCallum (1983) and Sims (1995). Here, I exclude this equilibrium for simplicity. Also, this equilibrium seems unrealistic.

while in the second case, the inflation rate depends on the total government liabilities and fiscal policy. [10]

In contrast to past research on the monetary instrument problem, active or passive fiscal policy is specified in order to guarantee a unique equilibrium under a particular monetary policy. First, a constant money growth rate policy implies that the second root (R) lies outside the unit circle and the monetary authority cannot change the money growth rate in order to collect seignorage and inflation tax in response to government deficit shocks. In this case, to guarantee a unique equilibrium, fiscal policy should be passive or the fiscal parameters should lie in the range of $|1/\beta-\gamma| < 1$ (the second root lies inside the unit circle). That is, fiscal policy should adjust net tax level enough to satisfy the government budget constraint in response to government deficit changes.

However, in the case of a constant interest rate policy, active fiscal policy should be specified for a unique equilibrium. Since the second root (0 since $\alpha_m=0$) lies inside the unit circle, the first root should lie outside the unit circle to achieve a unique equilibrium ($|1/\beta-\gamma| > 1$). That is, fiscal policy does not adjust enough so that the monetary authority should collect seignorage or inflation tax to satisfy the government budget constraint.

3. Constant Money Growth Rate-Passive Fiscal Regime vs. Constant Interest Rate-Constant Tax Policy Regime

In this section, the constant money growth rate-passive fiscal and the constant interest rate-constant tax regimes are compared.[11] I assume that the policy objective is

[10] In the first case, the equilibrium inflation rate is determined from equations (14), (15), and (17) without referring to equations (16) and (18). Equations (16) and (18) determine net tax level and the real government debt only. In contrast, in the second case, the equilibrium inflation rate is determined from equations (15), (16), (17), and (18). Equation (14) determines the growth rate of money only.

[11] The constant tax policy is chosen as the representative case of active fiscal policies since the constant tax policy implies that the fiscal authority does not change tax in response to government deficit changes. In

minimizing the variance of inflation under different sources of disturbances, given the steady state inflation rate.[12] Five kinds of disturbances are considered -- money demand shocks, aggregate demand shocks (discount rate shocks), endowment shocks, monetary policy shocks, and fiscal policy shocks.[13] Finally, the policy authorities are assumed not to have information on the current disturbances except for the case that monetary policy shocks and fiscal policy shocks are considered as discretionary policy on current disturbances.[14]

3.1. Equilibrium

From the linearized system (equations (14)-(18)), solutions are obtained following Sims (1995). The complete solutions in both regimes are reported in Appendix 2. In the case of the constant money growth rate-passive fiscal regime, the solution for inflation is:

$$\pi_t = -\left(1-\frac{1}{R}\right)\left(V_t - V_{t-1}\right) - \left(Y_t - Y_{t-1}\right) + \frac{1}{h}\eta_t + \frac{1}{R}\left(K_t - K_{t-1}\right) \tag{19}$$

In this regime, the effects are the same as those in the conventional models in which Quantity Theory of Money and Ricardian Equivalence hold. A positive money demand and a positive endowment shock decrease the inflation rate while a positive aggregate demand shock increases the inflation rate. In the next period, since the money growth

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addition, solution is simple under the constant tax policy, compared to other active fiscal policies. In Section 4, more general forms of active fiscal policies are analyzed.

12 The mean of inflation rate (or steady state inflation rate) is not considered as a policy objective since both regimes (in general, any policy combination that can achieve a unique equilibrium in this model) can achieve any steady state inflation rate by adjusting the constant term (ρ). Refer to Section 2.2.

13 The optimal policy rule is not formally derived since the objective of the analysis is to reexamine the monetary instrument problem and Poole (1970)'s results by considering the implication of government budget constraint and fiscal policy. Further major insights do not seem to be obtained by deriving optimal policy rule formally.

14 In some cases, we do not discuss the lagged effects. If the policy authority is allowed to set up a feedback rule on lagged variables which offsets the lagged effects, then only contemporaneous effects matter. In addition, once no serial correlations of the structural disturbances are assumed, there are only few cases where the lagged effects change the qualitative conclusion in the present analysis. Therefore, in some cases where the lagged effects do not change the results much, we do not discuss the lagged effects.

rate and the steady state inflation are constant, the price level returns to the original growth path. That is, the absolute values of the first and second period inflation rates are the same, but they have opposite signs. However, monetary policy shocks (money growth rate shocks) have only contemporaneous effects. In response to monetary policy shocks, the price level does not return to the original growth path in the second period because the money stock jumps from the original growth path. Notably, fiscal policy (neither fiscal policy shocks nor the steady state values of b, τ, γ_0, and γ) does not have any effects.[15]

In the case of the constant interest rate-constant tax regime, the solution for inflation is:

$$\pi_t = -\left(\frac{\beta m(R-1)}{m+bR}\right)V_t - \left(\beta + \frac{\beta m(R-1)}{m+bR}\right)Y_t + Y_{t-1} + \frac{1}{R}\eta_{t-1} - \left(\frac{\beta R}{m+bR}\right)\mu_t + \beta K_t - K_{t-1} \quad (20)$$

For this regime, detailed explanations are provided since the effects are different from those in conventional models. In response to a positive money demand shock (V_t), since the interest rate is constant, the money growth rate and the real money balance (from equation (14)) increase, and seignorage increases. Since net tax is constant in this regime, the monetary authority should offset the increase in seignorage by negative inflation tax in order to satisfy the government budget constraint. Therefore, the current inflation rate decreases. The size of the inflation rate decrease $\left(\frac{\beta m(R-1)}{m+bR}\right)$ depends on the steady state real balance (m) and the steady state real value of total nominal government liabilities or overall inflation tax base (money and gross interest payment of bonds, m+bR). This is because the real money balance determines the initial increase in seignorage and the total government liabilities determines the amount of inflation tax

[15] Fiscal policy does not have effects on any variables except for fiscal variables. Refer to Appendix 1.

that is collected by the inflation decrease. The larger the steady state real value of nominal government bond holdings is, the smaller the changes in the inflation rate.

A positive aggregate demand shock (K_t) increases the current inflation rate, which generates a positive inflation tax today. This increase in inflation tax should be offset by negative inflation tax. Therefore, in the next period, there is a deflation.[16] Note that the changes in the inflation rate (β) do not depend on the size of nominal government liabilities. This is because the initial inflation is generated from the disturbance, not from the monetary reaction to a disturbance changing government deficit in order to collect inflation tax.

Endowment shocks (Y_t) alter both goods market and money market equilibria, so the effects are the combination of the effects of a negative aggregate demand and a positive money demand shock.

Fiscal policy shocks affect the inflation rate contemporaneously. In response to a positive net tax shock (μ_t), the monetary authority generates a deflation and collects a negative inflation tax (the size of which depends on the steady state real value of total government liabilities ($m+bR$)) to satisfy the government budget constraint. Monetary policy shocks (η_t) have only lagged effects on the inflation rate. They alter only the composition of government liabilities (an increase in bond holdings and a decrease in the real money balance, caused by an increase in the nominal interest rate), but not the size of the total government liabilities, which determines the current inflation rate. In the next period, the inflation rate increases and inflation tax is collected to offset the increased debt service. The steady state government liabilities do not affect the size of the lagged inflation change because both the inflation tax collection and the increased debt service are affected by the steady state government liabilities and they are canceled out.

3.2 Comparison

[16] Note that the discounted value of the deflation in the next period is equal to the current inflation.

In this section, two regimes are compared, with minimizing the variance of inflation as the policy objective. Table 1 (a) reports results when the steady state inflation rate is one ($\pi=1$) and the steady state government bond holdings are zero ($b=0$). Table 1 (b) reports general results. In each case, contemporaneous and lagged effects are reported separately. The first column shows the type of disturbances. The second and the fourth columns report the inflation responses in the constant money growth-passive fiscal and the constant interest rate-constant tax regimes, respectively. The third column under 'rel. var.' compares the size of inflation responses (absolute values) in two regimes. '$>$' ('$<$') implies that the inflation response is larger (smaller) in the constant money growth-passive fiscal regime than in the constant interest rate-constant tax regime. 'b,' 'π,' and 'b, π' in Table 1 (b) imply that the results depend on the steady state value of b, π, and both b and π, respectively.

The effects of non-policy disturbances are examined first.[17] When $\pi=1$ and $b=0$, the contemporaneous effects on the inflation rate are the same in both regimes. However, in some sense, the constant interest rate policy is better in the presence of money demand shocks while the constant money growth rate policy is better in the presence of aggregate demand shocks, as in Poole (1970), even though the reasoning is different; The difference in two regimes results from different methods of financing government deficit changes.

First, in terms of the lagged effects, results are similar to those of Poole (1970). In the presence of aggregate demand (money demand) shocks, the lagged inflation response is smaller (larger) in the constant money growth rate-constant tax regime (the constant interest rate-active fiscal regime), and the constant money growth rate-constant tax regime (the constant interest rate-active fiscal regime) is better. In addition, in the presence of money demand shocks, the variance of inflation can be reduced by non-zero

[17] We skip the results on the endowment shocks since the inflation response in the presence of endowment shocks are the same as the one in the presence of both a negative aggregate demand and a positive money demand shock.

steady state government bond holdings in the constant interest rate-constant tax regime, since non-zero steady state government bond holdings increase inflation tax base (shown in Table 1 (b) and equation (20)).[18][19] As discussed in the previous section, these differences result from the different method of financing government deficit changes in each regime.

Table 1 (b) shows that in general, which regime is better depends on the value of π or/and b. The effects in each regime depends on the steady state inflation rate.[20] More interestingly, the effects in the constant interest rate-constant tax regime depends on the steady state real value of nominal government bond holdings (We provide detailed explanations for this in the next section.). Again, the difference between these two regimes results from different methods of financing government deficit changes.

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[18] Note that the fiscal authority can choose different values of the steady state government bond holdings by choosing γ_0 and γ. Refer to the steady state relations in Section 2.2 and footnote 5.

[19] We discuss how serial correlation of shocks (without any counter-cyclical policy on lagged effects) affects the results (when π and b are zero). In the constant money growth rate-passive fiscal regime, in response to non-monetary policy shocks, the price level returns to the original growth path in the next period. However, in the constant interest rate-constant tax regime, the price level does not. First, in the case of money demand shocks, the lagged effect is zero in the constant interest rate-constant tax regime. In this case, a negative (a high positive) correlation of the shocks results relatively large reduction of the variance of the inflation rate in the constant money growth rate-passive fiscal regime (the constant interest rate-constant tax regime). Second, in the case of aggregate demand shocks, the lagged effect becomes even larger than the current effect in the constant interest rate-constant tax regime. In this case, any serial correlation does not make the constant interest rate-constant tax regime better than the constant money growth rate-passive fiscal regime.

[20] In the constant money growth rate-passive fiscal regime, as the steady state inflation rate (and the steady state nominal interest rate) increases, the interest rate elasticity of money demand decreases (equation (14)). So, the inflation rate changes more in response to money demand shocks. But in response to aggregate demand shocks, the inflation rate changes less since less offsetting effects work as a result of the decrease in the interest rate elasticity of money demand. (A positive aggregate demand shock increases demand for current consumption, so the current inflation increases but the current increase in inflation is offset by an increase in the demand for real money balance due to the increase in the demand for current consumption.)

In the constant interest rate-constant tax regime, as the steady state inflation rate increases, the elasticity of inflation with respect to the real money balance increases and the inflation response increases in the presence of money demand shocks. (We can see this elasticity by transforming equation (16) to an equation with m_t instead of h_t.)

Therefore, the fundamental difference between two regimes results from whether equation (14) or equation (16) is used to determine the price level, that is, from different methods of financing government deficit changes.

We can also compare these two regimes based on the effects of policy shocks. First, those shocks are interpreted as implementation errors, and we choose the regime that does not allow those errors to affect inflation much. In the constant money growth rate-passive fiscal regime, implementation errors in fiscal policy do not affect the inflation rate. In the constant interest rate-constant tax regime, implementation errors in monetary policy have only lagged effects on inflation rate, which may be offset by fiscal discretion. Therefore, when there are implementation errors in fiscal policy (monetary policy), the constant money growth rate-passive fiscal regime (the constant interest rate-constant tax regime) is better. That is, it is better for the authority with less implementation errors to be active.

Second, those shocks are interpreted as discretion or counter reactions to current disturbances under the assumption that policy authorities have some information on current disturbances and structural parameters.[21] Since only the active policy can have contemporaneous effects on the inflation rate, only the active authority can effectively perform discretionary policy against current disturbances. Even with current information on disturbances, the passive authority cannot reduce the variance of inflation. Therefore, if both policy authorities have current information on the disturbances and structural parameters but one has better information than the other, then it is better for the authority with better information to be active.

In summary, Poole (1970)'s basic results do not necessarily hold in this model. Even when the results are similar, the reasoning is different: the results are based on the role of monetary and fiscal policies in financing government deficit changes and satisfying the government budget constraint in each regime. In addition, there are some interesting findings. First, it is better for the authority with superior information and less implementation errors to be active. Second, the variance of inflation depends on the

[21]Of course, when both authorities have exact information on current disturbances (also if they know the structural parameters), then they can perfectly offset changes in inflation rate due to current disturbances.

steady state real value of nominal government debt (bonds) in the constant interest rate-constant tax regime. In the next section, the latter result is discussed in details.

3.3. Nominal Government Debt (Bonds) Reduces the Variance of Inflation

In the constant interest rate-constant tax regime, an increase in the steady state real value of nominal government debt (bonds) reduces the variance of inflation in the presence of money demand shocks, endowment shocks, and fiscal policy shocks.[22] Any shocks altering the present value of the government deficit are financed by inflation tax in this regime, which in turn depends on total nominal government liabilities. When nominal government bond holdings increase, total nominal government liabilities increase, and the size of inflation required collecting the given amount of inflation tax decreases. In Woodford (1995)'s term, when there are larger nominal government liabilities, wealth effects from holding the government liabilities per changes in the inflation rate become larger and the size of changes in the inflation rate becomes smaller when there is a shock that needs different level of equilibrium inflation rate.

Then, we may claim that larger steady state nominal government debt is better for reducing the variance of inflation since the steady state nominal government debt does not have any effects on the variance of inflation in other cases (the constant interest rate-constant tax regime in the presence of aggregate demand shocks and monetary policy shocks, and the constant money growth rate-passive fiscal regime in the presence of any types of shocks).[23]

[22]When we interpret the fiscal policy shocks as implementation errors, if we assume that implementation errors become larger as the steady state tax level increases, then there may be an optimal size of nominal government debt (in the presence of fiscal policy implementation errors) since an increase in the steady state government debt decreases the variance of inflation through smaller inflation given the required inflation tax but increases the variance of inflation through larger steady state government tax level and larger implementation errors. For example, if we assume that the implementation errors (the size of fiscal policy shocks) are proportional to the steady state government tax level, then the optimal size of government debt is $m(\beta R-1)/(R(1-\beta))$.

[23] The strategy of large steady state nominal government debt to reduce the variance of inflation may be especially valuable when policy authorities do not have an exact idea on the current regime or the nature of disturbances in the economy. Other strategies need some information on the nature of major disturbances in the economy and if the information is not precise, then there may be some costs. In

Some past research suggests that large government deficits are responsible for high inflation when it is monetized or even when it is financed by government bonds. As Leeper (1991) emphasized, the distinction between two policy regimes in this paper is based on the method of financing deficit changes, that is, the marginal finance method, and not based on the average level of finance. In the model, a negative μ_t, and government deficit changes due to other structural shocks, increase the inflation rate in the constant interest rate-constant tax regime since the monetary authority finance the deficit changes by collecting inflation tax, but the steady state inflation does not change and the inflation rate increase is only temporary.[24] On the average finance level, in the passive monetary-active fiscal regime, government deficits are already backed by seignorage and direct taxation.[25] Therefore, the finding can be rephrased as: a larger

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contrast, this strategy works without any costs. In addition, this strategy does not require any knowledge on structural parameters. In other words, this strategy is not an empirical matter in the present model.

On the other hand, this result may serve as a favorable argument for non-indexed government bonds. If the government indexes some factions of bonds, then the policy authority cannot collect inflation tax from the indexed bond holdings. Therefore, in response to disturbances changing government deficits, larger changes in inflation is required and the variance of inflation becomes larger.

Bohn (1988) also provides a reason for non-indexation. Under the assumption that welfare losses from distortionary taxation is a convex function of the tax rate, a smooth path for tax rate changes or a smaller variance of tax rates is optimal. In the case of real shocks that increase government deficits and increase inflation, inflation tax compensates for the government deficits. The larger the nominal government bonds, the larger the inflation tax, the smaller the variance of direct tax, and the larger the welfare. Therefore, in his model, government nominal debt is a device for hedging. As he emphasized, his argument is based on assumptions on distortionary taxation.

In contrast, in the present analysis, even without the assumption of distortionary taxation, non-indexed government bonds can be better than indexed government bonds. But, the assumption of distortionary direct taxation may work as a counter argument against non-indexed bonds in the present model. As in equations (11) and (13), larger steady state government bond implies larger steady state direct taxation and there may be some penalties or distortions when the steady state nominal government bonds increase. Therefore, in the model with distortionary direct taxation, the optimal size of government debt requires that the marginal costs of nominal government debt (from distortion) and the marginal benefit of nominal government debt (from reducing the variance of inflation) are equalized. In addition, as discussed in a previous footnote, if the implementation errors of fiscal policy increase as the size of steady state government direct taxation increases, then this increase in implementation errors will generate larger variance of the inflation rate, and thus there will be another penalty incurred with larger steady state nominal government debt.

[24] Remember that any steady state level of inflation can be achieved by different values of ρ.

[25] Refer to footnote 5.

nominal government debt is better for reducing the variance of inflation in the passive monetary-active fiscal regime when the government debt is fully backed.[26]

4. Extension: Active Monetary-Passive Fiscal Regime vs. Passive Monetary-Active Fiscal Regime

This section extends the analysis to general comparison between the active monetary-passive fiscal regime and the passive monetary-active fiscal regime by allowing α_m, α_r, and γ to be non-zero.

When non zero α_r ($1/(R(R-1)) \geq \alpha_r > -1/R$) is allowed, the inflation rate in the active monetary-passive fiscal regime (after normalizing α_m as $1/h$) is:

$$\pi_t = -\left(1-\frac{1}{R}\right)\left(V_t - V_{t-1}\right) - \alpha_r(R-1)V_t - \left(Y_t - Y_{t-1}\right) + \eta_t + \frac{1}{R}\left(K_t - K_{t-1}\right) - \alpha_r(R-1)K_t$$

Now, the relative effects of money demand shocks and aggregate demand shocks change. There is a tradeoff. By increasing the negative (positive) weight on interest rate, the variance of inflation due to money demand shocks decreases (increases), but the variance of inflation due to aggregate demand shocks increases (decreases). The minimum variance in the presence of money demand shocks is achieved when α_r is close to -1/R while the minimum variance in the presence of aggregate demand shocks is achieved when α_r is 1/(R(R-1)). We can derive the optimal policy that reduces overall variance of inflation once the relative variance of each shock is specified. The result is consistent with Poole (1970), in the sense that a combination policy (using both interest rate and money growth rate) dominates the pure interest rate pegging or the

[26]Aiyagari and Gertler (1985) suggested that in the region of a particular type of active fiscal policy (their "non-Ricardian regime"), inflation rate and the price level are higher. The exact definition of their non-Ricardian regime is different from the definition of an active fiscal policy in this paper. Again, any steady state inflation rate can be achieved in both regimes of the present analysis.

pure money (growth rate) pegging. Also, note that the fiscal parameters and fiscal policy shocks still do not have any effect on the inflation rate in this regime.

In the case of the passive monetary-active fiscal regime, we report the result from allowing either non-zero α_m or non-zero γ, but not both since the solution is very complicated when both parameters are allowed to be non-zero and since some basic intuitions can be obtained without doing that.

When non-zero α_m ($(R-1)/(h(R+1)) \geq \alpha_m \geq -1/h$) is allowed, the inflation rate in the passive monetary-active fiscal regime (after normalizing α_r as $1/R$) is:

$$\pi_t = \sum_{s=1}^{t}\left(\frac{h\alpha_m R}{R-1-h\alpha_m}\right)^{t-s}\left[\begin{array}{l} -\dfrac{\beta m(R-1)}{m+bR}V_s - \dfrac{h\alpha_m(R-1)(m+\beta m+bR)}{(R-1-h\alpha_m)(m+bR)}V_{s-1} + \dfrac{h\alpha_m(R-1)}{R-1-h\alpha_m}V_{s-2} \\ -\dfrac{\beta R(m+b)}{m+bR}Y_s - \left(\dfrac{h\alpha_m(R-1)(m+\beta m+bR)}{(R-1-h\alpha_m)(m+bR)} + \dfrac{h\alpha_m}{R-1-h\alpha_m}\right)Y_{s-1} \\ +\dfrac{h\alpha_m R}{R-1-h\alpha_m}Y_{s-2} \\ +\beta K_s + \dfrac{h\alpha_m}{R-1-h\alpha_m}K_{s-1} - \dfrac{R-1-h\alpha_m(1+\beta)}{R-1-h\alpha_m}K_{s-2} \\ -\left(\dfrac{\beta R}{m+bR}\right)\mu_s - \left(\dfrac{h\alpha_m R}{R-1-h\alpha_m}\right)\left(\dfrac{\beta R}{m+bR}\right)\mu_{s-1} \\ +\dfrac{h\alpha_m(R-1)\beta R}{(R-1-h\alpha_m)(m+bR-bR\beta+mR\beta)}\mu_{s-2} + \dfrac{R-1}{(R-1-h\alpha_m)R}\eta_{s-1} \end{array}\right]$$

The current effect of each shock on the inflation rate is exactly the same as in the constant interest rate policy case and α_m does not affect contemporaneous inflation at all. Therefore, in terms of the contemporaneous response of the inflation rate, we do not observe any tradeoffs by changing α_m and a combination policy is not better than the pure interest rate pegging, which is different from the results of Poole (1970).[27]

[27] There are prolonged lagged effects of disturbances on the inflation rate. So, even in terms of lagged responses of the inflation rate, the pure interest rate pegging dominates the combination policy. The reason for the delayed effects is that the monetary authority still collects the same size of inflation tax in the current period, which needs the same size of inflation and the same size of the increase in the growth rate of money. But since the monetary authority controls only the combination of money growth rate and

On the other hand, as in the constant interest rate-constant tax regime, an increase in the steady state nominal government debt decreases the variance of the inflation rate in this regime, regardless of the value of α_m. In addition, monetary policy shocks still have only delayed effects on inflation rate.

When non-zero γ is allowed ($| 1/\beta - \gamma | > 1$), the inflation rate is:

$$\pi_t = -\left(\frac{\beta m \left(R - \frac{1}{\beta(\frac{1}{\beta} - \gamma)} \right)}{m + bR} \right) V_t - \left(\frac{\beta R \left(\frac{b}{\beta(\frac{1}{\beta} - \gamma)} + m \right)}{m + bR} \right) Y_t + Y_{t-1}$$

$$+ \left(\frac{\beta\gamma m}{(\frac{1}{\beta} - \gamma)(R - 1)(m + bR)} \right) \eta_t + \frac{1}{R}\eta_{t-1} - \left(\frac{\beta R}{m + bR} \right) \mu_t + \frac{1}{(\frac{1}{\beta} - \gamma)} K_t - K_{t-1}$$

Now changes in the fiscal parameter γ generate the tradeoff between reducing the variance of inflation due to money demand shocks and reducing the variance of inflation due to aggregate demand shocks. Also, by allowing non-zero γ, the variance of inflation may be reduced.[28] In the passive monetary-active fiscal regime, there are neither the tradeoff nor the possibility of reducing the inflation rate when monetary parameters are changed, but there are the tradeoff and the possibility when fiscal parameters are changed.

Also, as in the previous case, an increase in the steady state nominal government debt decreases the variance of the inflation rate, regardless of the value of γ. On the other hand, now monetary policy shocks have current effects on the inflation

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interest rate, the interest rate also changes. This change in the interest rate changes the debt service next period, and it is financed by inflation tax in the next period, which changes the debt service in the following period and so on.

[28] For example, when $1/\beta - 1 > \gamma > 0$, parts of the government deficit shocks are financed by direct tax in the next period. As a result, in response to money demand shocks, the amount of necessary inflation tax decreases, and variance of inflation decreases. In contrast, in the case of aggregate demand shocks, the variance of inflation increases. In response to aggregate demand shocks, real bond holdings decrease, which leads to direct tax decreases in the next period. Therefore, to offset the next period direct tax decrease, the increase in the current inflation should be larger than the previous case. When $1/\beta - \gamma$ is close to $1/(R\beta)$, the variance due to money demand shocks are minimized. As $|1/\beta - \gamma|$ increases, the variance due to aggregate demand shocks and endowment shocks become smaller.

rate, but the discretionary monetary policy offsetting the effects of current disturbances is still not simple. It requires the exact information on fiscal policy parameter γ, in addition to the information on disturbances and other structural parameters. Especially, the sign of the inflation rate response to monetary policy shocks is sensitive to γ. When the fiscal authority sets γ close to zero but the monetary authority is not sure of the sign of γ, the monetary authority cannot perform the appropriate discretionary policy even with the precise knowledge of the nature and size of current disturbances and other structural parameters.

In summary, previous findings can be generalized. First, the difference of two regimes results from the role of monetary and fiscal policies in financing the government deficit changes and satisfying the government budget constraint. Second, an increase in the steady state real value of the nominal government bonds decreases the variance of the inflation rate in the passive monetary-active fiscal regime. Third, it is better for the policy authority with better information and with less implementation errors to be active. In addition, there is a new finding – a combination policy is better than the pure interest rate or the pure money growth rate pegging in the active monetary-passive fiscal regime, but not in the passive monetary-active fiscal regime. Poole (1970)'s results and reasoning hold only in the active monetary-passive fiscal regime, not in the passive monetary-active fiscal regime.

5. Conclusion

By constructing an endowment economy model with various stochastic disturbances, which specifies fiscal policy explicitly to guarantee a unique equilibrium, the monetary instrument problem is examined, with minimizing the variance of inflation as the policy objective. As in Poole (1970), which regime is better depends on the nature of disturbances, and the choice of the optimal policy regime is an empirical matter. However, Poole (1970)'s exact results and reasoning do not apply, especially in the passive monetary-active fiscal regime. Difference of inflation responses in different regimes should be explained by the role of monetary and fiscal policies in financing the government deficit changes and satisfying the government budget constraint in each regime.

Future research on the comparison between the active monetary-passive fiscal and the passive monetary-active fiscal regime under more general setup may be interesting. First, comparison in the model with nominal rigidities using both output and inflation variability or formal welfare criterion as the policy objective seems to be fruitful.[29] Second, the model with distortionary taxation may be another direction.

[29] See Leeper (1993) for some simulation results.

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Table 1. Responses of Inflation (Deviation from the Steady State) to Structural Shocks

* The column under 'rel. var.' compares absolute value of the responses of the inflation rate (or the variance of the inflation rate) in the constant money growth rate-passive fiscal and the constant interest rate-constant tax regimes. 'π,' 'b,' and 'π,b' imply that the relative size depends on π, b, or both π and b. b is non-negative.

(a) $\pi = 1$, $b = 0$

1. contemporaneous effects

Shocks	Const. M gr.-Pass. Fiscal	rel. var.	Constant R-Constant Tax
MD (V_t)	$-(1-\beta)$	=	$-(1-\beta)$
AD (K_t)	β	=	β
AS (Y_t)	-1	=	-1
MP (η_t)	1	>	0
FP (μ_t)	0	<	$1/m$

2. lagged effects

Shocks	Const. M gr.-Pass. Fiscal	rel. var.	Constant R-Constant Tax
MD (V_t)	$(1-\beta)$	>	0
AD (K_t)	$-\beta$	<	-1
AS (Y_t)	1	=	1
MP (η_t)	β	>	0
FP (μ_t)	0	=	0

(b) General case

1. contemporaneous effects

Shocks	Const. M gr.-Pass. Fiscal	rel. var.	Constant R-Constant Tax
MD (V_t)	$-(1-\beta/\pi)$	π,b	$-m(\pi-\beta)/(m+b\pi/\beta)$
AD (K_t)	β/π	π	β
AS (Y_t)	-1	π,b	$-\beta-m(\pi-\beta)/(m+b\pi/\beta)$
MP (η_t)	$1/\pi$	>	0
FP (μ_t)	0	<	$1/(m/\pi+b/\beta)$

2. lagged effects

Shocks	Const. M gr.-Pass. Fiscal	rel. var.	Constant R-Constant Tax
MD (V_t)	$(1-\beta/\pi)$	>	0
AD (K_t)	$-\beta/\pi$	π	-1
AS (Y_t)	1	=	1
MP (η_t)	β/π	>	0

FP (μ_t)	0	=	0

Appendix 1. Solutions

1) Constant Money Growth Rate Policy-Passive Fiscal Regime

$$\pi_t = -\left(1-\frac{1}{R}\right)\left(V_t - V_{t-1}\right) - \left(Y_t - Y_{t-1}\right) + \frac{1}{h}\eta_t + \frac{1}{R}\left(K_t - K_{t-1}\right)$$

$$R_t = \left(1-\frac{1}{R}\right)K_t + \left(1-\frac{1}{R}\right)V_t$$

$$h_t = \frac{1}{h}\eta_t$$

$$E_t\pi_{t+1} = -\frac{1}{R}K_t + \left(1-\frac{1}{R}\right)V_t + Y_t$$

$$b_t = \sum_{s=1}^{t}\left(\frac{1}{\beta}-\gamma\right)^{t-s}\left[\begin{array}{l}\frac{(R-1)(m+bR-\beta mR)}{b\beta R^2}V_s + \frac{(m+bR-\beta mR)}{b\beta R}Y_s - \frac{1}{\beta}Y_{s-1}\\ +\frac{-m-bR+\beta mR}{b\beta R^2}K_s + \frac{1}{\beta}K_{s-1} - \frac{m+bR}{hbR\beta}\eta_s - \frac{1}{b}\mu_s\end{array}\right]$$

2) Constant Interest Rate Policy-Constant Tax Policy Regime

$$\pi_t = -\left(\frac{\beta m(R-1)}{m+bR}\right)V_t - \left(\beta + \frac{\beta m(R-1)}{m+bR}\right)Y_t + Y_{t-1} + \frac{1}{R}\eta_{t-1} - \left(\frac{\beta R}{m+bR}\right)\mu_t + \beta K_t - K_{t-1}$$

$$R_t = \frac{1}{R}\eta_t$$

$$h_t = \left(1-\frac{\beta m(R-1)}{m+bR}\right)V_t + \left(1-\beta-\frac{\beta m(R-1)}{m+bR})\right)Y_t + Y_{t-1}$$
$$-\frac{1}{(R-1)R}\eta_t + \frac{1}{R-1}\eta_{t-1} - \left(\frac{\beta R}{m+bR}\right)\mu_t + \beta K_t - K_{t-1}$$

$$E_t\pi_{t+1} = -K_t + \frac{1}{R}\eta_t + Y_t$$

$$b_t = -\frac{m}{bR}V_t + Y_t - \left(1+\frac{m}{bR}\right)K_t + \frac{m}{(R-1)bR}\eta_t$$

Appendix 2. Minimum State Variable Solution

The result of this paper is compared to the minimum state variable solution. McCallum (1983) suggested the “minimum state variable” or “bubble free” solution that is driven by the “fundamental” (but ignoring the government budget constraint and fiscal policy). In the present model, the solution can be obtained by assuming that expected value of money (or the price level) is fixed in every period. That is, $E_tM_{t+1} = M$ (or $E_tP_{t+1} = P$). The solution for the inflation rate in this case is:

$$\pi_t = -(Y_t - Y_{t-1}) + (K_t - K_{t-1})$$

We can see that Poole’s basic result holds by comparing the above equation with equation (19) under the constant money growth rate rule. That is, the constant money growth rate regime is better in the case of aggregate demand disturbances while the constant interest rate regime is better in the case of money demand shocks.

However, to fix the expected value of money ($E_tM_{t+1} = M$), the fiscal authority should keep the following rule.

$$\tau_t = \frac{M_{t-1} - B_t + RB_{t-1}}{MP_t}$$

That is, from the viewpoint of the current analysis, using a minimum state variable solution is equivalent to implicitly assuming a specific fiscal policy.