

# Vacancy Management IV: Quality of Real Estate Services

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After a truly long hiatus, we return to our series on this crucial property management topic. Readers are referred to prior vacancy management articles in our Summer 1989, Winter 1990, and Spring 1990 issues. A more technical coverage follows this two-page discussion.

The old expression “you have to spend money to make money” is applicable in any business enterprise. It is clear, for example, that a firm must spend more promoting its products or services if its revenues are to increase. Perhaps less obvious is the relationship between cash inflows and outflows, a relationship that is especially important in managing real estate because of that asset class’s unique spatial/locational attributes.

Indeed, some real estate managers view *operating statements* only as accounting devices, ignoring the insights these documents can offer into critical management issues. Because the operating statement’s simple algebra tells us that potential gross income minus allowances for vacancy and bad debts equals effective gross income (*EGI*), and that *EGI* minus operating expense (*OE*) equals net operating income (*NOI*), a manager might conclude that maximizing *NOI* involves simply keeping *EGI* as high, and *OE* as low, as possible. But *NOI* maximization requires a fuller understanding of fundamental interrelationships associated with the items in the operating statement.

For example, in altering rents we will change not only occupancy levels, but also the tenants’ propensity to generate bad debts; operating expenses will change as well. These effects might seem

somewhat obvious to anyone who has observed the market for rental real estate. Yet there are other, more complex interrelationships among operating statement items. Notably, changing *OE* in some dimensions affects the *quality* of the services associated with occupancy. Thus rents, occupancy levels, and bad debts may all be affected. Then, the feedback generated by these changes can cause further changes in *OE*. Yet as boggling as the various relationships may seem, they can be illustrated through an analytical device – a model – based on some straightforward economic principles. An important result revealed by our model is that the optimal response to a decrease in demand is to *reduce* quality.

### A Simple Microeconomic Model

The behavior of a company that owns an office property can be modeled like that of any other type of firm; the only special attribute is that there is some degree of spatial market power. In other words, the landlord’s actions can affect the rent that tenants are willing to pay; an office property is not like a farm commodity whose producer must be a *price-taker*. We can assume that, as more of the property’s units become occupied, the firm’s *EGI* first increases, but then decreases. *EGI* might be expected to decline over some range of higher occupancy because the manager achieves higher occupancy by reducing rents; if the drop in *EGI* from lower rent more than offsets the increase from higher occupancy, then *EGI* falls.

Of course rent per unit, or average *EGI* (*AEGI*), declines with the number of occupied units. Moreover, we might as-

sume that this relationship is *linear*, such that the amount by which rent must fall in order to increase occupancy by one unit remains constant as occupancy increases. We therefore represent the rent relationship (*AEGI*) in Figure 1 as a standard, downward-sloping linear demand curve.

We can also find the *change* in *EGI* that results from a one-unit change in occupancy. Those who have studied basic microeconomics will recognize this relationship as the standard *marginal revenue* curve; we will call it *marginal EGI*, or *MEGI*: the extra revenue that results from adding an additional unit of occupancy. Note that in Figure 1 the slope of the *MEGI* curve is twice as steep as the *AEGI*, or rent, curve (*MEGI* intersects the horizontal axis half as far out as *AEGI*).

The cost side is equally simple. We assume that *OE* is linearly related to the number of occupied units. While we also assume that *OE* rises with occupancy, this relationship does not necessarily have to hold. For example, a landlord might face *declining* expense with higher occupancy if each lease had a common area maintenance (*CAM*) term, under which the tenants pay to maintain common areas (such that the landlord must pay unoccupied units’ share). Ordinarily, however, higher intensity of use brings about higher costs, such that the change in *OE* resulting from a unit change in occupancy (*marginal operating expenses*, or *MOE*)<sup>1</sup> is positive. Moreover, because we have assumed a linear relationship between occupancy and expenses, *MOE* has a constant value. Figure 2 shows *MOE*, with its constant slope (shown as *b*), along with average operating expenses (denoted *AOE*).

One of the most important lessons from introductory economics is that we maximize profit by equating *marginal revenue* with *marginal cost*. For the firm providing office real estate services, we would maximize *NOI* by equating *MEGI* with *MOE*. If occupancy is less than  $U^*$ , the added *EGI* from increased occupancy exceeds the added operating expense (the height of *MEGI* exceeds *b* to the left of  $U^*$  in Figure 2), and therefore it is profitable to increase occupancy. However, for

Figure 1

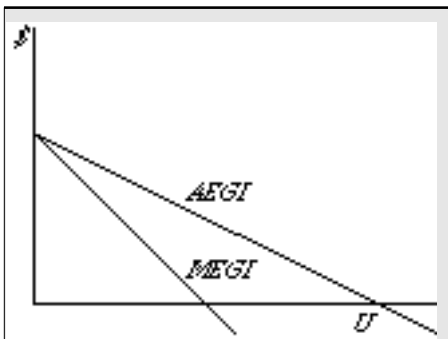
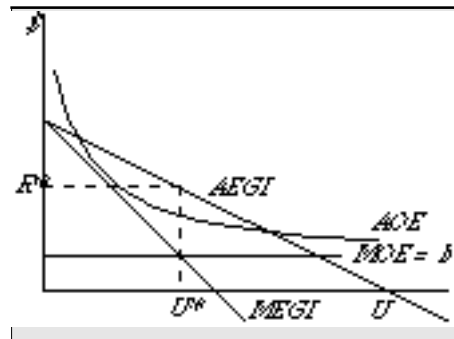
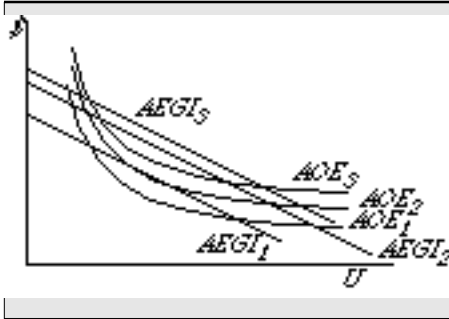


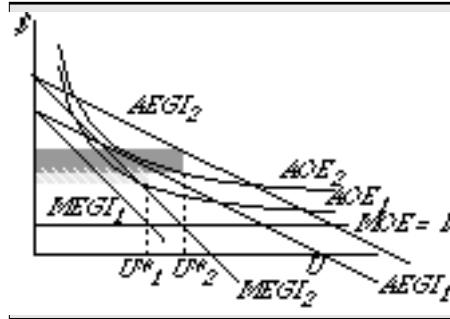
Figure 2



**Figure 3**



**Figure 4**



occupancy greater than  $U^*$  the addition to EGI is less than the accompanying added operating expense (MEGI's height is lower than  $b$  to the right of  $U^*$ ); the manager would actually earn a higher profit by reducing occupancy.

Thus, under conditions represented in Figure 2, optimal occupancy is at  $U^*$ , where added operating expenses are just equal to additional EGI. The optimal level of rent,  $R^*$ , is the EGI per unit at optimal occupancy  $U^*$ . Optimal rent therefore depends on influences that include the height of the demand curve and the slope  $b$  of the operating expense function.

### A Model That Incorporates Quality

Basic microeconomics focuses on the price/quantity relationship, but we can expand our analysis by incorporating a quality measure. Suppose that quality is represented by the height of the operating expense function; greater height indicates higher service quality regardless of occupancy. For example, replacing light bulbs in hallways on a regular, early schedule creates more cost (shown graphically as a higher AOE function) than changing them only after they burn out and tenants complain. Tenants' response to higher service quality should be to shift AEGI upward.

We can link the operating expense function's height with that of the demand (rent, or AEGI) curve through a model in which we assume that AEGI's intercept (rent that results in no occupied units, and zero revenue) increases at a decreasing rate as operating costs rise to reflect higher service quality. In Figure 3, an increase in average operating costs from  $AOE_1$  to  $AOE_2$  causes demand to rise from  $AEGI_1$  to  $AEGI_2$ . But a further increase in average operating costs by the same amount ( $AOE_2$  to  $AOE_3$ ) causes only a relatively small rent increase, from  $AEGI_2$  to  $AEGI_3$ .

Recall that our goal is to maximize NOI, the difference between rent per unit and average operating expenses per unit, multiplied by the number of units rented. In Figure 4, the firm facing rent of  $AEGI_1$  (and marginal rent of  $MEGI_1$ ) operates at an occupancy level of  $U^*_1$ . With average operating expenses of  $AOE_1$ , this firm's NOI is shown by the striped rectangle. What happens if the firm improves service quality such that average operating cost rises to  $AOE_2$ ? Rent rises to  $AEGI_2$ , and marginal revenue rises to  $MEGI_2$ . (Since we assume that the intercept of the operating expense function increases, but not the slope, MOE is unaffected by the operating expense increase). The firm's optimal occupancy level increases to  $U^*_2$ , and NOI is equal to the area represented by the shaded rectangle.

In this example, it seems clear that it is in the best interest of the firm to operate at higher-cost occupancy level  $U^*_2$  rather than at  $U^*_1$ , since this higher occupancy yields a higher NOI. But will additional, similar increases in service quality (and operating costs) continue to generate higher NOIs? The answer is no, because of the manner in which rent responds to

changes in operating costs: at some point NOI per unit will decrease with continued increases in quality (our treatment of AOE as rising more rapidly than AEGI should be intuitively appealing). Eventually, the loss from a lower NOI per unit on all occupied units will exceed the gain from increasing the number of units occupied.

One notable result that can be offered from a more detailed version of this analysis is that a decrease in demand (represented by a lower vertical intercept at each level of quality, or by a steeper slope) will reduce the optimal level of quality (see Table 1). The appropriate response to a decrease in demand (from an NOI standpoint) is to decrease quality. Yet conventional wisdom tells some managers to increase quality when demand decreases, to reduce the vacancy impact on the property. This potential for erroneous conclusions shows precisely why it is helpful to use models in analyzing these complex relationships. Of course, the model presented in this discussion may be flawed. If so, others may wish to suggest alternative models that deliver results more in line with the conventional wisdom on the market's appropriate quality response to a decrease in demand.

In conclusion, while we do not expect property managers to draw the kinds of graphs shown above, we hope that in understanding the microeconomic principles underlying the relationship between operating expenses and rent they will enhance their firms' performances. ■

1. As a 10th anniversary gift to our more careful readers, we offer the observation that the MOE ratio is related to the Current Unrealized Revenue Loss per Year (CURLY), Level Average Revenue Realized per Year (LARRY), and Shareholders' Effective Monthly Profit (ShEMP) measures.

**Table 1**

	$\frac{EGI}{U} = \alpha - \beta U$	$OE = a + \delta U$	$\alpha = \frac{\ln x - \ln x_0}{r}$	
			DECREASE DEMAND	
	base case	steeper slope	increase $a_0$	increase $r$
	$a_0$	1,000	1,100	1,000
	$b$	2,000	2,000	2,000
	$\beta$	500	500	500
	$r$	.0003	.0003	.00031
	$U^*$	9.5	8.2	8.8
	$NOI^*$	13,625	9,706	10,677
QUALITY INDEX	$a^*$	31,755	27,367	30,104
				28,353

## A Technical Analysis of the Quality of Real Estate Services

We have just seen that a firm acting optimally will select that level of rent, and therefore of occupancy, for which marginal operating expense is just equal to marginal effective gross income. The firm's resulting net operating income (NOI) was shown to be rent minus average operating expense at the optimal occupancy level, multiplied by the number of occupied units. This outcome was the best that the firm, facing particular demand and marginal operating expense curves, could attain. Of course, when we considered the relationship between the quality of real estate services and the level of rent, we saw that the firm potentially realizes many different optimal NOIs, one for each level of service quality. The key question therefore becomes which of the potentially optimal NOIs is truly optimal.

To identify this true optimal NOI, we must first specify the way in which rent responds to changes in the quality of real estate services. Second, we must identify precisely how average operating costs respond to changes in the quality of real estate services. Finally, after incorporating some basic mathematical equations, we can identify the truly optimal NOI. We can begin by reviewing the analytical framework of the previous article, while providing an extra measure of technical detail along the way.

### Seeking the Optimum

In the upper right quadrant of Figure 1, rent, or average effective gross income (AEGI), is represented as a downward-

sloping straight line. The relationship is shown as linear because the relationship between EGI and the proportion of units that are occupied is assumed to first increase and then decrease, in a manner represented graphically as a parabola, as the occupancy level increases. This result can be expressed mathematically as:

$$EGI = \alpha U - \beta U^2$$

an equation in which both  $\alpha$  and  $\beta$  are greater than zero and  $U$  is the number of occupied units. Dividing both sides by the number of occupied units,  $U$ , results in the rent, or AEGI, function's being a straight line:

$$R = AEGI = \alpha - \beta U$$

an equation in which  $\alpha$  is the vertical intercept and  $\beta$  is the slope, or steepness, of this demand "curve." Marginal effective gross income (MEGI), which has the same intercept as AEGI but a slope that is twice as steep, is the change in EGI attributable to a one-unit change in the number of occupied units. MEGI is computed as the first derivative of the EGI function:

$$\frac{dEGI}{dU} = \alpha - 2\beta U$$

Operating expenses are assumed to increase along a straight line as the number of occupied units increases:

$$OE = a + bU$$

an equation in which both  $a$  and  $b$  are, again, assumed to be positive. Marginal

operating expense, or the change in total operating expenses attributable to a one-unit change in occupancy, is computed as the following constant:

$$\frac{dOE}{dU} = b$$

The optimal occupancy is the level for which marginal operating expenses are just equal to marginal effective gross income. This relationship implies that

$$\alpha - 2\beta U^* = b$$

an equation for which  $U^*$  is the level of occupancy that maximizes NOI. Solving for  $U^*$  allows us to find the number of occupied units that maximizes NOI:

$$U^* = \frac{\alpha - b}{2\beta}$$

Substituting the optimal occupancy into the rent (demand) function, we find the optimal rent:

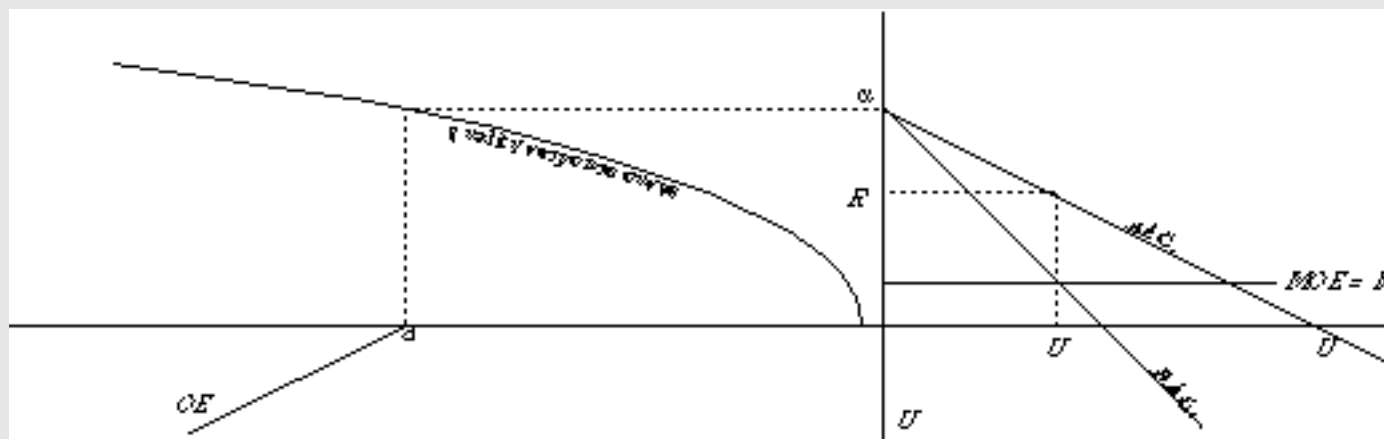
$$R^* = \alpha - \frac{\alpha - b}{2}$$

Note that rent is one of the primary decision variables in this model. Now that we have reestablished the way in which optimal occupancy and rent are determined, let us turn to the focus of this technical discussion: the effect of changes in the quality of real estate services on NOI.

### Total Operating Expenses & Demand

Total operating expenses,  $OE$ , are shown in the lower left quadrant of Figure 1.

Figure 1



A more conventional way to represent operating expenses graphically would be as a positive, increasing function in a quadrant for which the number of units appears on the horizontal axis and price is shown on the vertical axis. In the lower left quadrant of Figure 1, this conventional picture has been inverted, such that units appear on the vertical axis and price is on the horizontal axis. The intercept of the operating expense function, denoted  $a$ , can be thought of as the level of operating expenses incurred with 100% vacancy (0% occupancy).

Now, recall the relationship we have assumed between rent and operating expenses: that the intercept of the rent, or *AEGI*, function increases at a decreasing rate as the intercept of the operating expense function,  $a$ , increases. The *quality response curve* in the upper left quadrant of Figure 1 shows the intercept of the demand curve,  $\alpha$ , that corresponds to the intercept of the operating expense function,  $a$ . The fact that the quality response curve increases at a decreasing rate with increases in  $a$  can be represented symbolically as:

$$\alpha = f(a)$$

where  $f'(a) > 0$  and  $f''(a) < 0$

The meaning of the first statement is that the height of the rent function,  $\alpha$ , depends on the height of the operating expense function (in Figure 1, the horizontal intercept  $a$ ). The first portion of the latter statement,  $f'(a) > 0$ , simply means that as  $a$  increases,  $\alpha$  increases. The second portion,  $f''(a) < 0$ , means

that the size of the increase in  $\alpha$  gets smaller with each successive increase in  $a$ . (Using mathematical terminology, we would say that the first derivative is positive and the second derivative is negative.) In summary, the height of the demand curve, *AEGI*, increases at a decreasing rate with increases in the height of the operating expenses function.

### Average and Total Operating Expenses

Recall the definition of net operating income: rent per unit minus average operating expense per unit, multiplied by the number of units occupied. As discussed in the previous article, average operating expenses increase as service quality improves; this situation is shown graphically as an increase in the height of the operating expense curve,  $a$ . Yet while we can observe average operating expenses, how do we know exactly what the level of quality,  $\alpha$ , is?

Figure 2 demonstrates how, given a particular average operating expense curve, we can detect the level of quality, and consequently the height of the *AEGI* (or demand) curve associated with this level of quality. In that figure, the firm faces average operating expense curve  $AOE(a^\dagger)$ . The meaning of this notation is that average operating expenses depend on the horizontal intercept of the operating expense curve ( $a^\dagger$ , a magnitude that is yet to be identified). Let us randomly select some level of occupancy in Figure 2, perhaps four units. At this occupancy level, we know what average operating expenses are, and we also know what marginal operating expenses are. Now,

recall the formula for computing operating expenses:

$$OE = a + bU$$

Dividing each side of this equation by the number of occupied units, we can compute average operating expenses, or operating expenses per unit, as

$$\frac{OE}{U} = \frac{a}{U} + b$$

Rearranging algebraically, we can see that  $a$  divided by the number of units must be equal to

$$\frac{a}{U} = \frac{OE}{U} - b$$

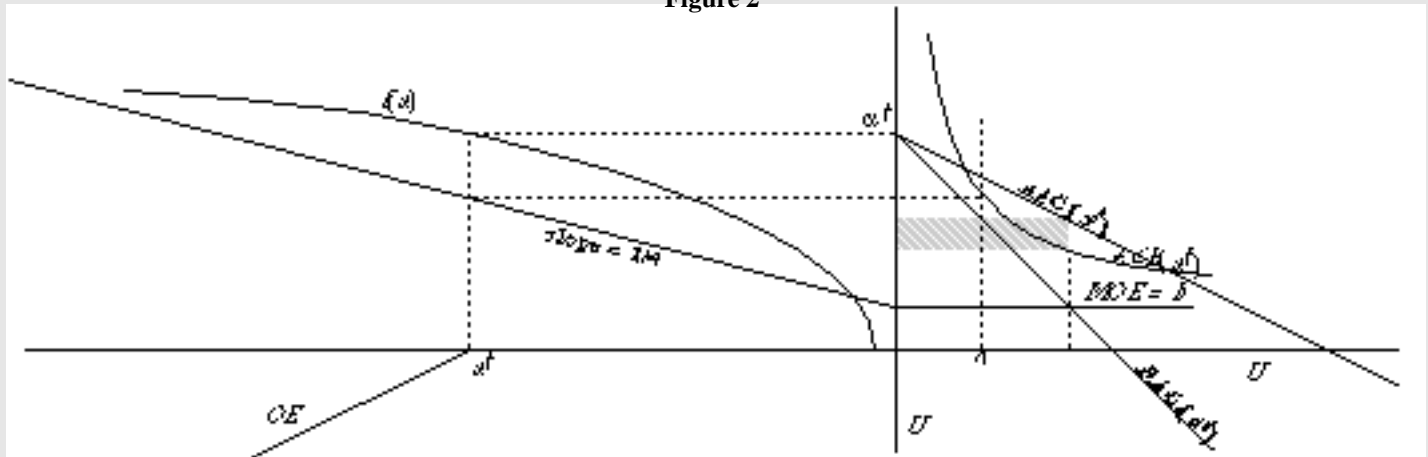
If  $U = 4$ , then

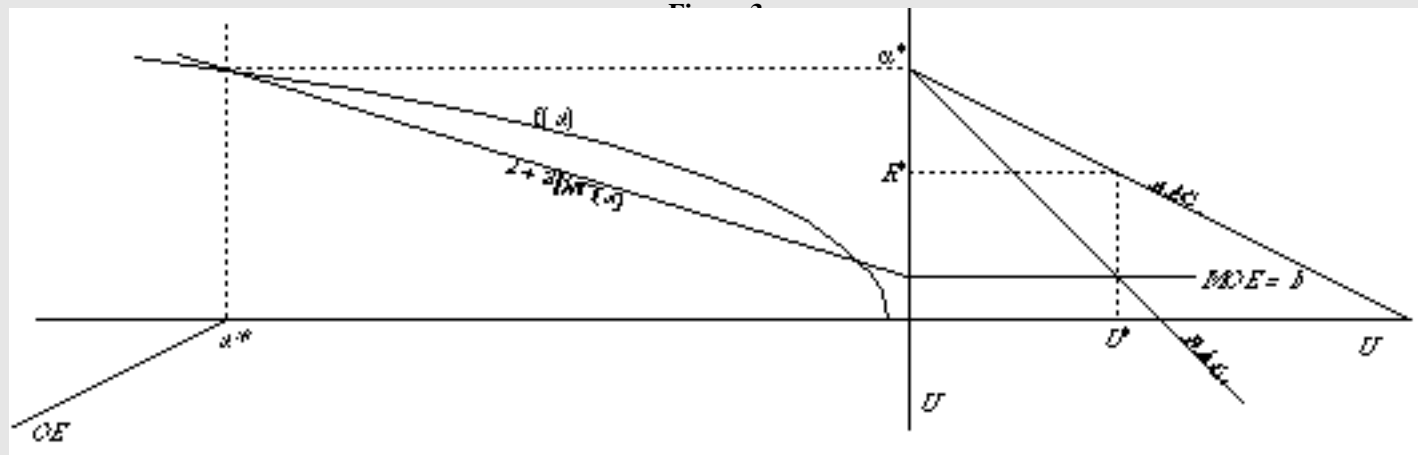
$$\frac{a^\dagger}{4} = \frac{AOE^\dagger}{4} - b$$

In Figure 2, we can identify  $a^\dagger$  as four times the difference between average and marginal operating expenses at a four-unit level of occupancy.

Another trick to identify  $a^\dagger$  is to draw a line that emerges from the vertical axis at  $b$ , with a slope equal to one divided by the number of units at which *AEGI* is measured. The result is shown in Figure 2 as the dark line with a slope equal to one-fourth. Given average operating expenses that prevail at four units of occupancy, we can read horizontally from the height of  $AOE(a^\dagger)$  to the point where this new line is crossed; we can verify visually that the resulting horizontal distance is  $a^\dagger$ . We can reach this same result algebraically

Figure 2





by rearranging terms in the previous equation to show

$$AOE^a = \frac{1}{4}a + b$$

Therefore, when an increase in total operating expenses causes average operating expenses to increase, we can observe this average amount at four units in order to determine the level of quality,  $a$ . Of course, we could have picked any number of units – for example, eight – at which to observe average operating expense. The slope of our line emerging from  $b$  would then be one-eighth, rather than one-fourth.

Now that we have identified  $a^*$  in Figure 2, we can use the quality shift function,  $f(a)$ , to identify the intercept of the rent function. We can then select the optimal level of rent, and the optimal level of occupancy, for a specified average and marginal effective gross income, and then calculate the maximum NOI associated with this level of service quality. This NOI level is represented graphically as the shaded rectangle in Figure 2.

If average operating expenses were higher at every level of occupancy, we would observe a new AOE curve corresponding to a higher level of quality of real estate services. But an increase in the level of quality, and therefore in observed operating costs, does not necessarily decrease total profit. Although there is a decline in net operating income *per unit*, the overall loss in NOI on previously occupied units (for which higher rents had previously been paid) is less than the increase to NOI that the manager realizes from the additional occupancy of units.

### The Maximum Optimal NOI

The firm attains its optimal NOI when it sets rent per unit equal to the value indicated along the demand function, at the level of occupancy for which marginal operating expense (MOE) is just equal to marginal effective gross income (MEGI). Of course, we have established that changes in the quality of real estate services affect both average operating expenses and the level of demand, such that there is a unique optimal NOI for every level of quality. We can identify the maximum from among all of the potential optimum NOIs by working with the definition of net operating income:

$$NOI^a = EGI^a - OE^a$$

or, substituting from our definitions of EGI and OE:

$$NOI^a = \alpha U - \beta U^2 - \left[ \frac{1}{4}a + bU \right]$$

Next, substituting the optimal occupancy,  $U^*$ , and the link between the intercepts of the rent and expense functions,  $\alpha = f(a)$ , into the NOI function and expanding produces the following relationship:

$$NOI^a = \frac{f^2(a)}{4\beta} - \frac{2bf(a)}{4\beta} - a - \frac{b^2}{4\beta}$$

This equation describes the NOIs that are optimal in light of each possible value for the intercept,  $a$ , of the operating expense function. However, at this stage of our analysis we still do not know how to choose the optimal level,  $a^*$ , of the operating expense function. In order to determine  $a^*$ , we must discover how NOI changes as  $a$  changes, and then set that level of change equal to zero. In this way (again, computing a first derivative), we

can find the maximum from among all the optimal NOIs:

$$\frac{dNOI^a}{da} = \frac{f(a) f'(a)}{2\beta} - \frac{f'(a)}{2\beta} = 0$$

We simplify this relationship algebraically, and set both sides of the equation equal to zero such that:

$$f(a) f'(a) - f'(a) = 2\beta$$

Rearranging so that we are able to show the optimal intercepts of both the rent function and the operating expense function produces

$$f(a) = b + \frac{2\beta}{f'(a)}$$

Of course, the left-hand side of this equation is simply our quality response curve,  $f(a)$ . Figure 3 provides a graphical representation of the right-hand side of this equation. Note the following problem: the quality response curve and the curve representing the right-hand side of the above equation intersect at two different points. One of these points generates the *minimum* NOI, while the other generates the *maximum* NOI. The OE curve and rent functions shown in Figure 3 are the ones that are associated with the maximum NOI. The optimal level of quality is  $a^*$ , and the optimal height of the rent function is  $\alpha^*$ . Corresponding to these values, the optimal level of occupancy for the firm is  $U^*$ , and the optimal rent to charge per unit is  $R^*$ . These particular values of occupancy and rent will yield the *maximum* optimal net operating income to the firm from among all the potential optimal NOIs. ■