Mortgage Mechanics, Part I: The Fixed Rate Mortgage Loan

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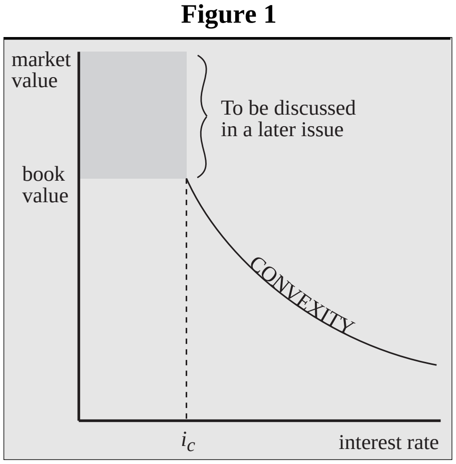
A reader heard that ILLINOIS REAL ESTATE LETTER planned to publish an article on "mortgage mechanics." He was relieved to know that mortgage mechanics indeed exist, and has since scheduled an appointment to have his points changed. We could not resist opening with this admittedly bad pun. By mortgage *mechanics*, we actually refer to various financial calculations, including the structural breakdown between principal and interest, in mortgage loan payments. Part of the payment on any loan represents a periodic return of principal (which can be zero in the case of an interest-only loan, or even negative in some lending situations); the remainder represents interest on the outstanding principal balance (or a lesser amount in the case of the graduated payment loan). The structure underlying mortgage loan payments is a topic of concern to many individuals who are studying real estate finance, working in the banking or real estate industry, or experiencing the home buying process for the first time.

Finding Unknown Values

For any type of loan, we can apply some basic financial formulas to compute the periodic payment (*debt service*) if we are told the principal amount borrowed, the interest rate, and the repayment period (*term*). Actually, we can compute values for any of the loan variables if we specify values for the others; any equation can be solved if it contains only one "unknown."

Figure 1



While a home borrower typically wishes to compute the monthly payment for a specified principal sum, she might instead wish to find the principal that could be serviced with a given monthly payment.

The Standard Fixed-Rate Loan

In this article, we examine the mechanics of the most basic, and in recent decades most popular, instrument of housing finance: the standard fixed-rate, fixed-payment mortgage loan (FRM). FRMs have dominated the US home lending market since the Federal Housing Administration (FHA) was established in 1934.

If the market rate drops below the contract rate, the value of the loan does not necessarily rise, because the lender faces the chance that the borrower will refinance at the lower rate and prepay the loan.

Under an FRM note, the interest rate is "locked in" when the loan is originated (or at some earlier commitment date); it remains fixed throughout the loan's life. The FRM offers some assurances to borrowers, or *mortgagors* (while also presenting them with the *tilt problem*, discussed below), but it poses substantial risks for lenders, or *mortgagees*.

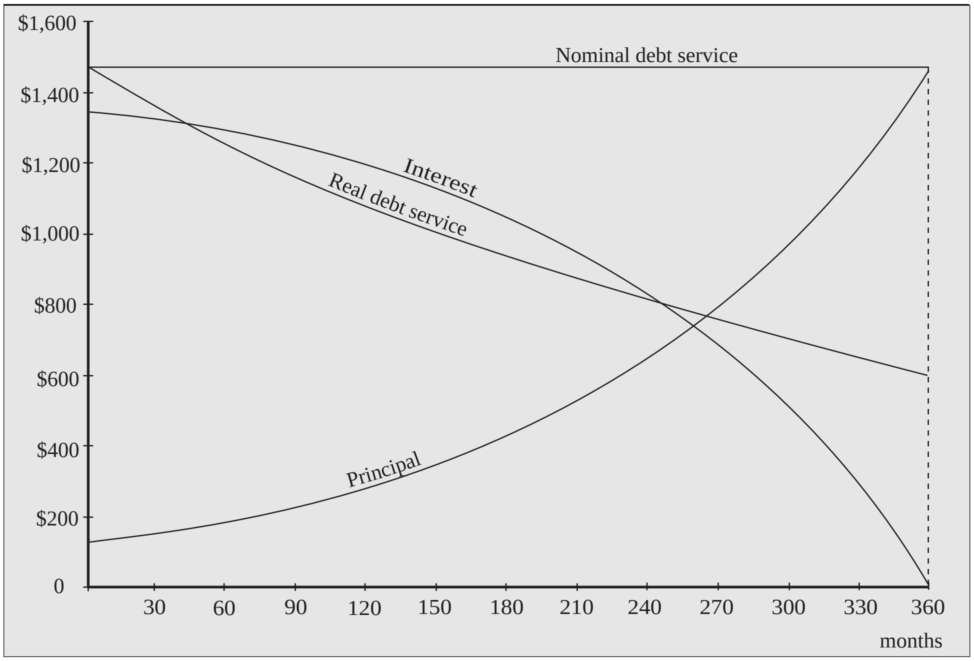
In fact, in the FRM arrangement, the lender bears *most* of the risk. The lender's risk is depicted in Figure 1. As the interest rate the lender could receive by lending today (the *market* interest rate) rises beyond the rate the lender agreed to accept when the loan was made (the loan's *contract* rate), i_c, the note's value falls, because the lender's rate of return is lower than it would be if the lender had waited to make the loan. This decline in value occurs at a decreasing rate; in other words, beyond i_c a given percentage decrease in the market interest rate has a greater impact on the loan value than would an equivalent increase in the rate. The function that describes this situation is said to be *convex*, as Figure 1 shows.

Yet the relationship is not symmetrical *around* i_c; if the market rate drops below i_c, the value of the loan does not necessarily rise. Rather, there is a chance that the borrower will refinance at the lower rate and prepay the loan, imposing an opportunity loss on the lender. The small darkly shaded area in Figure 1 helps to illustrate this *prepayment risk* situation. If the market rate falls below i_c, the loan's value is uncertain; it depends both on the possible existence of prepayment penalties (unusual – often even illegal, as in Illinois – for loans on single-family homes) and on the market's use of "points." (These topics will be discussed in a later *Mortgage Mechanics* offering.)

The mechanics of the FRM are such that each of the unchanging payments consists of differing proportions of interest and principal. Because interest is paid only on the remaining principal balance, the proportion of monthly debt service that represents interest is greatest at the beginning of the term. Similarly, because payments reduce the remaining principal balance little by little, the proportion of FRM debt service that represents repayment of principal (known as *amortization*) is greatest toward the end of the loan term. At the end of the term, the remaining principal balance is zero, and the loan is said to be *fully amortized*.

Details of the payment components for a loan are typically presented in a month-by-month debt service breakdown, shown in a lengthy document called an *amortization schedule*. While it would be cumbersome to present a full amortization schedule for examination, a graphical display is an effective tool for telling the same story. The amortization of a fixed-rate, fixed-payment loan is represented in Figure 2, where we illustrate relationships involving the *nominal* and *real* values of the FRM interest and

Figure 2



principal (and, in turn, total debt service) payments made over the loan term.

Debt Service and Its Components

Consider the case of a $200,000 home loan under which the borrower gives the lender a mortgage (pledge of the home's value) as security. The contract calls for monthly compounding of interest, and for equal monthly payments over a 30-year period, based on a stated annual interest rate of 8%. Because the FRM loan involves an unchanging payment stream (debt service) corresponding to a present lump sum (principal borrowed), it is an application of the *present value of an annuity* (PVA) concept known in accounting and finance. In lending terminology, we refer to the reciprocal of the PVA factor as the mortgage constant, a small decimal fraction by which we multiply the loan's initial principal in computing a monthly payment. This constant is the amount needed each month, stated as a percentage of the original principal balance, to ensure that the principal *and* all interest applicable to any remaining outstanding principal balance will be paid by the end of the loan term.

In the case at hand, we multiply the .007338 monthly mortgage constant by the $200,000 principal, for a $1,468 monthly payment. The mortgagor pays the mortgagee this amount every month during the term of the loan; $1,468 therefore is the *nominal* value of the monthly debt service (a current dollar measure, without any adjustment for inflation). We show the unchanging nominal debt service stream graphically as the horizontal line across the top of Figure 2. (The computations, based on some basic formulas relating to real estate finance, are explained in Table 1 on page 9.)

The part of the payment stream consisting of interest is represented by the downward sloping *concave* shaped curve. The interest portion of the $1,468 payment declines with each successive month. Interest paid in the first month, when the principal has not yet been reduced from its original level, is computed as $200,000 x (.08/12 months), or $1,333. But the interest component of the unchanging $1,468 payment then falls with each successive month, and the concavity implies that the decrease occurs at an increasing rate over time.

It is perhaps useful to view principal repayment as the *residual* component of debt service; it is simply what remains of each payment after interest has been taken into account. The amortization of principal for the loan described above is represented by the upward sloping, *convex* shaped curve in Figure 2. The principal portion of the $1,468 payment rises with each successive month, and the convexity implies that the increase occurs at an increasing rate over time.

The first debt service payment contains only $135 in principal amortization (the $1,469 total minus the $1,333 in interest), whereas the final payment consists almost entirely of the retirement of principal.

Surprising Outcomes

When first introduced to these ideas, people sometimes are surprised to learn how much time elapses before the majority of a month's debt service for a typical FRM loan is a repayment of principal. After all, an instinctive response might be that half of the principal should have been repaid halfway through the loan's term, an outcome that would be observed if half of each payment were interest and half were principal. The heavy weighting of interest in the early years' payments, however, causes the remaining balance to exceed half of the principal borrowed until a date far beyond half of the term.

In fact, for our 8%, $200,000 loan, the principal and interest proportions of debt service do not approximate each other until midway through the twenty-second year. Specifically, at an 8% interest rate, the majority of each debt service payment consists of interest up through the 256th of 360 debt service installments. In general, the switchover point depends on the contract interest rate and the term of the loan, occurring later, for any given term, if the contract interest rate is higher. (For rates lower than we typically see, such as 2%, principal amortization exceeds interest paid even in the first month.) Furthermore, repayment of the $200,000 principal accounts for only 38% of the debt service payment total over the loan term, with the balance attributable to interest. A size comparison of the areas under the principal and interest curves in Figure 2 illustrates these concepts.

Real and Nominal Values

The 8% contract interest rate on which payments are computed is a *nominal* rate, unadjusted for inflation (increases in the general level of prices) that may have occurred *since* the date when this rate was established. The rate does reflect *expectations* that borrowers and lenders held, on the day when the contract rate was established, regarding inflation that would occur over the loan term. According to the well-known Fisher Equation,

$$i = r + \pi_e + r\pi_e$$

where i is the observed nominal interest rate, r is the real rate, and π_e is the average expected rate of inflation. According to this relationship, the observed contract rate for an FRM loan is determined by a real rate of interest and a premium reflecting the average expected level of inflation over the loan's lengthy term. Assume, in the case at hand, that the 8% observed rate includes a 3% π_e premium for expected inflation. The real rate of interest r embodied in the 8% nominal rate i therefore is 4.854%; note that .04854 + .03 + (.04854) (.03) = .08.

Inflation is an important aspect of the analysis of fixed-rate mortgage loans for both the lender and the borrower. Consider first the lender's position; if inflation turns out to be greater (less) than had been expected, then the lender's real return is lower (higher) than had been anticipated. For example, based on our illustrative figures, if actual inflation averages less than 3% (perhaps only 2%) per year during the loan term, then the lender's 8% return exceeds the 6.951% (= .04854 + .02 + .04854 x .02) needed to keep the lender "whole" (based on 2% for actual inflation, and 4.854% for time value costs and default risks). Lower inflationary expectations thus result in a new, lower market interest rate, and the value of the existing loan can rise. Conversely, if actual inflation averages more than 3% (say 5%) per year, the lender's 8% return is less than the rate of 10.097% (= .04854 + .05 + .04854 x .05) needed to keep the lender "whole." Higher expected inflation leads to a new, higher market interest rate, and the value of the loan falls.

The Tilt Problem

What about the borrower's position? The answer is that if there is any positive inflation at all, the real value of the constant FRM payment falls over time (note that if a borrower's income rises to keep pace with actual inflation, the unchanging nominal loan payments become more affordable). The real value of the debt service payment stream is illustrated in Figures 2 and 3 by downward sloping curves, each of which equals the corresponding nominal debt service when the loan is originated but which gradually declines over time at a decreasing rate.

These inflationary effects cause a phenomenon in FRM loans known as the *tilt problem*, a name selected in recognition of the manner in which the real value of the debt service "tilts" during the term, such that the real cost of payments in the early years exceeds the real cost of later payments. Notice that near the beginning of a loan's life, the real debt service payment (which reflects the borrower's purchasing power) is close to the nominal payment. Then toward the end of the term, if there has been 3% inflation, the real debt service is *less than half* the nominal payment. In a high inflation environment, therefore, debt service ultimately becomes very inexpensive for the borrower (as occurred, for example, in the early 1980s for people who had obtained home loans in the mid 1970s).

The degree of tilt is affected by the contract interest rate, which reflects expected inflation. If a loan's contract rate is low, the reduced real cost of debt service that accompanies inflation is less dramatic for the borrower than would be realized if the contract rate were high. The higher the contract rate, the higher the initial debt service payment, and the more severe the tilt problem. The degree of tilt also rises as the rate of inflation ultimately realized increases.

The greater severity of the tilt problem under higher expected and realized inflation is shown in Figure 3. Along with the nominal and real debt service under a 3% inflationary expectation, the figure illustrates the case in which the lender expects inflation to average 5%. The contract interest rate therefore is slightly over 10% instead of 8%, and the nominal debt service rises to $1790 per month. The real value of the debt service payment tilts more severely if 5%, rather than 3%, inflation is initially expected and then ultimately realized; real payments start higher but become lower by the end of the loan term. Because the present values of these real debt service payment streams must equal one another, the total amount by which real payments at 3% inflation exceed those at 5% late in the loan term will be greater than the total amount by which

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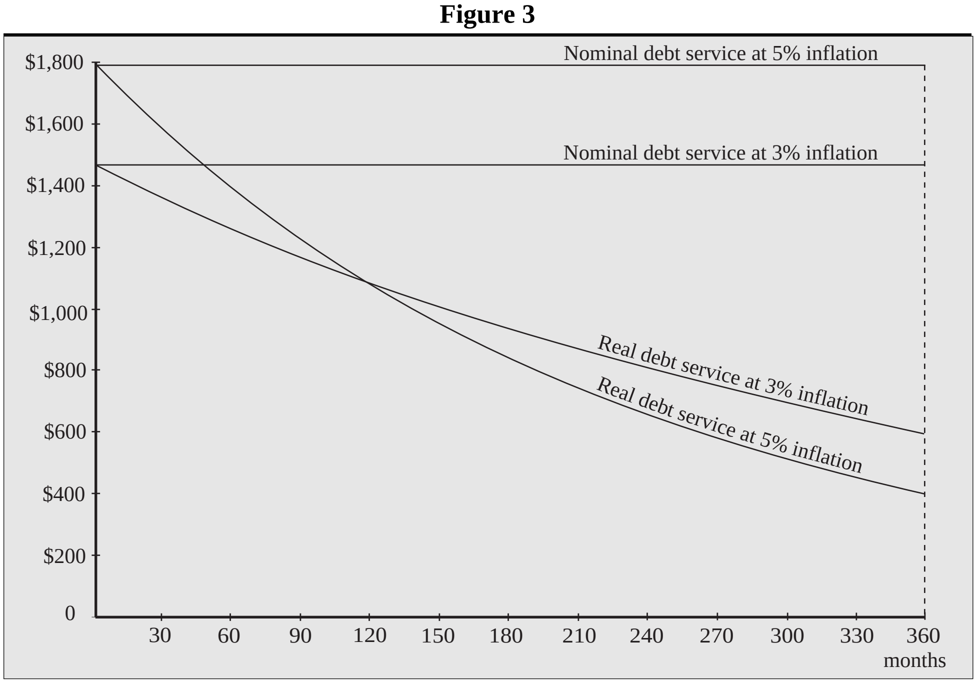
Figure 3



real payments under 5% inflation exceed those at 3% early in the loan term, due to time value of money considerations.

Problems and Solutions

An unfortunate accompanying result is that the borrower faces relatively high costs during the loan's first several years. The essence of the tilt problem is that real payments in the early years must be high enough to offset the eroded purchasing power, to recipients, of payments that will be made in the later years. Note that, in the context of our example (and as illustrated in Figure 3), even when inflation is only 3% the real cost of the initial payment is more than double the real cost of the final payment.

We concede that when inflation is low, as it has been lately, the market shows little concern for the tilt issue. But when inflation is more severe, the FRM's front-end loading of real costs discourages households from obtaining mortgage loans and, in turn, from buying homes. The situation raises troubling fairness questions, since the economically disadvantaged have few means at their disposal for dealing with the tilt problem. In fact, a convincing argument can be made that young families face housing affordability problems not (as is often argued) because they cannot amass down payments, but rather because the tilt problem forces them to subsidize their futures through major current financial sacrifices. Affordability issues thus might best be addressed through initiatives to encourage loan instruments that reduce or eliminate the tilt problem. Such instruments will be discussed in future *Mortgage Mechanics* articles. ■

This article is the first of several that will be devoted to the payment structures of various types of mortgage loans. The next offering in the series will be an examination of graduated payment and growing equity mortgage loans.

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Table 1

Given: LOAN = \$200,000, n = 360, i = 8%

Compute the mortgage constant:

To compute the monthly mortgage constant for a 30-year loan, apply the formula

$$MC = \frac{\frac{i}{12}\left(1+\frac{i}{12}\right)^{n}}{\left(1+\frac{i}{12}\right)^{n}-1} = \frac{\frac{.08}{12}\left(1+\frac{.08}{12}\right)^{360}}{\left(1+\frac{.08}{12}\right)^{360}-1} = .007337645$$

Find the monthly debt service payment:

PMT = mortgage constant x *LOAN*

PMT = .007337645 x \$200,000 = **\$1,467.53**

Find the outstanding loan balance:

BAL = $(1 - P_t)$ x *LOAN*, with P_t representing the proportion paid off after t periods.

To compute the proportion outstanding after 240 months $(1 - P_{240})$, apply the formula

$$P_t = \frac{\left(1+\frac{i}{12}\right)^{t}-1}{\left(1+\frac{i}{12}\right)^{n}-1} = \frac{\left(1+\frac{.08}{12}\right)^{240}-1}{\left(1+\frac{.08}{12}\right)^{360}-1} = .39522$$

BAL = (1 – .39522) x \$200,000 = **\$120,955.93**

Find the total principal amortized during some time interval:

$$PRIN_{t-k}^{t} = (P_t - P_{t-k}) \text{ x } LOAN$$

where $PRIN_{t-k}^{t}$ is the principal repaid between t and t – k periods

and $P_t - P_{t-k}$ is the proportion of the loan amortized between t and t – k periods.

To compute $P_{24} - P_{12}$ (proportion paid off during year 2), apply the formula

$$P_t - P_{t-k} = \frac{\left(1+\frac{i}{12}\right)^{t} - \left(1+\frac{i}{12}\right)^{t-k}}{\left(1+\frac{i}{12}\right)^{n}-1} = \frac{\left(1+\frac{.08}{12}\right)^{24} - \left(1+\frac{.08}{12}\right)^{12}}{\left(1+\frac{.08}{12}\right)^{360}-1} = .009046987$$

$PRIN_{12}^{24}$ = .009046987 x \$200,000 = **\$1,809.40**

Find the total interest paid during some time interval:

$$INT_{t-k}^{t} = (PMT \text{ x } k \text{ months}) - PRIN_{t-k}^{t}$$

where INT_{t-k}^{t} is the interest paid between t and t – k periods

INT_{12}^{24} = (\$1,467.53 x 12 months) – \$1,809.40 = **\$15,800.96**