Estimating Risk-Adjusted Costs of Financial Distress

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Finding the optimal, or value-maximizing, capital structure involves weighing the benefits of higher leverage against the costs. The main benefits of debt are the interest tax shield and, in the case of mature companies with limited growth opportunities, the reduction in the so-called "agency costs of free cash flow"—that is, the loss in value associated with managers' tendency to waste excess cash and capital on value-destroying projects instead of paying it out to debtholders. The downside of higher leverage is the increase in what academics refer to as the expected "costs of financial distress," the costs associated with the greater probability of default and bankruptcy.

To determine their optimal capital structure, managers need good estimates of the benefits and costs of debt. There are well-established methods for measuring the debt tax shield, with some estimates suggesting it can amount to 10-20% of a company's total value.[2] But estimating the cost of financial distress has proved much more difficult and elusive. Such costs include not only litigation fees and direct bankruptcy costs, but less quantifiable effects of financial trouble such as damage to the firm's reputation, the loss of key employees and customers, and, potentially the largest cost of all, the loss of value from foregone investment opportunities. And these since costs have a direct impact on recovery rates for investors, in a reasonably efficient market they should be reflected in lower bond and equity prices that reflect the possibility of financial distress.

In this paper, we propose a new approach for valuing expected financial distress costs—one that is premised on answering an important but simple question: what is the correct rate for discounting expected distress costs? Studies have estimated various costs associated with financial distress *when it takes place*, but when, or even if, these costs will be incurred is of course not known with certainty at the time of financing. As a result, the "correct" discount rate that reflects the true risk and uncertainty is difficult to derive.

We avoid this problem by adjusting not the discount rate, but instead the probabilities that distress actually occurs. Specifically, we propose using "risk neutral" probabilities that enable discounting at the risk-free rate. It is common practice to use historical default rates to forecast future defaults, and thus for estimating expected distress costs. This approach implicitly assumes that the risk of distress is primarily "idiosyncratic," meaning that the risk of default for each company is independent of the general economy and the defaults of other companies. In practice, of course, financial distress tends to happen during recessions and thus in times when investors' tolerance for risk is low. And for that reason, distress costs have a strong systematic component that must be taken into account.

We propose a simple way to incorporate systematic risk into the probability of default by using the risk premia implicit in corporate bond yield spreads. Our method begins with the well-documented finding that observed spreads are far greater than would be implied by historical default rates, and that at least part of the additional premia reflect a systematic risk premium. From the spreads, we are able to derive a risk-neutral market-implied probability of default that can be used to estimate the financial distress costs. To illustrate the practical import of our method, we provide a simple example in which financial distress costs that would be estimated at roughly 1.6% of firm value using the conventional method turn out to be as much as 5% after making our proposed adjustment for the increased systematic risk.

Valuing Distress Costs – the NPV formula

Financial distress, whether defined as a default, a bankruptcy, or simply a company struggling to meet all of its financial obligations, can impose significant costs. Studies have estimated the direct costs of litigation fees and bankruptcy to be about 3-5% of total firm value at the time of distress.[3] But of far greater consequence are the costs stemming from customers who are reluctant to buy from a distressed firm, high-quality employees who leave, managers who are distracted from running the business because of financing issues, and good investment opportunities that are passed over because of insufficient capital. One estimate puts the total loss in value attributable to financial distress in the range

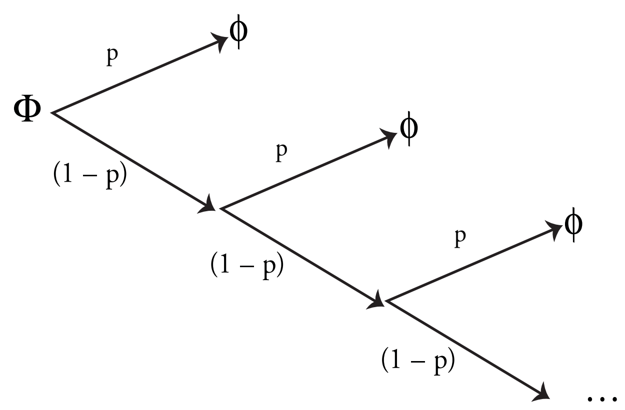
1. We wish to thank Don Chew and Jason Draho for many useful comments. All remaining errors are our own.

2. See John Graham, "Estimating the Tax Benefits of Debt," *Journal of Applied Corporate Finance*, Spring (2001), pp. 42-54.

3. See Jerold Warner, (1977), "Bankruptcy Costs: Some Evidence," Journal of Finance 32, pp. 337-347, and Lawrence Weiss, (1990), "Bankruptcy Resolution. Direct Costs and Violation of Priority of Claims," *Journal of Financial Economics* 27, pp. 255-311.

Figure 1 **Valuation Tree for Distress Costs**

This figure shows the valuation tree for financial distress costs. The probability p is the probability that the firm enters financial distress in each period, conditional on survival up to that period. φ represents the loss in value given distress, and Φ is the NPV of financial distress costs at date 0.



of 10-23% of pre-distress firm value.[4]

But, it's important to keep in mind that these costs materialize only if and when the firm gets into financial trouble. When a company is raising capital to finance investment, and perhaps altering its capital structure by issuing debt or equity, distress is only a possibility at some unknown future date. Nonetheless, the capital structure decision should still try to incorporate the expected distress costs. The challenge is to determine the net present value (NPV) of these costs, given the uncertainty about when and if they might arise.

In a paper published in the *Journal of Finance* in 2007, we developed a formal model to calculate the NPV.[5] We present here a simplified version of that model to illustrate the ideas and intuition, noting that it can be generalized to capture more detail.

Assume that a company has issued enough debt that it now has a BBB bond rating. The costs of financial distress *when they occur* are denoted by the letter ϕ, which can be thought of as a percentage of the current value of the firm. For example, if the firm is currently worth $5 billion and the distress costs are $500 million, then ϕ = 10%. In the calculations that follow, we will assume that ϕ is equal to 16.5% (which is the mid-point between 10% and 23% distress costs cited above).

To calculate the *present value* of distress costs, denoted by Φ, let's start by pointing out that the cost ϕ happens only in the event of distress. Starting today (year 0), we assume that distress will occur one year from now with probability p, and the probability that distress does not occur is thus 1 – p—though the firm could become distressed in future years. At year 1 there is again a probability p that distress costs occur in year 2, and so on.[6] The tree in Figure 1 summarizes these events.

Since this structure for the probability of distress does not change over time, the tree in Figure 1 that starts in year 1 is identical to the tree that starts in year 0. As a result, the NPV of distress calculated at year 1 will also be equal to Φ. Using this result, the NPV at year 0 is given by:

$$\Phi = \frac{p\phi + (1-p)\Phi}{1+r_\phi}, \qquad (1)$$

where r_ϕ is the appropriate discount rate for financial distress costs. Solving Equation (1) for Φ yields:

$$\Phi = \frac{p}{p+r_\phi}\phi, \qquad (2)$$

Equation (2) provides a simple formula to estimate the NPV of financial distress costs, provided we have estimates of the probability of distress p and the discount rate r_ϕ.

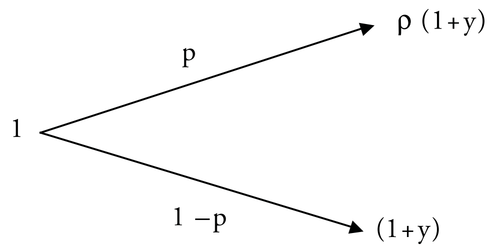
4. See Gregor Andrade and Steven Kaplan, (1998), "How Costly is Financial (not Economic) Distress? Evidence from Highly Leveraged Transactions that Become Distressed," *Journal of Finance* 53, pp. 1443-1493.

5. See Heitor Almeida and Thomas Philippon, (2007), "The Risk-Adjusted Cost of Financial Distress", *Journal of Finance* 62, pp. 2557-2586.

6. The analysis here assumes that all quantities are constant over time. For a discussion of the time-varying case see Almeida and Philippon, (2007).

Figure 2 **Valuation Tree, Perpetual Bond**

This figure shows the tree for the valuation of the perpetual bond described in the text. The bond's yield is y, which is equal to the bond's coupon. Thus the bond is priced at par (price = 1). The recovery rate in the event of default is given by ρ.



Estimating Distress Probabilities and Discount Rates

The standard approach in academic research is to use historical default rates to estimate p and a risk-free rate to discount financial distress costs.[7] Moody's has estimated that the yearly default probability for bonds of an initial BBB rating is equal to 0.53%.[8] Using this default rate with a risk-free rate of 5% and a loss of value given distress equal to 16.5% together in Equation (2) yields an NPV of distress equal to 1.6% of firm value.

The main problem with this approach, as suggested above, is that it ignores the systematic risk premia associated with distress and assumes that historical default rates are a good proxy for future expected default rates.[9] Since distress and default are more likely to occur in bad times, risk-averse investors should care more about financial distress than is implied by risk-free valuations. And this means that the NPV of distress for BBB bonds is likely to be larger than 1.6% of firm value.

The question we address here is how to incorporate the appropriate risk premia. While it's tempting to raise the discount rate, we argue that a better method is to adjust the probability of default and continue to discount at the risk-free rate. We are able to do that by relying on a very simple insight: Given the assumption that default "states" are the same as distress "states," the risk premium that is relevant for the valuation of distress costs is the same risk premium that is implicit in the valuation of corporate bonds. Thus, we can quantify the distress risk premium for a BBB firm by looking at the spreads at which BBB bonds trade in the market.

Bond Risk Premia

It is widely documented in academic research that the spread between corporate and government bonds is too large to be explained only by expected default rates and must therefore reflect an additional risk premium. To illustrate this point, assume that the debt issued by our BBB-rated firm is perpetual. The bond's promised coupon, which we call y, is assumed to equal the bond's yield (that is, the bond is priced at par). These assumptions imply that the bond price is constant over time. At the end of year 1, creditors receive (1+y) if there is no default (the bond's price plus year 1's coupon). In the event of default, creditors receive a fraction ρ—the recovery rate—of those cash flows, or ρ(1+y). Figure 2 depicts the perpetual bond's valuation tree.[10]

The bond's value can be expressed as the present value of future cash flows:

$$1 = \frac{(1 - p + p\rho)(1 + y)}{(1 + r_D)}, \qquad (3)$$

where r_D is the appropriate discount rate.

As with distress costs, the standard approach for pricing the bond is to assume that investors are risk neutral (so that r_D equals the risk free rate of 5%) and to use historical default probabilities to value the BBB-rated bond (p = 0.53%). Using a recovery rate ρ of 0.41, Equation (3) can be used to back out the BBB bond's yield.[11] This calculation gives y = 5.33%, or a BBB spread of 0.33% over the risk-free rate.

Although this represents a fair compensation for average historical losses on BBB bonds, it significantly underestimates

7. See Edward Altman, 1984, "A Further Empirical Investigation of the Bankruptcy Cost Question," *Journal of Finance* 39, pp. 1067-1089.

8. See Moody's, 2002, Default and Recovery Rates of Corporate Bond Issuers - A Statistical Review of Moody's Ratings Performance. This probability is the probability of default in year t conditional on survival until year t-1, for t = 1, 2…10 and for all bonds that were rated BBB at issuance (at t = 0). The probability is computed as an average over the years of 1970-2001, and over the horizons from 1 to 10 years. It corresponds to the probability p in Figure 1.

9. It also assumes that distress states and default states are the same: Equation (2) should include the distress costs that are incurred around the time of default. Capturing costs incurred long before default is more difficult (i.e., see Tim Opler and Sheridan Titman, (1994), "Financial Distress and Corporate Performance," *Journal of Finance* 49, pp. 1015-1040.

10. The assumptions about maturity, coupons and recovery simplify the calculation but do not significantly affect the results. See Almeida and Philippon (2007).

11. The average historical recovery rate of 0.41 is between 1982 and 2001 (Moody's (2002)).

the actual market spread. The average spread for 10-year BBB bonds is 1.9%, or six times larger than 0.33%.[12] Why? Mainly because investors assign a large risk premium to systematic default risk. And this risk premium in turn suggests that the standard approach to valuing distress costs using historical default rates and the risk-free rate is incorrect.

Risk-Adjusted Probabilities

The systematic default risk premium can be incorporated into the distress cost NPV calculation by using risk-adjusted probabilities.[13] As implied by their name, risk-adjusted probabilities incorporate risk premia by changing the relative weights that investors assign to different possible outcomes or "states." As a result, risk-adjusted probabilities do not represent the actual expected frequency of events in the real world. Instead, events that are expected to happen in bad states of the world (such as default) are assigned risk-adjusted probabilities that are larger than the historical ones.

If risk-adjusted probabilities are used to value the bond in Equation (3), then the risk-free rate must be used as the discount rate. This is because the risk adjustment is accomplished by changing the probability of default rather than by changing the discount rate, an adjustment that proves easier in most circumstances. Denoting the risk-adjusted probability of default by q, Equation (3) becomes:

$$1 = \frac{(1 - q + q\rho)(1 + y)}{(1 + r_F)}, \quad (4)$$

There are two ways to use Equation (4). Given an estimate for q, the bond's yield y can be backed out. This is how a bond trader, for example, might use Equation (4) because he is concerned with the bond's yield. Alternatively, and for our purposes, an observed yield can be used to estimate the implied risk-adjusted probability by rearranging Equation (4):

$$q = \frac{y - r_F}{(1 + y)(1 - \rho)}, \quad (5)$$

In Equation (5), the probability q incorporates the default risk premium that is implicit in the yield spread $y - r_F$. Using average historical data for BBB-rated bonds—a spread of 1.90% and recovery rate of 0.41% along with a risk free rate of 5%—gives a q of 3.03%, which is thus almost six times as large as the historical default probability p = 0.53%.

This calculation assumes that the observed 10-year spread of 1.90% for BBB bonds (computed using 10-year treasuries as a benchmark) is entirely due to default. But because Treasury securities are more liquid than corporate bonds, part of the spread can be viewed as a liquidity premium.[14] Also, Treasuries have a tax advantage over corporate bonds because they are not subject to state and local taxes.[15] These arguments suggest that the entire corporate yield spread cannot be attributed to default risk alone.

To isolate the component of the spread that is attributable to default risk, the part of the spread that is *not* likely to reflect default risk can be inferred from the spreads between short maturity AAA bonds and Treasuries.[16] It is unlikely that a 1-year maturity AAA corporate bond will ever default. Nevertheless, the average historical 1-year AAA spread in our data is 0.51%. Although this spread could reflect liquidity or tax considerations, it's very unlikely to reflect investors' perception of default risk.

Thus, in the case of our BBB with a total spread of 1.90%, we infer that the component of the spread attributable to default is 1.39%. Using this default spread in place of $y - r_F$ in Equation (5) we derive a risk-adjusted probability of default (free of tax and liquidity considerations) of 2.21%. This risk-adjusted probability can be used directly in Equation (2) in place of the historical probability p. Because the probability q incorporates the risk-adjustment, the risk-free rate is the correct discount rate in Equation (2). Note that under this approach it is not necessary to estimate default rates using historical data. This is an advantage to the extent that future default rates are expected to differ from historical ones.

Having calculated the risk-adjusted probability of default, we can now estimate the NPV of distress costs as follows

$$\Phi = \frac{q}{q + r_F}\phi, \quad (6)$$

where q is given by Equation (5). With q =2.21%, the NPV of distress is equal to 5.1% of firm value, and thus more than three times as high as the 1.6% NPV of distress before we incorporated the systematic risk premium.

NPV of Distress Costs for Different Bond Ratings

Using this basic model, it is easy to calculate the NPV of distress costs for companies with different bond ratings. The yield spreads reported in Table 1, which were obtained from

12. Since the bond valued in Equation (3) is perpetual, we recommend using long term spreads (typically 10 years or higher) to implement the model with real world data.

13. Equation (3) can be used to tell us what the discount rate rD has to be in order to justify the observed yield spread of 1.9%. Using y = 6.9% (the 5% risk free rate plus the 1.9% risk premium), we obtain a discount rate rD = 6.57%. Investors discount bond cash flows at a rate substantially larger than the risk free rate. But clearly, we cannot use 6.57% to discount distress costs, since the implicit rate for distress costs must be lower than the risk free rate. Adjusting the discount rate is not the right solution however. As we show next, it is much easier to adjust the probabilities.

14. See Long Chen, David Lesmond, and Jason Wei, (2004), "Corporate Yield Spreads and Bond Liquidity," Working paper, Michigan State University and Tulane University.

15. See Edwin Elton, Martin Gruber, Deepak Agrawal, and Christopher Mann, (2001), "Explaining the Rate Spread on Corporate Bonds," *Journal of Finance* 56, pp. 247-278.

16. See Long Chen, Pierre Collin-Dufresne, and Robert Goldstein, (2005), "On the Relation Between Credit Spread Puzzles and the Equity Premium Puzzle," Working paper, Michigan State University, U.C. Berkeley, and University of Minnesota.

Table I

The spread data for A, BBB, BB, and B bonds come from Citigroup's yieldbook, and refer to average corporate bond spreads over Treasuries for the period 1985 to 1995. Data for AAA and AA bonds come from Huang and Huang (2003). This table reports risk adjusted probabilities of default (q) calculated from bond yield spreads, as explained in the text. The probabilities p are the yearly historical probabilities of default (data from Moodys, averages 1970 to 2001). This table also reports our estimates of the NPV of the costs of financial distress expressed as a percentage of pre-distress firm value, calculated using risk adjusted probabilities (NPV (q)) and historical probabilities (NPV(p)). It also reports in the last row the increase in the NPV of distress costs that is associated with a rating change from AA to BBB. We use an estimate for the loss in value given distress of 16.5%, a recovery rate of 0.41, and a risk free rate of 5%.

Rating	10-year spread	q	p	NPV (q)	NPV (p)
AAA	0.63%	0.2%	0.1%	0.6%	0.3%
AA	0.91%	0.6%	0.1%	1.9%	0.3%
A	1.32%	1.3%	0.2%	3.4%	0.5%
BBB	1.90%	2.2%	0.5%	5.1%	1.6%
BB	3.32%	4.4%	2.4%	7.8%	5.3%
B	5.45%	7.7%	6.1%	10.0%	9.0%
			BBB minus AA	3.2%	1.3%

Citigroup's yield book for the period of 1985 to 2004, are the starting point. We use long-term spreads (10+ years) in our calculations and deduct the 1-year AAA spread of 0.51% to adjust for taxes and liquidity.

Column 3 of Table 1 shows the risk-adjusted probabilities of default obtained after computing Equation (5) for different ratings, using a recovery rate of 0.41 and a risk-free rate of 5%. For comparison purposes, the historical probabilities of default from Moody's are reported in column 4. As shown in the table, the risk-adjusted market-implied probabilities are larger than the historical ones for all ratings and maturities, in particular for investment-grade bonds.

Columns 5 and 6 show the NPV using the risk-adjusted probabilities (denoted by NPV(q)), and historical probabilities (NPV(p)), respectively. The significant differences, especially at higher credit ratings, clearly demonstrate that risk premium is a first-order issue in the valuation of distress costs. For example, distress costs for A-rated companies increase from 0.5% to 3.4% of firm value after adjusting for systematic risk. And an increase in leverage that moves a firm from AA to BBB increases its expected cost of distress by 3.2%, whereas the increase is only 1.3% using historical probabilities.

Conclusion

Recent events in financial markets show the relevance of systematic risk in determining the probability of bankruptcy and the costs of financial distress. The risk of bankruptcy for highly-levered companies will rise precisely when it is most disadvantageous: when it is harder to liquidate assets and more costly to raise new capital. Bond investors seem to be aware of these risks, and have usually demanded significant risk premia to hold debt securities issued by highly-levered firms. But since standard valuations of bankruptcy costs ignore these economy-wide risks, corporate managers who follow this practice will underestimate the actual expected costs of debt and may end up with excessive leverage in their capital structure.

In this article, we show how to adjust the valuation of financial distress costs to take such systematic risks into account. Our method uses observed bond spreads to back out the risk adjustment that managers should use to compute distress costs when making financing decisions. It produces distress costs that are substantially higher than those obtained through standard calculations. The key insight is that the market information built into default risk premia should be considered by corporate managers and incorporated into applied capital structure models.

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