DOES PRIOR PERFORMANCE AFFECT A MUTUAL FUND'S CHOICE OF RISK? THEORY AND FURTHER EMPIRICAL EVIDENCE*

by

Hsiu-lang Chen

and

George G. Pennacchi

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*Chen is from the Department of Finance, College of Business Administration, University of Illinois at Chicago, 601 South Morgan Street, Chicago, Illinois 60607 (telephone: 312-355-1024, fax: 312-413-7948, email: hsiulang@uic.edu). Pennacchi is from the Department of Finance, College of Commerce and Business Administration, University of Illinois at Urbana-Champaign, 1407 W. Gregory Drive, Urbana, Illinois 61801 (telephone: 217-244-0952, fax: 217-244-9867, email: gpennacc@uiuc.edu). We are grateful for valuable comments provided by Zoran Ivkovich, Jason Karceski, Matti Keloharju, and participants of the 2000 AFA Meetings and of seminars at the University of Notre Dame, the University of Illinois, the University of North Carolina at Chapel Hill, and the Federal Reserve Bank of Cleveland.
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Abstract

Recent empirical research on mutual fund tournaments examines the relation between prior performance, the structure of compensation, and a fund manager’s choice of portfolio risk. This paper models the portfolio decisions of a fund manager who competes in such a tournament. Explicit solutions for the manager’s portfolio choice are derived for compensation rules that can be either a concave, linear, or convex function of the fund’s performance relative to that of a benchmark. The model shows that assumptions made in prior empirical research may not hold. For particular compensation structures, a manager increases the fund’s “tracking error” as its relative performance declines. However, in contrast to the received wisdom, declining performance does not necessarily lead the manager to raise the variance of the fund’s return.

The paper develops a new empirical methodology for testing the relation between mutual fund performance and risk-taking. Unlike previous methods, it allows a fund’s risk to respond continuously to its relative performance and estimates risk-taking behavior for each individual mutual fund. This technique is applied to data on more than 4,000 equity mutual funds over the 1962 to 2001 time period. As predicted by the model, mutual funds are found to increase tracking error, but not return variance, as their performance declines. The effect is stronger for younger and smaller mutual funds, as well as those with relatively low front loads and high back loads.
I. Introduction

As mutual fund investing has grown, the management of mutual funds has come under closer scrutiny by financial economists. One strand of research examines potential agency problems between a mutual fund’s shareholders and its portfolio manager. Several studies investigate whether a manager might unnecessarily shift the fund’s risk in response to changes in its performance relative to other funds. This behavior is linked to the way the manager is compensated and to the actions of mutual fund investors. The manager’s compensation depends on her success in generating flows of new investments into the fund, while mutual fund investors “chase returns” by channeling investments into funds with better relative performance. This creates a situation described as a mutual fund “tournament” where portfolio managers compete for better performance, greater fund inflows, and, ultimately, higher compensation.

Inflows rise nonlinearly with a fund’s relative performance. Numerous studies document that mutual funds with the best recent performance experience a lion’s share of new inflows, but poorly performing funds are not penalized with sharply higher outflows.\(^1\) If the fund manager’s compensation rises in proportion to the fund’s inflows, this convex performance - fund flow relation produces a convex performance - compensation structure.\(^2\) Research, such as Sirri and Tufano (1998), notes that such compensation is similar to a call option, producing an incentive for the manager to raise the risk of the fund’s relative returns and increase the option’s value. To empirically test for the presence of this risk-taking incentive, studies have compared the behavior of a cross-section of mutual funds for which this risk-taking incentive is predicted to differ.

Chevalier and Ellison (1997) estimate the shape of the performance - fund flow relation and use it to infer different funds’ risk-taking incentives. They, like other studies, assume that a fund’s inflows respond primarily to its relative performance calculated over the previous calendar year. Thus fund managers compete in annual tournaments that begin in January and end

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\(^2\) The literature on mutual fund tournaments distinguishes between a fund’s investment advisor, the entity responsible for portfolio management, and the portfolio manager hired by the advisor. The typical investment advisor is paid a fixed fraction of the fund’s assets, assets that depend on both net fund inflows (external growth of assets) and the fund’s return (internal growth of assets). However, the portfolio manager’s compensation is assumed to depend only on his ability to generate extraordinary growth in fund assets, growth that depends on the fund’s return relative to (the average of) other funds’ returns. Common or systematic shocks to all funds’ returns (affecting internal asset growth) are not due to the individual manager’s portfolio selection ability and would not affect compensation. Hence, compensation is assumed to depend on relative, not absolute, performance.
in December. A fund’s risk-taking incentive over the final quarter of the year is assumed to be proportional to the estimated convexity of fund inflows measured locally around the fund’s September performance ranking. Using data on the portfolio compositions of mutual funds at the ends of September and December, they find that a fund tends to change the variance of its return relative to a benchmark return as the performance - fund flow relation would predict.

Another study by Brown, Harlow, and Starks (1996) tests whether a fund that performs relatively poorly at mid-year tends to raise the variance of its return over the latter half of the year more than does a fund that performs relatively well at mid-year. Calculating return variances using monthly data from a cross-section of mutual funds, they find support for the hypothesis that mid-year “losers” gamble to try to improve their relative end-of-year performance more than mid-year “winners.” Koski and Pontiff (1999) also use monthly returns data to calculate various measures of a fund’s risk, including the standard deviation, beta, and idiosyncratic risk of a fund’s returns. Their results are similar to those of Brown, Harlow, and Starks (1996) in that a mutual fund’s performance in the first six months of the year is negatively related to its change in risk between the first and second halves of the calendar year.

However, recent research by Busse (2001) suggests that some prior empirical findings may be spurious. Using daily, rather than monthly, data on mutual fund returns to calculate a fund’s variance of returns, he duplicates the tests in Brown, Harlow, and Starks (1996) and finds no evidence that mid-year poor-performing funds increase their return variances more than mid-year better-performing funds. If anything, better-performing funds appear to have slightly greater return variances in the second half of the year. Hence, the empirical evidence on how prior performance affects a mutual fund manager’s choice of risk appears mixed.

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3 This assumption is justified because sources of mutual fund information, such as Morningstar, Inc., typically compute relative fund performances using this calendar year period. Hence, flows of investor funds and, in turn, managerial compensation should be most sensitive to a fund’s calendar year performance. Empirical evidence in Koski and Pontiff (1999) indicates that changes in a fund’s risk are most strongly related to performance calculated over calendar years.

4 For example, the shape of the estimated fund flow relation for young mutual funds predicts that funds performing moderately well (poor) over the January to September period should choose lower (higher) risk over the final quarter of the year. However, the estimated fund flow relation for extremely good (poor) performers predicts that they should choose higher (lower) risk.

5 In addition to their test based on fund portfolio composition data, Chevalier and Ellison (1997) test the risk-taking incentives implied by their estimated performance - flow relation using monthly data on fund returns. They measure a fund’s risk as the variance of its return in excess of the return on a value-weighted market index of NYSE, AMEX, and NASDAQ stocks. The change in this risk measure from January through September versus October through December supports their estimated performance - flow relation. He also shows that if variances are calculated using monthly returns measured from the middle of each month, rather than from the beginning of each month as in Brown, Harlow, and Starks (1996), the evidence of risk-shifting disappears.
The current paper clarifies issues in this mutual fund tournament literature by providing new theoretical and empirical insights into risk-taking by mutual funds. First, it models the optimal intertemporal portfolio strategy of a mutual fund manager that faces the competitive “tournament” environment assumed by recent empirical work. Explicit solutions for this manager’s portfolio allocation are derived when her utility displays constant relative risk aversion and compensation is either a concave, linear, or convex function of the fund’s relative calendar-year performance.

An important implication of the model is that when the penalty for poor performance is limited so that the manager’s total compensation can never fall to zero, then the fund manager chooses to deviate more from the benchmark portfolio as the fund’s performance declines. In other words, when a fund is performing poorly it displays more “tracking error” than when it performs relatively well. It is not true, however, that the manager of a poorly performing fund necessarily increases the variance of the fund’s returns, as some previous empirical studies assume. Rather, under-performance can lead to more tracking error volatility but less variance of returns. Hence, the tests for shifts in returns variances performed by Brown, Harlow, and Starks (1996) and Busse (2001) may not determine whether fund managers truly are acting opportunistically.

Second, the paper develops a new, more powerful, empirical technique that is used to re-examine the evidence on risk-taking in the mutual fund industry. Our approach improves upon previous methods because it is consistent with a theory of managerial behavior, it allows a fund’s risk to change at each observation date not just once per year, and it can be applied to estimate risk-shifting incentives of individual mutual funds not only the entire industry. We apply this technique to data on more than 4,000 mutual funds that operated during the 1962 to 2001 period, a much larger sample than those of previous studies. As predicted by our model, most mutual funds are found to increase tracking error, but not return variance, as their performance declines. Based on these results, we also explore whether risk-shifting is systematically linked to mutual funds with particular characteristics.

The plan of the paper is as follows. Section II briefly discusses related theoretical work on the risk-taking incentives of mutual fund managers. In section III we present our model of a mutual fund manager whose compensation depends on the fund’s end-of-year relative performance. Throughout the year, the manager can rebalance the fund’s portfolio as its performance changes. His optimal portfolio choice is derived for a compensation schedule that, depending on its parameter values, can be concave, linear, or convex in the fund’s relative
performance. Section IV develops a new empirical method for testing the risk-taking behavior implied by our model. Section V describes our data, while section VI presents the empirical results regarding risk-taking by over 4,000 mutual funds. Concluding comments are in section VII.

II. Related Literature on a Portfolio Manager’s Choice of Risk

A growing literature analyzes the theoretical behavior of portfolio managers and investors. Many of these studies examine the link between a fund manager’s compensation contract and his choice of the fund’s portfolio. Grinblatt and Titman (1989) show how compensation contracts that include a bonus for good performance can produce adverse risk incentives. Mutual fund managers can maximize the present value of their option-like bonus by choosing a fund portfolio with excessive risk. Moreover, the fund manager could capture, without risk, the increased value of this bonus if she can hedge using her personal wealth.

Starks (1987) also considers the effects of a bonus contract, focusing on situations of asymmetric information between fund investors and fund managers. When investors cannot observe a manager’s choice of portfolio risk or the manager’s effort level, compensation contracts with symmetric payoffs dominate contracts that include a bonus. However, Das and Rangarajan (2001) show that the relative advantages of symmetric and bonus contracts can be reversed if investors’ choice of funds is made endogenous to the funds’ risk levels and compensation contracts. In their model, bonus contracts provide better risk-sharing between investors and fund managers when investors take account of a fund’s risk and contract choice.

Other research, such as Huberman and Kandel (1993), Heinkel and Stoughton (1994), and Huddart (1999), considers environments where fund managers possess different abilities that are unbeknownst to investors. In these game-theoretic models, there is typically an initial period when investors learn of managers’ abilities based on their relative performances, followed by a second period during which investors can switch their savings to those managers perceived to have the highest abilities. Hence, these “investor learning” models can explain the link between fund flows and prior performance. In addition, if managerial ability displays decreasing returns to scale, Berk and Green (2001) show that flows determine the relative sizes of funds such that, in equilibrium, mutual fund investors expect no future superior returns net of fund fees and expenses.

The model in the current paper differs from this previous work by focusing on how prior performance affects a fund manager’s choice of portfolio risk. It stresses the intertemporal reactions of fund managers that have been the basis of empirical studies, such as Brown, Harlow,
and Starks (1996), Chevalier and Ellison (1997), and Busse (2001). The model takes the structure of managerial compensation as given and studies a manager’s dynamic portfolio choice over an annual “tournament” evaluation period. Recent work by Carpenter (2000) and Cuoco and Kaniel (1999) is related to ours. These papers assume that the performance-related components of a portfolio manager’s compensation consist of options written on the portfolio’s value. Carpenter (2000) assumes a risk-averse manager is compensated in the form a fixed-fee plus a call option written on the value of the managed portfolio with an exercise price equal to the value of a benchmark asset. Cuoco and Kaniel (1999) is similar, but their model also allows the compensation contract to contain a penalty for poor performance in the form of the manager writing a put option on the managed portfolio.\footnote{Cuoco and Kaniel (1999) focus on the equilibrium asset pricing implications of different portfolio manager compensation contracts rather than on a manager’s incentive to shift the fund’s risk in response its performance. They consider an economy with individual investors and a fund manager who can choose among a bond and two stocks that are in fixed supply. Given the dividend process of each of the stocks, one of which represents an index and the other which represents a stock not in the index, they numerically solve for the equilibrium expected returns and volatilities of the stocks under different compensation rules.}

While we also assume that a portfolio manager’s compensation depends on the portfolio’s performance relative to a benchmark, the contract is not strictly in the form of standard call or put options. Rather than being an option-like, piece-wise linear function of performance, our contract assumes compensation is a smooth function that can be either concave, linear, or convex in relative performance. This is done to better capture the type of managerial compensation assumed in recent empirical studies of mutual fund tournaments, namely, compensation that depends on a fund’s assets under management and, hence, the manager’s ability to generate fund inflows.\footnote{The contract in Carpenter (2000) may better represent the compensation of a non-financial firm manager who receives stock options, while the contract in Cuoco and Kaniel (1999) may be most appropriate to the compensation of other portfolio managers, such as managers of pension funds.} Empirical evidence suggests that fund inflows are poorly modeled as a simple piece-wise linear function of performance. Moreover, our form of compensation is flexible enough to generate different equilibrium relationships between prior performance and a manager’s choice of risk, yet it leads to simple and intuitive closed-form solutions. This simplicity allows the model to directly guide our later empirical tests of mutual fund behavior. However, while our model differs from those of Carpenter (2000) and Cuoco and Kaniel (1999), we discuss below how all three produce the same qualitative results when similar assumptions are made.
III. Modeling a Mutual Fund Manager’s Portfolio Decisions

We now describe our model’s specific assumptions. A fund manager’s compensation is assumed to depend on the fund’s performance relative to a benchmark index. The fund’s portfolio can be invested partly in this benchmark index and partly in a set of “alternative” securities chosen by its fund manager. These alternative securities are defined as the portion of the fund’s total assets that accounts for the difference between the fund’s portfolio and one that is invested solely in the benchmark portfolio. The Appendix shows that the fund manager’s optimal choice of individual alternative securities is one where their relative portfolio proportions do not vary over time. This implies that the manager’s intertemporal portfolio choice problem can be transformed to one of allocating a portion of the fund’s portfolio to the benchmark index and the remaining portion to a single alternative composite security. Hence, we simplify the presentation by assuming at the start that the portfolio allocation problem involves only two types of securities: the benchmark index and a single alternative security.

Define $S_t$ as the value of the relevant benchmark index at date $t$ and $A_t$ as the date $t$ value of the alternative securities. $S_t$ and $A_t$ are assumed to follow the processes:

$$\frac{dS}{S} = \alpha_s dt + \sigma_s dz$$  \hspace{1cm} (1) $$dA/A = \alpha_A dt + \sigma_A dq$$  \hspace{1cm} (2)

where $\sigma_s dz \sigma_A dq = \sigma_{AS} dt$. For analytical convenience, $\sigma_A$, $\alpha_A$, $\sigma_s$, and $\sigma_{AS}$ are assumed to be constants. $\alpha_A$ and $\alpha_s$ may be time varying, as might be the case if market interest rates are stochastic. However, we require that the spread between their expected rates of return, $\alpha_A - \alpha_s$, be constant.

If the fund manager allocates a portfolio proportion of $1-\omega$ to the benchmark index and a proportion $\omega$ to the alternative securities, then the fund’s portfolio's value, $V$, follows the process:

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9 If the benchmark portfolio was the S&P500 index, the alternative securities could be a portfolio of small capitalization stocks that are not part of the S&P500 index. This is the assumption made in Cuoco and Kaniel (1999). However, in our model a particular fund manager’s alternative securities might instead be a portfolio of large capitalization stocks that are part of the S&P500 index but which the fund holds in different proportions to the S&P500’s value-weightings. For example, if a particular stock represented 3% of the benchmark portfolio but only 2% of the fund manager’s portfolio, the fund manager’s alternative securities would include a short position in that stock.

10 This “two-fund separation” result is similar to that of Merton’s (1971) case of lognormal asset prices.
\[
\frac{dV}{V} = (1-\omega)\frac{dS}{S} + \omega \frac{dA}{A} = \left[ (1-\omega)\alpha_s + \omega \alpha_A \right] dt + \left[ (1-\omega)\sigma_s dz + \omega \sigma_A dq \right]
\] (3)

Note that whenever \( \omega \neq 0 \), the fund’s return in (3) deviates from the benchmark return. We can also calculate the process followed by the fund’s relative performance. Define \( G_t \equiv V_t/S_t \) to be the date \( t \) ratio of the value of the fund’s portfolio to that of the benchmark. A simple application of Itô’s lemma shows that

\[
\frac{dG}{G} = \omega \left( \alpha_A - \alpha_s + \sigma_A^2 - \sigma_A \sigma_s \right) dt + \omega \left( \sigma_A dq - \sigma_s dz \right)
\] (4)

The fund manager is assumed to compete in a tournament for inflows into the fund. At the start of the tournament’s assessment period, \( G = 1 \) by definition, but then changes stochastically according to equation (4). Thus, \( G_t \) measures the date \( t \) ratio of the fund’s return to that of the benchmark since the start of the tournament, and hence \( D_t \equiv \ln(G_t) \) is the difference between the fund’s continuously-compounded return and that of the benchmark index since the beginning of the tournament.\(^\text{12}\) The tournament ends at date \( T \), which, for example, could be the last trading day of the calendar year. The manager’s compensation is a function of the fund’s relative performance at the end of the tournament, so that his compensation or “pay” can be written as \( P[G_T] \).\(^\text{13,14}\)

\(^\text{11}\) The Appendix derives the values of \( \alpha_A \) and \( \sigma_A \) in terms of the parameters of processes for \( n \) individual alternative securities.

\(^\text{12}\) We refer to relative performance as the ratio of returns, \( G_t \), rather than the difference in returns, \( D_t \), but this distinction is nonessential. The manager’s compensation function could be rewritten in terms of \( \ln G_t \), rather than \( G_t \). As will be shown, a manager’s optimal portfolio choice is independent of prior performance when compensation is proportional to a power of \( G_t \) rather than \( D_t \), so the ratio is a natural variable to use.

\(^\text{13}\) In practice, compensation may depend also on the performance of the overall equity (mutual fund) market. Karceski (2002) shows that a fund’s inflows are highest when the fund performs relatively well and, simultaneously, the overall stock market performs well. He studies the implications of this phenomenon for fund managers’ selection of high versus low beta stocks and the equilibrium effects on asset prices. Though our analysis omits this market effect, including would not fundamentally change how managers respond to prior relative performance. Rather, it would influence the average level of market (beta) risk chosen by fund managers.

\(^\text{14}\) The assumption that compensation depends only on the fund’s performance relative to a single index is a simplification for another reason. In general, a fund’s net inflows, and hence its manager’s portfolio choices, might depend on the final performances of each of the mutual fund’s competitors. Our simplified structure can be justified in an environment where there is a large (infinite) number of mutual funds that choose different “alternative” securities. Their relative performances over the year would be a smooth, approximately normally distributed, function around a mean performance. This would justify (as we do in our empirical work) using the “average” performance of all mutual funds as a sufficient statistic for comparing any given mutual fund’s performance.
The fund manager maximizes his expected utility of wealth (compensation) at the end of the tournament by choosing the fund’s asset allocation at each point in time during the assessment period.\footnote{By equating the manager’s compensation to his wealth, we assume that other sources of personal wealth are negligible and that the manager does not hedge his compensation risk via his personal portfolio. This is a standard assumption, though Grinblatt and Titman (1989) is an exception.} This maximization problem can be written as

\[
\text{Max}_{\omega(x) \in \mathbb{R}_{[t,T]}} E_t \left[ U \left( P[G_T] \right) \right]
\]  

subject to the process followed by \( G \) given in equation (4). If we define \( J(G, t) \) as the derived utility of wealth (or performance) function, then assuming that \( U(P[G_T]) \) is concave in \( G_T \), the solution to (5) satisfies the Bellman equation:\footnote{If \( U(P[G_T]) \) is not concave in \( G_T \), as may be the case if \( P[G_T] \) is convex, a solution may be characterized using a concavifying argument similar to Carpenter (2000).}

\[
0 = \text{Max}_{\omega} J_t + J_G \omega \alpha_g G + \frac{1}{2} J_{GG} \omega^2 \sigma^2_G G^2
\]

where \( \alpha_g = \alpha_A - \alpha_S + \sigma^2_S - \sigma_{AS} \) and \( \sigma^2_G = \sigma^2_A - 2\sigma_{AS} + \sigma^2_S \). The first order condition with respect to \( \omega \) implies that the portfolio proportion invested in the alternative securities is

\[
\omega^* = -\frac{J_G \alpha_g}{GJ_{GG} \sigma^2_G}
\]

Substituting this back into the Bellman equation, one obtains an equilibrium partial differential equation for \( J_t \):

\[
0 = J_t - \frac{1}{2} \left( \frac{J_G \alpha_g}{J_{GG} \sigma^2_G} \right)^2
\]

The solution to equation (8) must satisfy the boundary condition \( J(G_T, T) = U(P[G_T]) \), which }
Regarding the manager’s compensation schedule, we choose a specification that allows it to be either a concave, linear, or convex function of fund performance. This generality is important because the model is the basis of our empirical tests, and prior evidence suggests that compensation may not always be convex. While most empirical studies of mutual funds, such as Sirri and Tufano (1998), emphasize the convexity of fund flows and compensation to performance, Chevalier and Ellison (1997) find that the relation appears non-convex for certain types of mutual funds at particular performance levels. Moreover, a linear function of performance might better describe the compensation of other types of money managers. The following compensation function provides empirical flexibility yet leads to fairly simple and intuitive solutions to the manager’s portfolio choice problem.

\[
P[G_T] = \left( a + \frac{b}{c} G_T \right)^c
\]

where \( b > 0, c > 0, \) and \( a > -\frac{b}{c}. \) Note that when \( 0 < c < 1, \) the manager’s compensation is a concave function of performance, \( G_T. \) Compensation is linear in performance when \( c = 1, \) while when \( c > 1 \) the function is convex. If \( a \) is set equal to 1 and one takes the limit of \( P \) as \( c \) goes to infinity, the function becomes the exponential form \( P[G_T] = \exp(bG_T). \)

In the linear case of \( c = 1, \) the parameter \( a \) can be interpreted as a fixed component of a manager’s fee schedule, and, in principle, it could be positive or negative. For example, if the manager incurs fixed expenses (overhead) that are not explicitly reimbursed by the fund, then \( a \)

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17 In addition, other agency concerns between a mutual fund advisor (the entity that directly receives the advisory fees paid by the mutual fund) and the portfolio manager that it hires could affect the manager’s compensation. For example, poor performance by a portfolio manager may be personally more damaging than what would be predicted by fund flows if such performance results in the manager’s termination. Khorana (1996) and Chevalier Ellison (1999) provide empirical evidence that underperformance increases the probability that a manager is terminated. The threat of firing could make the manager’s compensation appear non-convex, at least when performance is poor.

18 Empirical evidence in Del Guercio and Tkac (1999) finds a performance - fund flow relation that appears linear for pension fund managers. They conclude that pension managers have less incentive to risk-shift than mutual fund managers.

19 The function could be further generalized by multiplication of a positive constant, that is,

\[
P[G_T] = \left( a + \frac{b}{c} G_T \right)^d \quad \text{where} \quad d > 0, \text{ but this extension has no effect on portfolio choice. If performance is defined as the difference in, rather than the ratio of, returns, then compensation is convex (concave) in } D_T \equiv \ln G_T \text{ whenever } a + b G_T \text{ is positive (negative). Since it will be shown that } a + (b/c)G_T \text{ is always positive in equilibrium, compensation can be a convex function of the difference in returns even when } 0 < c \leq 1. \text{ Hence, defining performance as the difference in returns expands the cases for which compensation is a convex function of performance.}
may be negative. In general, when $c \neq 1$, the parameter $a$ does not translate directly to a fixed fee, but its sign continues to determine whether total compensation has the potential to be non-positive. Lowering $a$ (possibly below zero) decreases total compensation, but sensible solutions to the manager’s portfolio choice problem require that $a > - b / c$. This restriction provides the manager with a feasible portfolio strategy that guarantees positive compensation (wealth) at the end of the tournament. The manager can avoid zero wealth (and infinite marginal utility) by investing solely in the benchmark portfolio for the entire assessment period, since then compensation equals $(a + G_T b / c)^\gamma = (a + b / c)^\gamma > 0$.

When $c \gamma < 1$, the boundary condition $U(P(G_T)) = (P(G_T))^\gamma / \gamma$ is a concave function of $G_T$, and an interior solution to the manager’s portfolio choice problem exists. This is always the case when $\gamma < 0$, that is, the manager’s risk-aversion exceeds that of logarithmic utility. However, if $0 < \gamma < 1$ and $c$ is sufficiently greater than 1 so that $\gamma c > 1$, then $U(P(G_T))$ is convex and the manager chooses $\omega$ to maximize the expected rate of return on $G$. From equation (4), this implies setting $\omega = +\infty$ if $\alpha_G > 0$, and $\omega = -\infty$ if $\alpha_G < 0$.

Assuming $c \gamma < 1$, the solution to equation (8) is

$$J(G, t) = \frac{1}{\gamma} \left( a + \frac{b}{c} G \right)^\gamma e^{-\theta(T-t)}$$

(10)

where $\theta = \frac{-c \gamma \alpha_G^2}{2(1 - c \gamma) \sigma_G^2}$. If equation (10) is substituted into equation (7), then the manager’s optimal proportion invested in the alternative securities is

$$\omega^* = \frac{\alpha_G}{(1 - c \gamma) \sigma_G^2} \left( 1 + \frac{ac}{bG} \right)$$

(11)

Note from the restriction $a > - b / c$ that the term $(1 + \frac{ac}{bG})$ is always non-negative in equilibrium, even when $a < 0$. To see this, suppose that $a$ is negative and that $G$ declines sufficiently from its initial value of unity, so that $(1 + \frac{ac}{bG})$ approaches zero. Then from equation (11) the manager’s optimal strategy is to invest fully in the benchmark portfolio. But at this point with $\omega^* = 0$, equation (4) implies $dG = 0$. With no further changes in the relative performance of the fund, the manager optimally prevents compensation from falling to zero.
Equation (11) shows that the manager chooses a long (short) position in the alternative securities whenever $\alpha_G$ is positive (negative), and the magnitude of the position is inversely related to risk aversion and independent of the tournament’s time horizon, $T-t$. For the special case of $a = 0$, so that compensation is proportional to a power of relative performance, $P[G_T] = (bG_T/c)^c$, the alternative securities’ portfolio weight is constant and invariant to changes in the fund’s performance. However, for the general case of $a \neq 0$, $\omega^*$ varies with changes in $G$. When $a > 0$, the manager moves closer to the benchmark portfolio with improvements in fund performance. The reverse occurs when $a < 0$. For the special case of $a = 1$ and $c \to \infty$, that is, compensation is the exponential form $P[G_T] = \exp(bG_T)$, equation (11) becomes

$$\omega^* = -\frac{\alpha_G}{\gamma bG \sigma_G^2}$$

so that the alternative securities’ portfolio weight responds inversely to relative performance.

We summarize the manager’s portfolio behavior with the following proposition.

**Proposition I**: If a fund manager’s utility displays constant relative risk aversion and has compensation given by (9), then when $c \gamma < 1$ an interior solution to the portfolio choice problem exists. Moreover, $\frac{\partial [\omega^*]}{\partial G} = -a \frac{c |\alpha_G|}{(1 - c \gamma)bG^2 \sigma_G^2}$, whose sign is opposite to that of the compensation parameter $a$. When $a$ is positive (negative), a decline in the fund’s relative performance leads the fund manager to deviate more (less) from the benchmark index.

The case $a > 0$ provides theoretical justification for Lakonishok, Shleifer, and Vishny’s (1992) argument that successful managers attempt to lock-in gains. Furthermore, such behavior is likely to increase with the convexity of compensation, since when $\gamma < 0$ one can show that a larger value of $c$ makes portfolio choice more sensitive to prior performance. In contrast, the $a < 0$ case leads to managerial risk-shifting behavior that is opposite to that assumed in recent empirical studies of tournaments. For this case, total compensation is not automatically bounded

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20 That portfolio choice is independent of the investment horizon is a common feature of standard portfolio choice problems such as Merton (1971). The solution in (11) is analogous to that of a standard portfolio choice problem where the alternative securities portfolio plays the role of a risky asset portfolio and the benchmark portfolio is the risk-free asset. From this perspective, $\alpha_G$ and $\sigma_G$ are the risky asset’s excess returns.
at zero, and a manager more closely matches the benchmark as performance declines to prevent zero compensation and infinite marginal utility.

Proposition I is consistent with the findings of Carpenter (2000) and Cuoco and Kaniel (1999). When they assume that compensation equals a positive component plus a call option written on the portfolio’s relative performance, they find that a manager increases tracking error as performance declines. This compares to our model’s case of $a > 0$, since in both instances the compensation rule is restricted to always be positive, and portfolio managers need not fear obtaining zero wealth when increasing their tracking error risk. In contrast, when Cuoco and Kaniel (1999) assume that compensation also includes the manager writing a put option on his performance, the manager reduces his tracking error as performance declines. Here, compensation includes a penalty for poor performance, and it compares to our case with $a < 0$, since in both instances the compensation structure is not constrained to be positive. Hence, we see that as performance declines, the manager more fully hedges against zero compensation by more closely matching the benchmark.

While our model is not rich enough to adequately judge how different compensation schedules affect the welfare of fund shareholders, the fact that a manager’s investment strategy is independent of past performance when $a = 0$ would appear to favor such a compensation rule. Since risk-shifting generates additional transactions costs, setting $a = 0$ avoids this expense.

Proposition I also has implications for the risk measures chosen by Brown, Harlow, and Starks (1996) and Busse (2001). Each of these studies test the following relation:

$$\frac{\sigma_{2L}}{\sigma_{1L}} > \frac{\sigma_{2W}}{\sigma_{1W}}. \quad (13)$$

where $\sigma_{ij}$ denotes the standard deviation of the rate of return on mutual fund $j$’s portfolio during the $i^{th}$ half of the year. Mutual fund $j = L$ is a “loser” that displayed relatively poor performance in the first half of the year, while mutual fund $j = W$ is a “winner” that had relatively good performance during the first half. The implication is that a mutual fund that is a mid-year loser should increase the variance of its fund more than a fund that was a mid-year winner.

Does our model imply the inequality in (13)? To answer this question, note that the proportion invested in the alternative securities that would minimize the variance of the mutual fund’s rate of return given by equation (3) is

\[ \frac{\sigma_{2L}}{\sigma_{1L}} > \frac{\sigma_{2W}}{\sigma_{1W}}. \]
\[ \omega_{\min} = \frac{\sigma^2_S - \sigma^2_{AS}}{\sigma^2_G} \]  

(14)

Written in terms of this variance minimizing portfolio allocation, the optimal portfolio allocation in (11) becomes

\[ \omega^* = \omega_{\min} \frac{\alpha_G}{(\sigma^2_S - \sigma^2_{AS})(1 - c\gamma)} \left(1 + \frac{ac}{bG} \right) \]  

(15)

This allows us to state the following proposition:

**Proposition II:** Suppose \( c\gamma < 1 \) and \( a > 0 \), so that an interior solution exists in which the manager invests more in the alternative securities as performance declines. If \( \frac{\alpha_G}{(\sigma^2_S - \sigma^2_{AS})} > 0 \) and

\[ \frac{\alpha_G}{(\sigma^2_S - \sigma^2_{AS})(1 - c\gamma)} \left(1 + \frac{ac}{bG} \right) < 1 \], so that \( \omega^* \) and \( \omega_{\min} \) are of the same sign but \( \omega^* \) is smaller in magnitude than \( \omega_{\min} \), then a decline in \( G \) which moves \( \omega^* \) farther from zero and closer to \( \omega_{\min} \) reduces the variance of the mutual fund’s return. Otherwise, a decline in \( G \) raises the variance of the fund’s return.

Therefore, relation (13), the basis of previous empirical tests, is not a necessary implication of our model. This could explain why Busse (2001) fails to find empirical support for it, even if mutual fund managers do, in fact, have an incentive to shift risk. Propositions I and II clarify that worsening performance may, indeed, cause a mutual fund manager to deviate more from the benchmark portfolio, but this could move the portfolio closer to one that minimizes variance. Hence, the correct empirical indicator of risk-shifting should be the variance of a fund portfolio’s return relative to the benchmark’s return (tracking error), not the portfolio’s total return variance. Indeed, perhaps the most publicized “gamble” by a mutual fund manager was one that increased tracking error but reduced total variance. In late 1995, Jeffrey Vinik shifted the portfolio of Fidelity’s Magellan Fund out of technology stocks and into allocations of 19% bonds and 10% cash. With the subsequent stock market rally, the bet turned sour and led to Robert Stansky replacing Vinik as the fund’s portfolio manager.

manager acts as if risk-aversion varies with performance.
IV. Empirical Methodology

This section outlines the empirical technique used to examine the link between a mutual fund’s performance and its choice of risk. Unlike previous tests that allow a fund’s risk to change only once per calendar year (see inequality (13)), our method assumes that risk can respond to calendar-year performance at each (monthly) observation date. This is more realistic and closer in spirit to our theoretical model where risk varies continuously with the fund’s relative performance. Moreover, the technique allows risk-shifting behavior to be estimated for each individual mutual fund, thereby enabling us to test whether risk-shifting appears stronger for particular types of funds. This technique is applied to two different time series for each mutual fund. The first is the fund’s rate of return process, $d\ln V$, while the second is its change in relative performance process, $dG$, or, equivalently, its rate of return in excess of that of the benchmark, $d\ln G$. Our tests analyze how these processes relate to the contemporaneous level of a fund’s relative performance since the beginning of each calendar year, $G$.

The empirical studies of Brown, Harlow, and Starks (1996) and Busse (2001) are essentially tests of whether a fund’s prior performance predicts its future variance of returns. Recall from our model that even if fund managers do shift risk by deviating more from the benchmark as relative performance declines, poor (good) performance does not necessarily predict higher (lower) variance of returns. Still, the link between calendar-year performance and a fund’s return variance may be of independent interest. Hence, we first re-examine this issue but with a new empirical method that allows fund managers to react to performance at each monthly observation date.

Let $R_t \equiv \ln \left( \frac{V_{t+1}}{V_t} \right) \approx d \ln V_t$ be an individual mutual fund’s rate of return from the beginning to end of month $t$. If current relative, calendar-year performance, $G_t \equiv V_t / S_t$, predicts the future variance of $R_t$, then a simple way of modeling such a relation is

\[ \text{(14)} \]

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21 Chevalier and Ellison’s (1997) empirical tests do use an appropriate risk measure since they calculate risk as the variance of a fund’s returns in excess of a benchmark portfolio.

22 Note that since tournaments are assumed to occur each calendar year, $G$ is reset to 1 at the beginning of January each year, even though we use multiple years of fund returns to estimate these processes.

23 While most research assumes that changes in a fund’s risk are due to managerial incentive problems, Koski and Pontiff (1999) propose another explanation for why a fund’s variance could be inversely related to its prior performance. If better performing funds receive greater cash inflows, then a fund’s return variance will decrease until its new (riskless) cash is fully invested in equities or until the fund increases its exposure by purchasing equity derivatives. Because transactions costs are mitigated by gradual, rather than immediate, purchase of stocks, and some mutual funds are restricted from holding derivatives, the variance of a fund’s returns may decline temporarily following a cash inflow. Hence, our empirical work on the link between performance and variance provides new evidence on this alternative hypothesis.
where $\mu_R$ is assumed constant and $\varepsilon_t \sim N(0,1)$. Thus, the month $t$ conditional variance of $R_t$ is $(c_0 + c_1 G_t)^2$. Since $\partial \text{Var}_t[R_t]/\partial G_t = 2c_1 (c_0 + c_1 G_t)$, without loss of generality we restrict the standard deviation $(c_0 + c_1 G_t)$ to be positive when maximum likelihood estimates of equation (16) are obtained for each mutual fund. Hence, if $c_1 < 0$ ($c_1 > 0$), then variance is a decreasing (increasing) function of performance.$^{24}$ A negative $c_1$ would support the results of Brown, Harlow, and Starks, while a non-negative $c_1$ would be consistent with the findings of Busse (2001).

Our second set of estimates analyzes the process followed by an individual mutual fund’s change in relative performance, $dG_t$. An analysis of $dG_t$ is arguably more interesting and meaningful than studying the variable $R_t$ because our theoretical model places strong restrictions on the mean and variance of changes in a fund’s relative performance, not of its return. As shown in the Appendix, the model implies that the equilibrium $dG_t$ process satisfies

$$dG_t = \mu_G (d_0 + d_1 G_t) + (d_0 + d_1 G_t) \eta_t$$

(17)

where $dG_t = G_{t+1} - G_t$ refers to the change in relative, calendar-year performance from the beginning to end of month $t$, $\mu_G = |\alpha_G|/\sigma_G$, $d_0 = \frac{ac}{b} \frac{|\alpha_G|}{(1-c\gamma)\sigma_G}$, $d_1 = \frac{|\alpha_G|}{(1-c\gamma)\sigma_G}$, and

$\eta_t \sim N(0,1)$. Because $\mu_G$ and $d_0 + d_1 G_t$ are of the same sign, to make comparison of parameter estimates across mutual funds uniform, we assume that they are both non-negative.$^{25}$

$^{24}$ The restriction $(c_0 + c_1 G_t) > 0$ is innocuous in that it does not change the maximized value of the likelihood function which depends only on the variance of the return process, $(c_0 + c_1 G_t)^2$. If, instead, $(c_0 + c_1 G_t) < 0$ then the maximum likelihood estimates of $c_0$ and $c_1$ are identical except that their signs are reversed. In this case $c_1 > 0$ ($c_1 < 0$) would indicate that the variance of the fund’s portfolio decreases as performance declines. We impose this restriction simply to make a uniform comparison of the parameter estimates across funds.

$^{25}$ As discussed in the previous footnote, assuming $(d_0 + d_1 G_t) > 0$ is of no quantitative or qualitative significance since it does not change the maximized value of the likelihood function. If, instead, $(d_0 + d_1 G_t) < 0$ and $\mu_G < 0$, the maximum likelihood estimates of $d_0, d_1$, and $\mu_G$ are identical but with opposite signs. In this case, the qualitative interpretations for $d_0$ and $d_1$ would be exactly the reverse of those discussed in the text.
Estimating the parameters of equation (17) is equivalent to estimating those of the fund’s “excess return” or “tracking error” process

$$R_t - R_{St} \approx dD_t = \left(\frac{d_0}{G_t} + d_1\right) + \left(\frac{d_0}{G_t} + d_1\right) \eta_t$$

where $R_{St} = \ln(S_{t+1}/S_t)$ is the benchmark portfolio’s rate of return from the beginning to end of month $t$. Note that $R_t - R_{St} \equiv \ln(V_{t+1}/V_t) - \ln(S_{t+1}/S_t) = d \ln V - d \ln S = d \ln G = dD$.

From the definitions of $d_0$ and $d_1$, $\text{sgn}(d_0) = \text{sgn}(a)$ and $d_1 \geq 0$, where $a$ is the fixed parameter in the manager’s compensation schedule. By inspection of the stochastic component of equation (18), it is clear that a positive (negative) value of $d_0$ implies a negative (positive) performance - tracking error relation. For the special case of proportional compensation ($a = d_0 = 0$), the volatility of the excess return process is simply $d_1$, which is independent of performance. In contrast, the special case of exponential compensation implies $d_0 > 0$ and $d_1 = 0$, so that the fund’s excess return volatility is inversely proportional to performance. These results confirm Proposition I and the optimal allocation rules given in equations (11) and (12).

Equations (16) and (17) form the basis of our empirical work. The next section discusses the source of the data used to test these equations.

V. Data Description

Our information on mutual fund returns and characteristics come from the Center for Research in Security Prices (CRSP) Survivor-Bias Free Mutual Fund database. The sample covers mutual funds that operated during the period from January 1962 to September 2001. We selected domestic equity funds whose investment style could be broadly classified as either a “growth” fund or a “growth and income” fund.26 We also chose only those funds that reported returns for at least 36 continuous months, this condition being necessary to obtain reasonable parameter estimates of each fund’s returns process. These selection criteria gave us 2,658 growth (G) funds, 606 growth and income (GI) funds, and 879 “style-mixed” (SM) funds, the latter category being funds whose reported investment style was either growth or growth and

26 Mutual funds with a Wiesenberger or Investment Company Data, Inc. (ICDI) objective of “Aggressive Growth,” “Growth,” “Maximum Capital Gains,” “Small Capitalization Growth,” or “Long Term Growth,” are classified as growth funds. Mutual funds with an objective of “Growth and Income” or “Growth with Current Income” are classified as growth and income funds. Index funds are excluded.
income during only part of their life. Each of these three fund groups was given a different benchmark index, \( S \), set equal to an equally-weighted average of the returns on all funds within the group, including funds that lacked the minimum 36 months of returns.

Figure 1 shows the sample’s number of G, GI, and SM mutual funds in operation during each month of our sample period. The number of funds grew rapidly over the 1990’s and peaks approximately three years prior to the sample period’s end since no new funds were added during the last 36 months. This reflects the parameter estimation constraint that a fund report at least three years of returns.

Table 1 presents summary statistics of our mutual fund sample, broken down by fund style. Age is defined as the number of years from the fund’s inception date until the date it expired or, for surviving funds, the end-of-sample date of September 2001. Row 1 shows that G and GI funds have, on average, similar ages of approximately 8.6-8.7 years, whereas style-mixed funds, whose definition is conditioned on a style change having occurred, are much older. On average, G funds have greater turnover, front loads, and expense ratios than GI funds, whereas GI funds have relatively greater back loads. Manager tenure is the average number of years that the same individual or group manages a fund’s portfolio. Average manager tenure is similar for G and GI funds, but somewhat longer for SM funds.

As an indicator of a fund’s size relative to all other sample funds, we computed an asset size score by assigning a rank score from 0 (smallest) to 1 (largest) for each fund according to its total net asset value at the end of each year. This score is then averaged over each year of the fund’s life. On average, GI funds are only slightly larger than G funds, while the average SM size is the highest of the three groups, probably reflecting the greater average age of SM funds. G funds, however, tend to be the fastest growing funds while SM funds are the slowest. Finally, somewhat greater than 80 % of our sample’s G and GI funds were in operation (surviving) at the end of our sample period, whereas only 73 % of SM funds had not been liquidated or merged out of existence.

VI. Results

The first set of results are maximum likelihood estimates of the parameters describing the individual mutual funds’ returns, equation (16). Though not a relationship that is predicted by our model, we estimate this equation because it is in the spirit of tests performed by Brown, Harlow, and Starks (1996) and Busse (2001). Estimates are obtained for each of the 4,143 funds subject to the constraints that the mean and standard deviation of \( R \), be positive, that is, \( \mu_R \geq 0 \).
and $c_0 + c_1 G_t \geq 0$. Table II summarizes these estimates for the three fund style groups and for all funds combined. The first five columns of the table give the minimum, first quartile, median, third quartile, and maximum for each parameter’s point estimates for the sample of individual funds. Column six reports the proportion of estimates that is strictly greater than zero. Columns seven and eight give the proportions of estimates that are significantly positive and negative, respectively, at the five percent confidence level.

Row one of Table II reports the distribution of estimates for the sum of the two parameters, $c_0 + c_1$. Since the average of $G_t$ across funds is approximately 1, $c_0 + c_1$ can be interpreted as a mutual fund’s average or unconditional monthly rate of return standard deviation. The median estimates of this monthly standard deviation for the G, GI, and SM groups are 5.85, 4.51, and 4.30, respectively. However, of most interest are the estimates of the parameter $c_1$, which would be negative if a fund increases its rate of return variance as its performance declines. Importantly, the point estimate for $c_1$ is positive for over 70% of all funds, and positive and statistically significant for over 37% of the funds. For less than 5% of the funds is the $c_1$ estimate significantly negative. These results are qualitatively the same for each of the three style groups.

To gauge the economic significance of these $c_1$ estimates, consider a one standard deviation decline in $G_t$, which when computed across all funds and months during the sample period equals approximately 8.2%. The fund with the 25th percentile estimate of $c_1 = -0.0051$ would respond by increasing its annual standard deviation of return by a negligible 0.14% (-0.082x(-0.0051)x$\sqrt{12}$), while the fund with the 75th percentile estimate of $c_1 = 0.0332$ would decrease its annual standard deviation of return by 0.94% (-0.082x(0.0332)x$\sqrt{12}$). These volatility changes are 0.8% and -5.2%, respectively, of the median unconditional volatility of the funds in our sample. Hence, the response of portfolio volatility to performance is mild for most funds.

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27 The restriction that the expected return on each fund be positive is a relatively loose condition that was not binding for the great majority of funds. See Table 2. The latter constraint was imposed as two constraints, $c_0 + c_1 \text{Max}[G_t] > 0$ and $c_0 + c_1 \text{Min}[G_t] > 0$, where Max[$G_t$] and Min[$G_t$] refer to a particular fund’s maximum and minimum value of $G_t$ over its sample period.

28 As a comparison, over the January 1962 to September 2001 sample period, the CRSP value weighted portfolio had a monthly return standard deviation of 4.48%. From January 1975 to September 2001, the standard deviations of monthly returns of the S&P500, S&P500 Growth, and S&P500 Value portfolios were 4.40%, 4.89%, and 4.22%, respectively.

29 The percentage standard deviations of $G_t$ for the separate G, GI, and SM fund groups are 8.8, 3.4, and 7.7, respectively (not reported in tables).
These findings, in particular the evidence of relatively more positive than negative estimates of $c_1$, run counter to the results of Brown, Harlow, and Starks (1996) who use Morningstar monthly returns data and a different empirical methodology.\textsuperscript{30} Rather, our results support Busse (2001) who employs the same methodology as Brown, Harlow, and Starks (1996) but uses daily returns data. Since Busse (2001) argues that his use of daily data produces more efficient estimates of the volatility of funds, our paper’s consistency with his suggests that our new, arguably more efficient, estimation method compensates for lower frequency data.

As we emphasized earlier, our model shows that a poorly-performing fund does not necessarily choose to increase the variance of its portfolio returns, implying that the sign of $c_1$ is ambiguous. However, the theory is unambiguous regarding the form of a mutual fund’s change in relative performance process, $dG_t$, whose parameters are the same as the process followed by the fund’s excess return or tracking error, $dD_t = R_t - R_{St}$. Recall that the conditional standard deviation of $R_t - R_{St}$ equals $(d_0/G_t + d_1)$. The model predicts that $d_1$ is non-negative, while if $d_0$ is positive (negative), tracking error volatility increases (decreases) with underperformance.

To investigate this implication of the model, we calculated a second set of maximum likelihood parameter estimates based on equation (17), which is equivalent to the excess return process in equation (18). For each of the 4,143 funds, numerical maximization was carried out subject to the theoretical restrictions $\mu_G \geq 0$ and $d_0 + d_1 \max[G_t] > 0$.\textsuperscript{31} The results are summarized in Table III. It gives the distribution of estimates for the sum $d_0 + d_1$, which is approximately a fund’s unconditional monthly standard deviation of tracking error. The median tracking error volatilities for G, GI, and SM style funds are 3.15 %, 1.70 %, and 2.39 %, respectively. The G and SM median volatilities are slightly higher than, and GI median volatility is comparable to, median tracking error volatilities found by Chan, Chen, and Lakonishok (2002).\textsuperscript{32}

Next, consider the estimates of $d_1$, summarized in the third row of each fund style panel of Table III. 87 % of all funds have a positive estimate of $d_1$, and the estimate is positive and statistically significant for over 70 % of all funds. This implies that behavior consistent with exponential compensation, the case of $d_1 = 0$, can be ruled out for most mutual funds. Most

\textsuperscript{30} The qualitative nature of our results is not sensitive to our choice of CRSP data. An earlier version of this paper that used Morningstar data also found significantly more positive than negative estimates of $c_1$. The results using the (survivorship-biased) Morningstar data are available from the authors upon request.

\textsuperscript{31} As was done in the estimation of the funds’ return processes, the latter constraint was imposed as two constraints, $d_0 + d_1 \max[G_t] > 0$ and $d_0 + d_1 \min[G_t] > 0$.

\textsuperscript{32} They use the S&P500 index and two other benchmarks constructed from Fama-French and Sharpe style models to compute mutual funds’ tracking errors. Their median tracking error volatilities for these three benchmarks are 1.95, 1.51, and 1.41, respectively.
interesting are the statistics on $d_0$, reported in the second row of each panel in Table III. About 88% of the funds have estimated values of $d_0$ that are positive and almost 70% of the estimates are positive and statistically significant. As mentioned earlier, that a large majority of funds appear to have positive values for $d_0$ suggests that most funds do, indeed, increase tracking error volatility as performance declines. However, for typical funds, the size of this effect is not huge. For example, a one-standard deviation 8.2% decline in $G_t$ would lead the funds having the median and 75th percentile estimates of $d_0 = 0.0064$ and $0.0153$ to increase their annual tracking error volatilities by 0.18% and 0.43%, respectively. As a proportion of the median unconditional tracking error volatility of the funds, these changes represent increases of 1.88% and 4.50%, respectively.

Taken together, the evidence on $d_0$ and $d_1$ shows that the great majority of funds raise tracking error volatility as performance declines, but the magnitude of this effect is not as large as would be predicted by an exponential compensation schedule. Still, this is solid support for the type of tracking error risk-shifting found by Chevalier and Ellison (1997), but not the type of total volatility risk-shifting reported by Brown, Harlow, and Starks (1996). Let us now investigate in more detail which types of funds are more likely to engage in shifting their tracking error risk.

Columns one and two of Table IV report the averages of various fund characteristics for two different sets of mutual funds. The first set are the approximately 88% of all funds whose point estimates of $d_0$ are positive. These are the funds that tend to increase their tracking error as performance declines. The second set are the remaining mutual funds whose negative estimate of $d_0$ implies that their tracking error falls as performance declines. The third column of Table IV shows the differences in the averages of the two sets’ characteristics, and the last column indicates whether the difference is statistically significant.

The evidence in Table IV accords with our intuition regarding which funds are more likely to gamble following poor performance. Funds that raise their tracking error as performance declines are significantly younger and smaller, and have shorter manager tenures and a lower probability of survival, than other funds. Also, they tend to have relatively low front loads but high back loads, a possible indication that they are anxious to attract new money but wish to dissuade old money from leaving. However, Table IV also shows that these funds average slower asset growth, which could indicate that their gambles are often unsuccessful.

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33 Chevalier and Ellison (1997) also find that younger funds have a greater tendency to increase tracking error risk following moderately poor performance.
may also reflect the likelihood that these younger funds face greater competition for investors’ savings than funds with a longer track record.

VII. Conclusion

By modeling the incentives of a manager engaged in a mutual fund tournament, the situation assumed by recent empirical studies, we have provided greater insights into the manager’s optimal portfolio strategy. Perhaps the most important implication of our model is that a mutual fund’s tracking error volatility, not its total return volatility, is what a portfolio manager increases as the fund’s performance declines. More specifically, there is an ambiguous connection between prior performance and a fund’s total return variance, and this could explain the conflicting results of some recent empirical studies that have attempted to test for such a link.

This study also developed a new, more powerful technique for estimating the relation between a mutual fund’s prior performance and its risk-taking. The method allows a fund’s risk to respond to its relative performance at each observation date, not just once per calendar year. Also, unlike previous methods, it is able to estimate risk-shifting behavior for individual mutual funds, thereby allowing for differences between funds. We illustrated this technique by applying it to the monthly returns of more than 4,000 growth and growth and income mutual funds over the period 1962 to 2001.

As our theory predicts might happen, the empirical results show that most funds do not increase their variance of returns as their relative performance declines. If anything, there appears to be a positive relationship between performance and portfolio variance, opposite to the received wisdom, but consistent with newer evidence by Busse (2001). However, our results do provide strong support for the theory’s predicted inverse relation between relative performance changes and tracking error volatility. Moreover, we find that younger and smaller mutual funds and those with low front loads but high back loads appear more likely to increase their tracking error following a performance decline.
Appendix

This appendix shows that equation (2) in the text is the equilibrium process for a portfolio of individual alternative securities that are chosen optimally by the fund manager. The appendix then derives the equilibrium process for a mutual fund’s relative performance when the manager’s compensation takes the general form given in equation (9) in the text.

As before, assume that the benchmark index follows equation (1) in the text, but let there be \( n \) different alternative securities. The date \( t \) value of the \( i^{th} \) security, \( A_i(t) \), is assumed to follow the process

\[
dA_i / A_i = \alpha_i dt + \sigma_i dq_i \quad i = 1, \ldots, n
\]

(A.1)

where \( \sigma_dz \sigma dq = \sigma_S dt \) and \( \sigma dq = \sigma dt \) for all \( i, j = 1, \ldots, n \). It is assumed that \( \sigma_S, \sigma, \sigma_S, \) and \( \sigma \) are constants. \( \alpha \) and \( \alpha \) may be time varying but the spread between their expected rates of return, \( \alpha - \alpha_S \), is assumed to be constant.

If the fund manager allocates a portfolio proportion of \( w_i \) to alternative security \( i \) and \( 1 - \sum_{i=1}^{n} w_i \) to the benchmark index, then the fund’s portfolio value, \( V \), follows the process

\[
dV / V = \left(1 - \sum_{i=1}^{n} w_i\right)dS / S + \sum_{i=1}^{n} w_i dA_i / A_i
\]

\[
= \left(\left(1 - \sum_{i=1}^{n} w_i\right)\alpha_S + \sum_{i=1}^{n} w_i \alpha_i\right) dt + \left(\left(1 - \sum_{i=1}^{n} w_i\right)\sigma_S dz + \sum_{i=1}^{n} w_i \sigma_i dq_i\right)
\]

(A.2)

Let \( w = [w_1 \ w_2 \ldots \ w_n]' \) denote the \( nx1 \) vector of portfolio weights. A simple application of Itô’s lemma shows that the fund’s relative performance, \( G \equiv V/S \), follows the process

\[
dG / G = \sum_{i=1}^{n} w_i \left(\alpha_i - \alpha_S + \alpha_S^2 - \alpha_S\right) dt + \sum_{i=1}^{n} w_i \left(\sigma_i dq_i - \sigma_S dz\right)
\]

\[
= w'\alpha_g dt + w'\left(\sigma_g dz\right)
\]

(A.3)

where \( \alpha_g \) is an \( nx1 \) vector whose \( i^{th} \) element equals \( \alpha_{gi} \equiv \alpha_i - \alpha_S + \alpha_S^2 - \alpha_S \) and \( \sigma_g dz \) is an \( nx1 \) vector whose \( i^{th} \) element equals \( \sigma_{gi} dz_i \), where \( \sigma_{gi} \equiv \sigma_i^2 + \sigma_S^2 - 2\sigma_S \) and \( dz_i \equiv (\sigma_i dq_i - \sigma_S dz) / \sigma_{gi} \).

The Bellman equation for the manager’s optimal portfolio choice problem is then

\[
0 = \text{Max}_w J + J_G w'\alpha_g G + \frac{1}{2} J_{GG} w'\Omega w G^2
\]

(A.4)

where \( \Omega \) is an \( nxn \) matrix whose \( i^{th} \) element equals \( \Omega_{ij} \equiv \sigma_{ji} + \sigma_S^2 - \sigma_S - \sigma_{ji} \). The first order conditions with respect to the \( w_i \)’s are

\[
J_G \alpha_g + J_{GG} \sum_{j=1}^{n} w_j \Omega_{ij} = 0 \quad i = 1, \ldots, n
\]

(A.5)
The $n$ linear equations in (A.5) can be solved to obtain

$$w_i = -\frac{J_G}{J_{GG}} \sum_{j=1}^{n} v_{ij} \alpha_{ij} \quad (A.6)$$

where $v_{ij}$ is the $i,j^{th}$ element of $\Omega^{-1}$. If we define $\delta_i$ as the portfolio proportion invested in alternative asset $i$ relative to the portfolio proportion invested in all $n$ alternative securities then

$$\delta_i = \frac{w_i}{\sum_{k=1}^{n} w_k} = \frac{\sum_{j=1}^{n} v_{ij} \alpha_{ij}}{\sum_{k=1}^{n} \sum_{j=1}^{n} v_{kj} \alpha_{kj}}$$

which is independent of the fund’s relative performance, $G$. Since the optimal amounts invested in the individual alternative securities are constant shares of the total amount invested in alternative securities, the fund manager’s problem can be simplified to one of choosing between two different assets at each point in time. One asset is the benchmark portfolio following the process in equation (1) of the text and the other is the alternative security portfolio following the process given in equation (2), where $\alpha_A \equiv \sum_{i=1}^{n} \delta_i \alpha_i$, $\sigma^2_A \equiv \sum_{i=1}^{n} \sum_{j=1}^{n} \delta_i \delta_j \sigma_{ij}$, and

$$dq \equiv \sum_{i=1}^{n} (\delta_i \sigma_i / \sigma_A) dq_i.$$

We next derive the equilibrium process for a mutual fund’s relative performance. For a given asset allocation, the fund’s relative performance follows the process of equation (4) of the text. This can be re-written as

$$dG = \omega^* \alpha_A G dt + \omega^* G \sigma_G dx \quad (A.8)$$

where $dx = (\sigma_A dq - \sigma_G dz) / \sigma_G$ is a standard Brownian motion process and $\omega^*$ is the portfolio manager’s optimal proportion invested in the alternative asset. Using equation (11) to substitute for $\omega^*$ gives

$$dG = \left(\frac{ac}{b} + G\right) \left(\frac{\alpha_G^2}{(1-c\gamma)\sigma_G^2} dt + \left(\frac{ac}{b} + G\right) \frac{\alpha_G}{(1-c\gamma)\sigma_G} dx \right)$$

which can be re-written as

$$dG = \mu_G \left(d_0 + d(G)\right) dt + \left(d_0 + d(G)\right) dx \quad (A.9)$$

where $\mu_G = |\alpha_G| / \sigma_G$, $d_0 = \frac{ac}{b} \left(\frac{\alpha_G}{(1-c\gamma)\sigma_G}\right)$ and $d_1 = \frac{\alpha_G}{(1-c\gamma)\sigma_G}$. With no loss in generality $(dx$ can be redefined as $-dx)$ the standard deviation in (A.9) can be assumed positive implying

---

34 Equation (A.6) holds even when one of the alternative securities is assumed to earn a risk-free return of $r$. For example, if the $i^{th}$ security is risk-free, then $\alpha_{ij} \equiv r - \alpha_S + \sigma^2_S$ and $\Omega_{ij} = \sigma^2_S$. If there are no redundant alternative securities, then $\Omega$ is a positive-definite, non-singular matrix, and its inverse exists.

35 Equation (17) in the text is a discrete-time (monthly) analogue of (A.10).
that \( d_0 + d_1 G > 0 \) and also \( \mu_G = |\alpha_G|/\sigma_G > 0 \). Thus, one can conclude that
\[ \text{sgn} (d_0) = \text{sgn} (a) \text{ and } d_1 \geq 0. \]
Since \( a > 0 \) implies tracking error increases with underperformance, a positive value of \( d_0 \) implies a negative tracking error - performance relation.

Tests of (A.10) are equivalent to tests of the fund’s relative (excess) return process, \( d\ln G \). Using Itô’s lemma and (A.10) implies

\[
d \ln G = \left( \frac{d_0}{G} + d_1 \right) \left( \mu_G - \frac{1}{2} \left( \frac{d_0}{G} + d_1 \right) \right) dt + \left( \frac{d_0}{G} + d_1 \right) dx \quad \text{(A.11)}
\]

This is the continuous-time analogue of equation (18) of the test since
\[
d \ln G = d \ln V - d \ln S \approx \ln \left( V_{t+1}/V_t \right) - \ln \left( S_{t+1}/S_t \right) \equiv R_t - R_{St} ,
\]
where \( R_t \) and \( R_{St} \) are defined as the returns on the fund’s portfolio and the benchmark portfolio, respectively. (A.11) shows that the volatility of the fund’s relative return equals \( \left( \frac{d_0}{G} + d_1 \right) \). Hence, under proportional compensation, \( d_0 = 0 \) and the volatility of the fund’s relative return is independent of performance. If compensation were exponential, \( d_0 > 0 \) and \( d_1 = 0 \) so that the fund’s relative return volatility would be inversely proportional to performance, \( G \).

---

36 Recall that \( \left( \frac{a c}{b} + G \right) \) is always positive in equilibrium and an interior solution requires \( c \gamma < 1 \).

37 The limiting case of exponential compensation requires \( a = 1 \) and \( c \rightarrow \infty \), implying \( d_0 > 0 \) and \( d_1 = 0 \).
References


Figure 1

Number of Growth Mutual Funds

Number of Growth&Income and Style-Mixed Mutual Funds

- Growth&Income Funds
- Style-Mixed Funds
Table I

Summary Statistics of Mutual Fund Sample

<table>
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<tr>
<th>Fund Characteristic</th>
<th>Number of Funds</th>
<th>Growth Mutual Funds</th>
<th>Quartiles</th>
<th>Quarters</th>
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<td>5.00</td>
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<td>1.21</td>
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<td>0.53</td>
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<td>2.21</td>
<td>0.00</td>
<td>0.00</td>
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<td>0.00</td>
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<td>All Mutual Funds</td>
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</tr>
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</table>

Note: Statistics are based on a sample taken from the CRSP Survivor-Bias Free US Mutual Fund Database covering the period January 1962 to September 2001. Mutual funds with a Wiesenberger or Investment Company Data, Inc. (ICDI) objective of “Aggressive Growth,”
“Growth,” “Maximum Capital Gains,” “Small Capitalization Growth,” or “Long Term Growth,” are classified as growth funds. Mutual funds with an objective of “Growth and Income” or “Growth with Current Income” are classified as growth and income funds. Style-mixed funds are mutual funds with a style of growth or growth and income for only part of their lives. Index funds as well as funds with less than 36 consecutive monthly returns are excluded. This sample contains 2658 growth funds, 606 pure growth-income funds, and 879 style-mixed funds in this study. Age is defined as the number of years from the fund’s inception date until the date it expired or, for surviving funds, the end-of-sample date of September 2001. Manager tenure is the average number of years that an individual manages a fund’s portfolio. A fund’s asset size score is computed by assigning a rank score from 0(smallest) to 1(largest) for each fund according to its total net asset value at the end of each year. This score is then averaged over each year of the fund’s life. A fund’s asset growth is calculated as the average annual log change in total net assets, and the proportion surviving is the proportion of total sample funds in operation as of September 2001.
Table II

Parameter Estimates for Mutual Funds’ Rate of Return Processes

\[ R_t = \mu_R + (c_0 + c_1 G_t) \varepsilon_t \]
\[ \varepsilon_t \sim N(0,1) \]

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<thead>
<tr>
<th></th>
<th>Distribution of Parameter Estimates</th>
<th>Summary Statistics</th>
</tr>
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<tr>
<td></td>
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<tr>
<td>Growth Mutual Funds</td>
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<tr>
<td>( c_0 + c_1 )</td>
<td>0.0139</td>
<td>0.0493</td>
</tr>
<tr>
<td>( c_0 )</td>
<td>-0.2807</td>
<td>0.0246</td>
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<tr>
<td>( c_1 )</td>
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<tr>
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<td>( c_1 )</td>
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<td>( c_1 )</td>
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<tr>
<td>( \mu_R )</td>
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Note: Maximum likelihood estimates are for the monthly return processes of 2,658 Growth, 606 Growth and Income, and 879 Style-Mixed mutual funds having at least 36 monthly return observations during the January 1962 to September 2001 sample period. The theoretical restrictions \( \mu_R \geq 0 \) and \( c_0 + c_1 G_t \geq 0 \) are imposed.
### Table III

#### Parameter Estimates for Mutual Funds’ Performance Changes

\[ dG_t = \mu_G (d_o + d_1 G_t) + (d_o + d_1 G_t) \eta_t \]

or

\[ R_t - R_{st} \approx dD_t = \left( \frac{d_o}{G_t} + d_1 \right) \left[ \mu_G - \frac{1}{2} \left( \frac{d_o}{G_t} + d_1 \right) \right] + \left( \frac{d_o}{G_t} + d_1 \right) \eta_t \]

\[ \eta_t \sim N(0,1) \]

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<th>Parameter Estimates</th>
<th>Quartiles Distribution of Parameter Estimates</th>
<th>Summary Statistics</th>
</tr>
</thead>
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<tr>
<td></td>
<td>Min</td>
<td>25%</td>
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<td><strong>Growth Mutual Funds</strong></td>
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<tr>
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Note: Maximum likelihood estimates are for the monthly return processes of 2,658 Growth, 606 Growth and Income, and 879 Style-Mixed mutual funds having at least 36 monthly return observations during the January 1962 to September 2001 sample period. The theoretical restrictions \(\mu_G \geq 0\) and \(d_o + d_1 G_t \geq 0\) are imposed.
Table IV

Differences in the Characteristics of Risk-Shifting Mutual Funds

\[ dG_t = \mu_G (d_0 + d_1 G_t) + (d_0 + d_1 G_t) \eta_t, \]

or

\[ R_t - R_{s_t} \approx dD_t = \left( \frac{d_0}{G_t} + d_1 \right) \left[ \mu_G - \frac{1}{2} \left( \frac{d_0}{G_t} + d_1 \right) \right] + \left( \frac{d_0}{G_t} + d_1 \right) \eta_t \]

\[ \eta_t \sim N(0,1) \]

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<th>( d_0 &lt; 0 )</th>
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<th>( t )-value</th>
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