Syllabus

Introduction

This course attempts to provide accessible yet rigorous introduction to continuous-time methods in economics and finance. The motivation rises from two main sources. First, there are a large number of interesting problems in economics and finance for which continuous-time (or a continuum of control variables) is a natural way of thinking about the issue. Second, in many cases discrete time version of the problem doesn’t allow for analytical solutions while deriving and completely characterizing analytical solutions in continuous-time is relatively straight-forward.

The objective of the course is to students a wide expose to tools and applications that can be utilized in the dissertation and further research or that are widely applied by non-academic employers such as central banks and investment firms. Clearly, understanding these models is crucial for understanding data generating processes in areas where these employers operate.

We start with deterministic models, and then review probability theory. We then move to continuous-time stochastic processes and show how can characterize their behavior by reducing them into martingales. Finally, we move to applications in economics and finance. A byproduct of the course is a journey to the world of continuous-time asset pricing; in this journey both equilibrium and arbitrage models are covered.

Contents

The course will cover 10 different topics in the following order. I expect you to read emphasized references.

1. Deterministic Optimal Control

2. Review of Probability
   - Measure Spaces, Random Variables, Expected Values, Conditional Expectation.

3. Stochastic Processes
   - Definition, Kolmogorov Existence Theorem, Markov Processes, Brownian Motion.
4. **Stochastic Integrals**

5. **Reduction to Martingale Gains from Trade**

6. **Changes of Variables**
   - Itô’s Lemma, Local Time, Girsanov’s Theorem.
   - Øksendal (1998, Chapter 4), Karatzas and Shreve (1991, Section 3.3).

7. **Stochastic Differential Equations**
   - Øksendal (1998, Chapter 5), Karatzas and Shreve (1991, Sections 5.1 and 5.2).

8. **The Black-Scholes Model**

9. **Equilibrium Asset Pricing**
   - The Consumption CAPM, The Term Structure of Interest Rates.

10. **Stochastic Optimal Control and Applications**
    - Merton’s Problem, Exchange-Rate Target Zones, \((S, s)\) Rules, Irreversible Investment.

**Organization**

The lectures will be held on Tuesdays and Thursdays at 3:00p.m.–4:50p.m. in 126 Wohlers Hall. The office hours are on Tuesdays and Thursdays at 2:00–3:00p.m. in 469 Wohler’s Hall. In case you can’t make it to the office hours, I am happy to meet with you some other time. If you have any questions or problems at any time during the course, please contact me at seppala@uiuc.edu.

The materials for this course will be available on the course’s web page, located at:

http://www.cba.uiuc.edu/seppala/econ490/

There is going to be two midterms and a final. Final is going to cover only the material presented after the second midterm. The midterms count for 40% of the grade, the problem sets 20%, and the final counts for 40% of the grade.
Readings

The required textbooks for the course are


There are several other books and articles that are (at least partially) covered during the course. I recommend you to take a look at