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The dynamics of mergers and acquisitions in oligopolistic industries

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ABSTRACT

This paper embeds an oligopolistic industry structure in a real options framework in which synergy gains of horizontal mergers arise endogenously and vary stochastically over time. We find that (i) mergers are more likely in more concentrated industries; (ii) mergers are more likely in industries that are more exposed to industry-wide shocks; (iii) returns to merger and rival firms arising from restructuring are higher in more concentrated industries; (iv) increased industry competition delays the timing of mergers; (v) in sufficiently concentrated industries, bidder competition induces a bid premium that declines with product market competition; and (vi) mergers are more likely and yield larger returns in industries with higher dispersion in firm size.

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1. Introduction

Despite extensive research on mergers and acquisitions, some important issues in the takeover process remain unclear. Most models tend to focus on firm characteristics to explain why firms should merge or restructure and do not endogenize the timing and terms of a takeover deal or the synergy gains arising from a deal. In addition, most real options models of investment under uncertainty consider symmetric firms in a competitive industry when they study product market competition (see, e.g., Dixit and Pindyck, 1994; Miao, 2005), while real options models of mergers and acquisitions typically abstract from industry characteristics. On the other hand, most studies in the industrial organization literature (see, e.g., Salant et al., 1983; Perry and Porter, 1985; Farrell and Shapiro, 1990) build models of exogenous mergers to examine their welfare implications under different policy settings. Yet little work has been done on the implications of industrial organization for the dynamics of mergers. Thus, an important and heretofore overlooked aspect of bid premiums, merger returns, terms, and timing is their relation to industrial organization.

We embed takeovers in an industry equilibrium model with heterogeneous firms that allows us to study the effect of strategic product market considerations on: (i) the endogenous synergy gains from merging; (ii) the joint determination of the timing and terms of mergers; and (iii) the returns to both merging and rival firms.¹ The model also provides novel insights into how the interaction between industry competition and bidder competition for a scarce target influences bid premiums in takeover deals. To our knowledge, this paper presents the first *unified framework* to examine the link between

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E-mail addresses: dhackbar@illinois.edu (D. Hackbarth), miaoj@bu.edu (J. Miao).¹ Weston et al. (2003) provide a detailed review of the literature on mergers and acquisitions.

firm heterogeneity, industry structure, and the timing, terms, and returns of takeovers, and to relate the outcome of takeover contests with multiple bidders to product market competition.

There is ample empirical evidence on industry characteristics affecting mergers and acquisitions in practice. Mitchell and Mulherin (1996), Andrade et al. (2001), Andrade and Stafford (2004), and Harford (2005) find that takeover activity is driven by industry-wide shocks, and is strongly clustered by industry. Borenstein (1990) and Kim and Singal (1993) document that airline fares on routes affected by a merger increase significantly over those on routes not affected by a merger. Kim and Singal (1993) and Singal (1996) also identify a positive relation between airfares and industry concentration. Eckbo (1985) documents that acquirers and targets earn positive returns, while Singal (1996) reports in addition that returns to rival firms are positively related to changes in industry concentration. This non-exhaustive list of empirical research strongly supports the notion that mergers and acquisitions not only affect product market outcomes and vice versa but also have substantial anticompetitive effects.²

Against the backdrop of the industry-level evidence, we study the timing of a horizontal merger of two firms in an asymmetric industry equilibrium. To analyze the role of industry characteristics for the takeover process, we embed an oligopolistic industry structure similar to Perry and Porter's (1985) in a dynamic real options model of mergers and acquisitions. As a result, synergy gains arise endogenously due to oligopolistic (Cournot) competition.³ In particular, we specify a tangible asset that helps firms to produce output at a given average cost. The merged entity operates the tangible assets of the two merging partners, so it is larger and its average and marginal costs are lower. This allows us to capture *economically relevant and realistic asymmetries* caused by the merger of a subset of firms. In addition to a different *cost structure* for the merged firm, all firms face a different *industry structure* after a merger of two firms. Recognizing these economic effects, we derive closed-form solutions to industry equilibria in a real options model of takeovers in which the merger benefits, the merger timing, and the merger terms are determined endogenously based on an option exercise game featuring both cost reduction and changes in product market competition.

Our model delivers a number of predictions that are in line with the available evidence. First, we analyze shocks to industry demand, which relax the assumption of an exogenously given market size in previous analyses. For our model with a stochastic market size, we demonstrate that a merger occurs the first time the demand shock hits a trigger value from below. In other words, control transactions emerge as an endogenous product market outcome from rising demand shocks in the model and hence cyclical product markets generate procyclical takeover activity. Second, our real options model formalizes the folklore that mergers are more likely in industries that are more exposed to or more sensitive to industry-wide demand shocks. All of these implications are consistent with the evidence reported by, e.g., Mitchell and Mulherin (1996) and Maksimovic and Phillips (2001).

Third, cumulative stock returns are higher for the smaller merging firm (e.g. target) than the larger merging firm (e.g. acquirer) when the firms have identical merger costs. The intuition for this interesting finding is that the smaller firm benefits relatively more from a merger in our industry equilibrium and hence enjoys a larger relative synergy gain (i.e. return). Because mergers typically involve acquisitions of a small firm by a large firm (see, e.g., Andrade et al., 2001 or Moeller et al., 2004), this implication supports the available evidence without reliance on additional assumptions, such as asymmetric information or misvaluation.

Moreover, our model has several novel testable implications regarding the timing and terms of mergers, the returns to merging firms, and the bid premium in contested deals, which demonstrate the importance of relaxing the assumption of exogenous synergy gains, such as scale economies. First, the model's industry equilibrium reveals that cumulative stock returns are determined by an anticompetitive effect, two size effects, and a hysteresis effect. While the latter two effects are shared by other real options models of mergers, the anticompetitive effect is unique to our analysis.⁴ Due to this effect, we find that returns to merging and rival firms arising from restructuring are higher in more concentrated industries. Intuitively, returns are under certain conditions positively related to anticompetitive profit gains, which are positively related to industry concentration.

Second, and perhaps surprisingly, increased product market competition delays the timing of mergers in our real options model. Notably, this result is *contrary* to some conclusions in recent research on irreversible investment under uncertainty. Grenadier (2002), for instance, emphasizes that more industry competition increases the opportunity cost of waiting to invest and thus accelerates option exercise. In his symmetric industry model, anticompetitive profits result from exogenously reducing the number of identical firms that compete in the industry. Firms are allowed to be asymmetric in our model, however, and anticompetitive profits result from endogenous synergy gains of combining two firms. Crucially, these synergy

² Eckbo (1983), Eckbo (1985), and Eckbo and Wier (1985) find little evidence that challenged horizontal mergers are anticompetitive. McAfee and Williams (1988) question whether event studies can detect anticompetitive mergers. Andrade and Stafford (2004), Holmstrom and Kaplan (2001), Shleifer and Vishny (2003) differentiate the takeover activity in the 1960s and 1970s and the more recent merger waves of the 1980s and 1990s.

³ Unlike earlier real options models of mergers, we do not assume exogenous synergy gains from merging, such as economies of scale or efficiency-enhancing capital reallocation. There are several recent cases of mergers that match our modeling approach closely. These cases were investigated by the Department of Justice and approved if they were not likely to reduce competition substantially despite of the substantial change of the cost structure and the industry structure. For example, Whirlpool acquired Maytag in 2006 and gained market power in the appliance industry. Whirlpool also substantiated large cost savings. Another example is the \$2.6 billion deal of 2008 between two major US air carriers, Delta and Northwest. This merger reduced the combined carrier's operating costs without being viewed to damage competition substantially.

⁴ The anticompetitive effect also affects the merger terms. In Section 4.1, we show that a large acquirer (small target) demands a lower (higher) ownership share in the merged firm if the industry is less competitive.

gains are lower in more competitive industries, *ceteris paribus*, because a merger of two firms has a smaller effect on output price increases in these industries. These lower synergy gains from exercising the merger option offset the opportunity cost of waiting to merge. Thus, by relaxing some key features of previous work, our real options model implies that firms in more competitive industries may optimally exercise their option to merge later.

Third, our model sheds light on if and how multiple sources of competition (i.e. bidder competition and industry competition) in our real options framework influence *bid premiums* in control transactions. Intuitively, bidder competition puts the target in an advantageous position and allows target shareholders to extract a higher premium from the bidding firms. Hence one might be tempted to conclude that bidder competition generally raises the price of a contested deal irrespective of industry competition. However, we demonstrate that in our model the degree of industry competition plays a central role for if and how a bid premium emerges in equilibrium. Suppose a large firm bidder and a small firm bidder compete for a small firm target. When their merger costs are similar, the large firm bidder wins the takeover contest, which is consistent with the stylized fact that targets are on average smaller than acquirers. Notably, only in a sufficiently concentrated industry, bidder competition speeds up the takeover process and leads the large firm acquirer to pay a bid premium to discourage the competing small firm competitor. This bid premium decreases with industry competition. By contrast, in a more competitive industry, the small firm bidder does not matter, and the equilibrium of the option exercise game corresponds to the one without bidder competition. If the two bidding firms are identical, then bidder competition will squeeze away all surplus to the bidders and allow target shareholders to extract all merger surplus.

Fourth, our analysis provides new insights into the role of dispersion in firm size for timing and returns of mergers. We find that, all else equal, takeovers are more likely in industries in which firm size is more dispersed. In our industry framework, the endogenous synergy gains from merging increase when the size difference of the two merging partners' tangible assets rises. Because of this economic effect, the model also predicts that returns to merging and rival firms arising from restructuring are higher in industries with more dispersion in firm size.

Our work contributes to a growing body of research using the real options approach to analyze mergers and acquisitions. Lambrecht (2004), Morellec and Zhdanov (2005), Hackbarth and Morellec (2008), Margsiri et al. (2008), and Bernile et al. (in press) analyze the timing and terms of takeovers under exogenous synergy gains in that takeovers provide either economies of scale or result in a more efficient allocation of resources. To isolate the effect of industry competition on mergers as much as possible, our model abstracts from these exogenous synergy gains. Lambrecht briefly analyzes a duopoly model and shows that market power enhances symmetric firms' exogenous synergy gains. In contrast, we consider an oligopolistic Cournot–Nash equilibrium with an arbitrary number of asymmetric firms competing with each other in the product market which allows for endogenous synergy gains. More recently, Leland (2007) considers purely financial synergies in motivating acquisitions when timing is exogenous, while Morellec and Zhdanov (2008) explore interactions between financial leverage and takeover activity with endogenous timing. None of these studies considers the relation of oligopolistic industry equilibria and takeover activity. Lastly, our paper is also related to Grenadier (2002) who studies the effects of industry competition on the exercise of real options in oligopolistic industries. But he does not study takeovers.

Other complementary but less related models study industry dynamics and linkages between incentives to merge and entry or investment; see especially Gowrisankaran and Holmes (2004) or Pesendorfer (2005) and Qiu and Zhou (2007). Unlike our study, these authors typically focus on welfare implications of mergers as well as long-run industry equilibria (i.e., if mergers towards monopoly are profitable or not). Instead, we focus on the merger timing equilibrium of two firms in an industry without entry where the merger in turn affects industry equilibrium. Our paper is also related to Gorton et al. (2009), who propose a theory of mergers that combines managerial merger motives with an industry level regime shift that may lead to some value-increasing merger opportunities. They also emphasize the importance of the distribution of firm size in an industry. Unlike our analysis, they do not consider industry competition and its interaction with bidder competition in a dynamic environment. All these articles are silent on the role of industry structure for merger timing, terms, and returns, or bid premiums, which are central to our analysis.

The remainder of the article is organized as follows. Section 2 presents the model. Section 3 derives industry equilibrium without merger and analyzes the incentive to merge. Section 4 studies equilibrium mergers with a single bidder, while Section 5 studies equilibrium mergers with multiple bidders. Section 6 concludes. Proofs are relegated to Appendix A.

2. The model

We consider an industry with infinitely-lived firms whose assets generate a continuous stream of cash flows. The industry consists of N heterogeneous firms that produce a single homogeneous product, where $N \geq 2$ is an integer. Each firm i initially owns an amount $k_i > 0$ of physical capital.

To focus on the timing and returns of mergers and acquisitions and to keep the industry analysis tractable, we follow Perry and Porter (1985), Shleifer and Vishny (2003), and others by assuming that firms can grow through takeovers, but not through internal investments.⁵ In addition, we assume that productive capital does not depreciate over time. The industry's total capital stock is in fixed supply and equal to K . Therefore, the industry's capital stock at each

⁵ See e.g. Bernile et al. (in press) on takeovers and entry or Margsiri et al. (2008) on takeovers and internal growth.

time t satisfies

$$\sum_{i=1}^N k_i = K. \quad (1)$$

The cost structure is important in the model. We denote by $C(q, k_i)$ the cost function of a firm that owns an amount k_i of the capital stock and produces output q . The output q is produced with a combination of the fixed capital input, k_i , and a vector of variable inputs, z , according to a smooth concave production function, $q = F(z, k_i)$. Then the cost function $C(q, k_i)$ is obtained from the cost minimization problem. To isolate the role of product market competition on mergers, assume that the production function F has constant returns to scale. This implies that $C(q, k_i)$ is linearly homogeneous in (q, k_i) . For analytical tractability, we adopt the quadratic specification of the cost function, $C(q, k_i) = \frac{1}{2} q^2 / k_i$. This cost function may result from the Cobb–Douglas production function $q = \sqrt{k_i} z$, where z may represent labor input. Note that both the average and marginal costs decline with the capital asset k_i . Salant et al. (1983) show that if average cost is constant and independent of firm size, a merger may be unprofitable in a Cournot oligopoly with linear demand. It is profitable if and only if duopolists merge into monopoly. As Perry and Porter (1985) point out, the constant average cost assumption does not provide a sensible description of mergers.

The productive capital (or tangible asset) plays an important role in the model. It allows us to capture economically relevant and realistic asymmetries caused by the merger of a subset of heterogeneous firms. Notably, the merged entity faces a different optimization problem as a result of a change in production costs (i.e., average and marginal costs decline with the amount of productive capital employed) and important strategic considerations (i.e., increased market power). We now integrate these effects into a dynamic real options model of mergers. To do so, let the industry's inverse demand at time t is given by the linear function

$$P(t) = aY(t) - bQ(t), \quad (2)$$

where $Y(t)$ denotes the industry's demand shock at time t observed by all firms, $Q(t)$ is the industry's output at time t , and a and b are positive constants; a represents exposure of demand to industry-wide shocks, and b represents the price sensitivity of demand. For all $t > 0$, we assume that the industry's demand shock is governed by the geometric Brownian motion process

$$dY(t) = \mu Y(t) dt + \sigma Y(t) dW(t), \quad Y(0) = y_0, \quad (3)$$

where μ and σ are constants and $(W_t)_{t \geq 0}$ is a standard Brownian motion defined on $(\Omega, \mathcal{F}, \mathcal{P})$. To ensure that the present value of profits is finite, we make the following assumption:

Assumption 1. The parameters μ , σ , and r satisfy the condition $2(\mu + \sigma^2/2) < r$.

Following Shleifer and Vishny (2003), we consider a subset of two firms $i (i=1,2)$ with capital stocks k_i , which can negotiate a takeover deal at time $t > 0$ if it is in their shareholders' best interest.⁶ To this end, we assume that each firm i 's strategy space is restricted to the optimal exercise strategy of its merger option. For each firm i , this strategy is given by a threshold y_i^* , such that the merger is executed the first time when $Y(t)$ exceeds y_i^* . The second-stage negotiation problem then reduces to identifying the merger terms, ξ_i , which will induce both firms to exercise their merger option at the globally efficient merger threshold, y^* . In reality, control transactions are costly. We therefore assume that each merging firm i incurs a fixed lump-sum cost $X_i > 0$ for $i=1,2$. This cost captures fees to investment banks and lawyers as well as the cost of restructuring.

Finally, we assume that all firms in the industry are Cournot–Nash players and that management acts in the best interests of shareholders. We also assume that shareholders are risk-neutral and discount cash flows by $r > 0$. Thus, all decisions are rational and value-maximizing choices.

3. Industry equilibrium without merger

To incorporate an oligopolistic industry structure into a real options framework of mergers, we need to work backwards. That is, we first characterize industry equilibrium when asymmetric firms play Cournot–Nash strategies without mergers. This allows us to derive the firms' value functions for a given industry structure. Next, we examine the incentive to merge, which is based on the merging firms' net gains in these value functions resulting from a change in industry structure at an arbitrary time.

3.1. Equilibrium characterization and firm strategies

Let $q_i(t)$ denote the quantity selected by firm i at time t . Then firm i 's instantaneous profit is given by

$$\pi_i(t) = [aY(t) - bQ(t)]q_i(t) - \frac{1}{2} q_i(t)^2 / k_i, \quad (4)$$

⁶ So far, few models endogenize multiple mergers of heterogeneous firms (see, e.g., Qiu and Zhou, 2007). In a setup with symmetric firms, Kamien and Zang (1990) find that firms remain independent when merger decisions are endogenous. However, the multiplicity of equilibria in their model limits the robustness of their results. Similarly, our real options model can only produce reliable results for an exogenously assigned sequence of multiple mergers.

where

$$Q(t) = \sum_{i=1}^N q_i(t) \tag{5}$$

is the industry output at time t . Given the instantaneous profits, we can compute firm value, or the present value of profits

$$V_i(y) = \mathbb{E}^Y \left[\int_0^\infty e^{-rt} \pi_i(t) dt \right], \tag{6}$$

where $\mathbb{E}^Y[\cdot]$ denotes the conditional expectation operator, given that the current industry shock takes the value $Y(0)=y$.

We define strategies and industry equilibrium as follows. The strategy $\{(q_1^*(t), \dots, q_N^*(t)) : t \geq 0\}$ constitutes an industry (Markov perfect Nash) equilibrium if, given information available at date t , $q_j^*(t)$ is optimal for firm $j = 1, \dots, N$, when it takes other firms' strategies $q_i^*(t)$ for all $i \neq j$ as given. Because firms play a game in continuous time, there could be multiple Markov perfect Nash equilibria, as is well known in game theory. Instead of finding all equilibria, we will focus on the equilibrium in which firms adopt static Cournot strategies. As is well known, the static Cournot strategies that firms play at each date constitute a Markov perfect Nash equilibrium. The first proposition characterizes this industry equilibrium.⁷

Proposition 1. *The strategy*

$$q_i^*(t) = \frac{\theta_i}{1+B} \frac{aY(t)}{b} \tag{7}$$

constitutes a Cournot–Nash industry equilibrium at time t for firms $i = 1, \dots, N$, where

$$\theta_i = \frac{b}{b+k_i^{-1}} \quad \text{and} \quad B = \sum_{i=1}^N \theta_i. \tag{8}$$

In this equilibrium, the industry output at time t is given by

$$Q^*(t) = \frac{B}{1+B} \frac{aY(t)}{b}, \tag{9}$$

and the industry price at time t is given by

$$P^*(t) = \frac{aY(t)}{1+B}. \tag{10}$$

Note that this proposition also characterizes the equilibrium after a merger, once we change the number of firms and the capital stock of the merged firm. A merger brings the capital of two firms under a single authority and thus reduces production cost.

We use the subscript M to denote the merged entity. The merged firm owns capital $k_M = k_1 + k_2$. By Proposition 1, the values of θ_i for the non-merging firms $i \geq 3$ are unaffected by the merger. We can verify that the value of θ for the merged firm satisfies

$$\max\{\theta_1, \theta_2\} < \theta_M = \frac{\theta_1 + \theta_2 - 2\theta_1\theta_2}{1 - \theta_1\theta_2} < \theta_1 + \theta_2. \tag{11}$$

Thus, the value of B in Proposition 1, which determines total industry output, changes after the merger to

$$B_M = B + \theta_M - \theta_1 - \theta_2 < B. \tag{12}$$

By Eqs. (9), (10), and (12), we conclude that the merger causes total output to fall and industry price to rise. In addition, we can use Eqs. (12) and (7) to show that

$$\max\{q_1^*(t), q_2^*(t)\} < q_M(t) < q_1^*(t) + q_2^*(t), \tag{13}$$

where $q_M(t)$ is the output produced by the merged firm at time t . This result implies that the merged firm produces more than either of the two merging firms, but less than the total output level of the two. The analysis highlights the tension of a merger: after a merger, the industry price rises, but the merged firm restricts production. Thus, a merger may not always generate a profit gain.

In order to analyze mergers tractably, we follow Perry and Porter (1985) and consider an oligopoly structure with small and large firms. Specifically, we assume that the industry initially consists of n identical large firms and m identical small firms. Each large firm owns an amount k of capital and incurs merger costs X_l if it engages in a merger. Each small firm owns an amount $k/2$ of capital and incurs merger costs X_s if it engages in a merger. In this case, Eq. (1) becomes $nk + mk/2 = K$, and hence we can use $m = 2Kk^{-1} - 2n$ to replace m in the analysis below.

We assume that either two small firms can merge (symmetric merger) or a small firm and a large firm can merge (asymmetric merger). In the former case, a merger preserves the two-type industry structure, but the number of small firms

⁷ Proofs for all propositions are given in Appendix A.

and large firms changes. In the latter case, firm heterogeneity increases. That is, all non-merging small or large firms remain identical, but the merged entity owns more physical capital than a large firm, destroying the two-type industry structure.

We do not consider mergers between two large firms or further mergers over time. This assumption makes our analysis tractable and permits us to focus on the key questions of how product market competition interacts with bidder competition and how product market competition influences the timing and terms of mergers as well as merger returns. One justification of our assumption may be related to antitrust law. White (1987, p. 16) writes that: “[the Horizontal Merger] Guidelines use the Herfindahl-Hirschman Index (HHI) as their primary market concentration guide, with concentration levels of 1,000 and 1,800 as their two key levels. Any merger in a market with a post-merger HHI below 1,000 is unlikely to be challenged; a merger in a market with a post-merger HHI above 1,800 is likely to be challenged (if the merger partners have market shares that cause the HHI to increase by more than 100), unless other mitigating circumstances exist, like easy entry. Mergers in markets with post-concentration HHI levels between 1,000 and 1,800 require further analysis before a decision is made whether to challenge”. In our model, a merger of two large firms would raise industry concentration to a higher level than a merger of two small firms or a merger between a small firm and a large firm. The industry concentration level following a merger of two large firms is more likely to cross the regulatory threshold, and thus such a merger is more likely to be challenged by antitrust authorities.⁸

As an example, the two largest office superstore chains in the United States, Office Depot and Staples, announced their agreement to merge on September 4, 1996. Seven months later, the Federal Trade Commission voted 4 to 1 to oppose the merger on the grounds that it was likely to harm competition and lead to higher prices in “the market for the sale of consumable office supplies sold through office superstores”. (see Dalkir and Warren-Boulton, 2004.)

3.2. Firm valuation and incentive to merge

We first consider a symmetric merger of two small firms. Define

$$\Delta(n) \equiv (b+k^{-1})(b+2(Kb+1)k^{-1})-b^2n > 0. \quad (14)$$

Note that the argument n in this equation indicates that there are n large firms in the industry. This notation is useful for our merger analysis below because the number of large firms changes after a merger option is exercised. It follows from Eq. (1) that $Kk^{-1} > n$, which implies that both $\Delta(n)$ and $\Delta(n+1)$ are positive. We use Proposition 1 and Eq. (6) to compute firm values.

Proposition 2. Consider a symmetric merger between two small firms in the small-large oligopoly industry. Suppose Assumption 1 holds. The equilibrium value of the type $f = s, l$ firm is given by

$$V_f(y; n) = \frac{\Pi_f(n)y^2}{r-2(\mu+\sigma^2/2)}, \quad (15)$$

where

$$\Pi_s(n) = \frac{a^2(b+k^{-1})^3}{\Delta(n)^2}, \quad (16)$$

$$\Pi_l(n) = \frac{a^2(b+2k^{-1})^2(b+k^{-1}/2)}{\Delta(n)^2}. \quad (17)$$

After a merger between two small firms, there are $n+1$ identical large firms and $m-2$ identical small firms in the industry. Thus, the value of the large firm after a merger is given by $V_l(y; n+1)$. It follows from Proposition 2 that the benefit from merging is given by

$$V_l(y; n+1) - 2V_s(y; n) = \frac{[\Pi_l(n+1) - 2\Pi_s(n)]y^2}{r-2(\mu+\sigma^2/2)}. \quad (18)$$

The term $\Pi_l(n+1) - 2\Pi_s(n)$ represents the profitability of an anticompetitive merger. For there to be takeover incentives, this term must be positive. After a merger, the number of small firms in the industry is reduced, and hence the industry's market structure is changed. As the output of two small firms prior to a merger exceeds the output of the merged firm, an incentive to merge requires that the increase in industry price be enough to offset the reduction in output of the merged entity. The conditions for takeover incentives may be summarized as follows.

Proposition 3. Consider a symmetric merger between two small firms. Let $\Delta(n)$ be given in Eq. (14) and define the critical value

$$\Delta^* \equiv \frac{b^2}{1-A}, \quad (19)$$

⁸ For an industry with two large firms and a price sensitivity of $b=0.5$, our model's pre-merger HHI equals 1,378 when $k=0.2$, $K=1$, and $a=100$. This concentration measure rises to 1,578 (1,734) following a merger of two small firms (a merger of a small and a large firm) compared to a post-merger HHI is 2,022 after a merger of two large firms. See Appendix A for derivations.

where A is given by

$$A \equiv (b + 2k^{-1}) \sqrt{\frac{b + k^{-1}/2}{2(b + k^{-1})^3}} \tag{20}$$

(i) If

$$\max_n \Delta(n) = \Delta(0) < \Delta^*, \tag{21}$$

then there will always be an incentive to merge. (ii) If

$$\min_n \Delta(n) = \Delta(K/k-1) > \Delta^*, \tag{22}$$

then there will never be an incentive to merge. (iii) If

$$\Delta(0) > \Delta^* > \Delta(K/k-1), \tag{23}$$

then when n is high enough so that $\Delta(n) < \Delta^*$, there will be an incentive to merge.

In Proposition 3, (21) and (23) provide two conditions for takeover incentives in our Cournot–Nash framework.⁹ That is, when the increase in price outweighs the reduction in output so that the net effect leads to an increase in instantaneous profits, the two small firms have an incentive to form a large organization. These two conditions depend on the industry demand function through the price sensitivity b , and the size of a large firm k , and on the industry structure through the number n of large firms prior to the restructuring. Moreover, the proposition shows that there could be no incentives to merge if the condition in Eq. (22) holds. It is straightforward to show that there is always an incentive to merge when the industry consists of a small-firm duopoly.¹⁰ A similar result is obtained by Perry and Porter (1985) and Salant et al. (1983) in a static industry model.

Alternatively, after a merger between a large firm and a small firm, the industry consists of $n-1$ identical large firms, $m-1$ identical small firms, and a huge merged firm. The merged entity owns capital $k_M = k + k/2 = 3k/2$. We use Proposition 1 and Eq. (6) to derive firm values.

Proposition 4. Consider an asymmetric merger between a large firm and a small firm in the small-large oligopoly industry. Suppose Assumption 1 holds. After this merger, the equilibrium firm value is given by

$$V_f^a(y; n-1) = \frac{\Pi_f^a(n-1)y^2}{r-2(\mu+\sigma^2/2)} \quad \text{for } f = s, l, M, \tag{24}$$

where

$$\Pi_s^a(n-1) = \frac{a^2(b+k^{-1})^3}{\left[\Delta(n) - b^2 \left(1 + \frac{2}{2+3bk}\right)\right]^2}, \tag{25}$$

$$\Pi_l^a(n-1) = \frac{a^2(b+2k^{-1})^2(b+k^{-1}/2)}{\left[\Delta(n) - b^2 \left(1 + \frac{2}{2+3bk}\right)\right]^2}, \tag{26}$$

$$\Pi_M^a(n-1) = \frac{3a^2(b+2k^{-1})^2(b+k^{-1})^2(3b+k^{-1})/(3b+2k^{-1})^2}{\left[\Delta(n) - b^2 \left(1 + \frac{2}{2+3bk}\right)\right]^2}. \tag{27}$$

The benefit from an asymmetric merger is given by

$$V_M^a(y; n-1) - V_s(y; n) - V_l(y; n) = \frac{[\Pi_M^a(n-1) - \Pi_s(n) - \Pi_l(n)]y^2}{r-2(\mu+\sigma^2/2)}. \tag{28}$$

For there to be an incentive to merge, the expression in Eq. (28) must be positive. As in Proposition 3, we have the result.

Proposition 5. Consider an asymmetric merger between a small firm and a large firm in the small-large oligopoly industry. Let $\Delta(n)$ be given in Eq. (14) and let the critical value Δ^* take the value

$$\Delta^a \equiv \frac{b^2}{1-D} \left(1 + \frac{2}{2+3bk}\right), \tag{29}$$

⁹ Similar conditions have been derived by Perry and Porter (1985) in their static model.

¹⁰ One can verify that condition (21) is satisfied for $k=K$, $n=0$, and $m=2$, which is a limiting case of our model.

where D is given by

$$D \equiv \frac{(b+k^{-1})(b+2k^{-1})}{3b+2k^{-1}} \sqrt{\frac{3(3b+k^{-1})}{(b+k^{-1})^3+(b+2k^{-1})^2(b+k^{-1}/2)}} \tag{30}$$

Then parts (i)–(iii) in Proposition 3 apply here.

To facilitate the analysis for a single bidder, we make the following assumption.

Assumption 2. Suppose $\Pi_M^a(n-1) - \Pi_S(n) - \Pi_I(n) > 0$ and $\Pi_I(n+1) - 2\Pi_S(n) < 0$ so that there is an incentive to merge between a large firm and a small firm, but no incentive to merge between two small firms.

A sufficient condition for this assumption is that case (i) or (iii) holds in Proposition 5, but both cases are violated in Proposition 3. We invoke an alternative assumption for the analysis of multiple bidders.

Assumption 3. Suppose $\Pi_I(n+1) - 2\Pi_S(n) > 0$ and $\Pi_M^a(n-1) - \Pi_S(n) - \Pi_I(n) > 0$ so that there is an incentive to merge between a large and a small firm and between two small firms.

A sufficient condition for this assumption is that case (i) or (iii) holds in both Propositions 3 and 5. The following lemma is useful for our later merger analysis:

Lemma 1. Under Assumptions 2 or 3, the profit differentials $\Pi_M^a(n-1) - \Pi_S(n) - \Pi_I(n)$ increase with the parameters a and n , and the profit ratios $[\Pi_M^a(n-1) - \Pi_S(n) - \Pi_I(n)] / \Pi_S(n)$ and $[\Pi_M^a(n-1) - \Pi_S(n) - \Pi_I(n)] / \Pi_I(n)$ increase with the parameter n .

To interpret this lemma, recall that the parameter n represents the number of large firms in the industry prior to a merger. This parameter proxies for industry concentration. A higher value of n represents a higher level of industry concentration. The parameter a represents the exposure of industry demand to the industry-wide shock. A higher value of a implies that the increase in industry price is higher in response to an increase in the exogenous demand shock. Lemma 1 then demonstrates that under certain conditions the anticompetitive gains from a merger measured in terms of either profit differential or profit ratio are larger in more concentrated industries. In terms of profit differential, these gains are also greater in industries that are more exposed to industry-wide shocks.

Example 1. The conditions in Assumption 2 or 3 are not easy to check analytically. We thus use a numerical example to illustrate them and Propositions 3 and 5. The baseline parameter values are: $b=0.5$, $k=0.2$, and $K=1$. By Eq. (1), these values imply that the numbers of large and small firms must satisfy $2n+m=10$.

Fig. 1 shows the profit differentials of symmetric and asymmetric mergers for a wide range of parameter values of b and n . Both profit differentials increase monotonically with the number of large firms n , but non-monotonically with the price sensitivity parameter b . Intuitively, this non-monotonicity results from two opposing effects of a decline in b ; that is, it raises price, but reduces output. Thus, change in the price sensitivity parameter has an ambiguous effect on the profit differentials, because it depends on whether the price effect or the quantity effect dominates.

From the lower left end of the surfaces in Fig. 1, we find that asymmetric mergers can be profitable, while symmetric mergers are not, so Assumption 2 is satisfied. This happens, for example, when $b=0.5$, and n takes values from 1 to 5. When $b=0.4$, and n takes values from 1 to 5, however, both symmetric and asymmetric mergers are profitable, so Assumption 3 is satisfied.

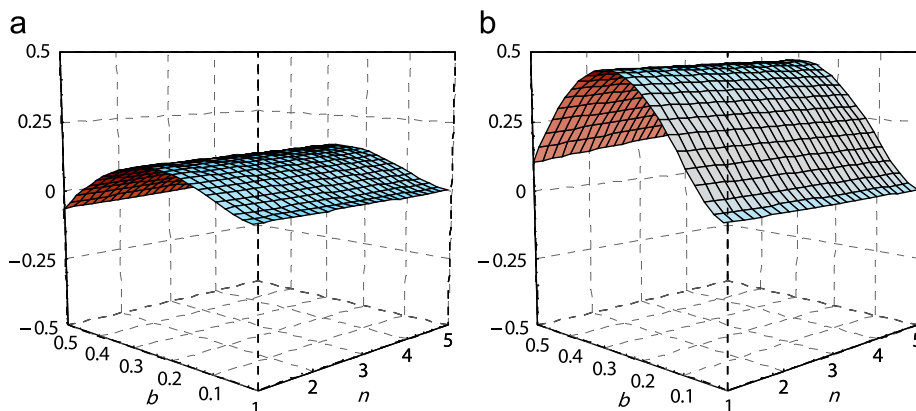


Fig. 1. Profit differentials in symmetric and asymmetric mergers. This figure depicts the profit differentials $\Pi_I(n+1) - 2\Pi_S(n)$ and $\Pi_M^a(n-1) - \Pi_S(n) - \Pi_I(n)$ as a function of the price sensitivity parameter b and the number of large firms n when $a=100$, $K=1$, and $k=0.2$.

4. Merger equilibrium with a single bidder

After having characterized the industry equilibrium, we now derive the merger equilibrium for the case of a single bidder in this section and for the case of multiple bidders in the next section. In particular, we solve for the optimal merger timing and terms in both cases and compute merger returns and bid premiums.

4.1. Merger policies

In this subsection, we analyze the timing and terms of a merger between a small firm target and a large firm bidder. We suppose Assumption 2 holds, so that two identical small firms do not have an incentive to merge, but a large firm and a small firm do have an incentive to merge.¹¹

We assume the acquirer submits a bid in the form of an ownership share of the merged firm's equity. Given this bid, both acquirer and target shareholders select their value-maximizing merger timing. In equilibrium, the merger timing chosen by the acquirer and the target are the same. The merger offers participants in the deal an option to exchange one asset for another. That is, they can exchange their shares in the initial firm for a fraction of the shares of the merged firm. Thus, the merger opportunity is analogous to an exchange option (Margrabe, 1978). The equilibrium timing and terms of the merger are the outcome of an option exercise game in which each participant determines an exercise strategy for its exchange option. We first solve for the equilibrium, and then show that the equilibrium timing is globally optimal.¹²

To solve for the equilibrium, we first consider the exercise strategy of the large firm bidder. Let ξ_l denote the ownership share of the large firm in the merged entity. Then $1 - \xi_l$ is the ownership share of the small firm target. The merger surplus accruing to the large firm is given by the positive part of the (net) payoff from the merger: $[\xi_l V_M^a(y; n-1) - V_l(y; n) - X_l]^+$, where $V_M^a(y; n-1)$ and $V_l(y; n)$ are given in Propositions 2 and 4. When it considers a merger, the large firm trades off the stochastic benefit from merging against the fixed cost X_l of merging. Since firms have the option but not the obligation to merge, the surplus from merging has a call option feature.

Let y_l^* denote the merger threshold selected by the large firm. The value of this firm's option to merge, denoted $OM_l(y, y_l^*, \xi_l; n)$ for $y \leq y_l^*$, is given by

$$OM_l(y, y_l^*, \xi_l; n) = \mathbb{E}^Y \{ e^{-r\tau_{y_l^*}} [\xi_l V_M^a(Y(\tau_{y_l^*}); n-1) - V_l(Y(\tau_{y_l^*}); n) - X_l] \}, \tag{31}$$

where $\tau_{y_l^*}$ denotes the first passage time of the process (Y_t) starting from the value y to the merger threshold y_l^* selected by the large firm. By a standard argument (e.g., Karatzas and Shreve, 1999), we can show that

$$OM_l(y, y_l^*, \xi_l; n) = [\xi_l V_M^a(y_l^*; n-1) - V_l(y_l^*; n) - X_l] \left(\frac{y}{y_l^*} \right)^\beta, \tag{32}$$

where β denotes the positive root of the characteristic equation

$$\frac{1}{2} \sigma^2 \beta(\beta-1) + \mu\beta - r = 0. \tag{33}$$

Note that it is straightforward to prove that $\beta > 2$ under Assumption 1. Eq. (32) admits an intuitive interpretation. The value of the option to merge is equal to its share of the merger benefits net of the merger costs, $[\xi_l V_M^a(y_l^*; n-1) - V_l(y_l^*; n) - X_l]$, generated at the time of the merger multiplied by a discount factor $(y/y_l^*)^\beta$. This discount factor can be interpreted as the Arrow-Debreu price of a primary claim that delivers \$1 at the time and in the state the merger occurs.

The optimal threshold y_l^* selected by the large firm maximizes the value of the merger option in Eq. (32). Thus, it satisfies the first-order condition

$$\frac{\partial OM_l(y, y_l^*, \xi_l; n)}{\partial y_l^*} = 0. \tag{34}$$

Solving this equation yields

$$y_l^*(\xi_l; n) = \sqrt{\frac{\beta X_l}{\beta - 2} \frac{r - 2(\mu + \sigma^2/2)}{\xi_l \Pi_M^a(n-1) - \Pi_l(n)}}. \tag{35}$$

¹¹ This is reasonable because deals often involve a small firm being acquired by a large firm. In Andrade et al. (2001) sample of 4,256 deals over the 1973–1998 period, the median target size is 11.7% of the size of the acquirer. Moeller et al. (2004) measure relative size as transaction value divided by acquirer's equity value, and report averages of 19.2% (50.2%) for 5,503 small (6,520 large) acquirers between 1980 and 2001.

¹² Our analysis follows similar steps as in Lambrecht's (2004) case of friendly mergers where both managements first decide on the timing of the merger and subsequently negotiate how to divide the merger gains.

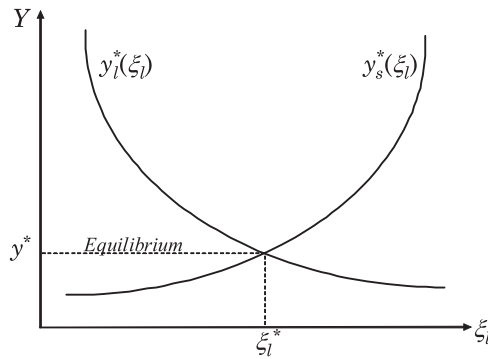


Fig. 2. Reaction functions with a single bidder. The figure plots the reaction functions of the bidder and the target as a function of the bidder's ownership share. The decreasing (increasing) line represents the acquirer's (target's) strategy. The crossing point of the reaction functions represents the option exercise equilibrium of Proposition 6.

It follows that, as a function of ξ_l , y_l^* declines with ξ_l . This function gives the merger threshold for a given value of the ownership share ξ_l . Fig. 2 illustrates this function.

We next turn to the exercise strategy of the small firm target, which can be solved in a similar fashion. The value of the small firm target's option to merge is given by

$$OM_s(y, y_s^*, \xi_l; n) = [(1 - \xi_l)V_M^a(y_s^*; n - 1) - V_s(y_s^*; n) - X_s] \left(\frac{y}{y_s^*}\right)^\beta. \tag{36}$$

The optimal exercise strategy y_s^* selected by the small firm target satisfies the first-order condition

$$\frac{\partial OM_s(y, y_s^*, \xi_l; n)}{\partial y_s^*} = 0. \tag{37}$$

Solving this equation yields

$$y_s^*(\xi_l; n) = \sqrt{\frac{\beta X_s}{\beta - 2} \frac{r - 2(\mu + \sigma^2/2)}{(1 - \xi_l)\Pi_M^a(n - 1) - \Pi_s(n)}}. \tag{38}$$

This equation implies that, as a function of ξ_l , the merger threshold y_s^* selected by the small firm target rises with ξ_l . Fig. 2 charts y_s^* as a function of ξ_l .

In equilibrium, the negotiated ownership share must be such that the bidder and the target agree on merger timing, or $y_l^* = y_s^*$. Using this condition, we can solve for the equilibrium timing and terms of the merger. The crossing point of the reaction functions in Fig. 2 characterizes the option exercise equilibrium.

Proposition 6. Consider an asymmetric merger between a large firm and a small firm. Suppose that Assumptions 1 and 2 hold.

(i) The value-maximizing merger policy is to merge when the industry shock (Y_t) reaches the threshold value

$$y^*(n) = \sqrt{\frac{\beta(X_s + X_l)}{\beta - 2} \frac{r - 2(\mu + \sigma^2/2)}{\Pi_M^a(n - 1) - \Pi_l(n) - \Pi_s(n)}}. \tag{39}$$

(ii) The merger threshold y^* declines with a and n . (iii) The share of the merged firm accruing to the large firm is given by

$$\xi_l^*(n) = \frac{X_l}{X_s + X_l} + \frac{\Pi_l(n)X_s - \Pi_s(n)X_l}{(X_s + X_l)\Pi_M^a(n - 1)}. \tag{40}$$

(iv) The ownership share ξ_l^* declines with n .

Parts (i) and (ii) of Proposition 6 characterize the merger timing. As the values of the option to merge for both firms increase with the realization y of the industry shock, a merger occurs in a rising product market. Thus, consistent with empirical evidence documented by Maksimovic and Phillips (2001), cyclical product markets generate procyclical mergers. This result is also consistent with Mitchell and Mulherin's (1996) empirical finding that industry shocks contribute to the merger and restructuring activities during the 1980s.

As in most real options model, the merger threshold y^* given in (39) determines the merger timing and merger likelihood. A higher value of the merger threshold implies a larger value of the expected time of the merger and a lower probability of merger within a given time horizon.¹³ To interpret Eq. (39), we use Propositions 2 and 4 to rewrite it as

$$V_M^a(y^*, n-1) - V_I(y^*; n) - V_S(y^*; n) = \frac{\beta(X_S + X_I)}{\beta - 2} > X_S + X_I. \tag{41}$$

Eq. (41) implies that, at the time of the merger, the benefit from the merger exceeds the sum of the merger costs $X_S + X_I$. This reflects the option value of waiting. Because mergers and acquisitions are analogous to an irreversible investment under uncertainty, the standard comparative statics results from the real options literature (e.g., Dixit and Pindyck, 1994) apply to merger timing. For example, an increase in the industry’s demand uncertainty delays the timing of mergers, and an increase in the drift of the industry’s demand shock speeds up the timing of mergers.

Our novel comparative statics results are related to industry characteristics. First, part (ii) of Proposition 6 implies that the optimal merger threshold declines with the parameter a . Since the parameter a represents the exposure of the industry demand to the exogenous shock, one should expect to observe more mergers and acquisitions in industries where demand is more exposed to or more sensitive to exogenous shocks. The intuition behind this result is that an increase in the parameter a raises industry demand for a given positive shock. Thus, it increases anticompetitive gains $\Pi_M^a(n-1) - \Pi_I(n) - \Pi_S(n)$ as shown in Lemma 1, thereby raising the benefits from merging. This result is consistent with empirical evidence in Mitchell and Mulherin (1996), who report that the industries experiencing the most merger and restructuring activity in the 1980s were the industries exposed most to industry shocks.

We next turn to the effect of industry concentration on merger timing. Part (ii) of Proposition 6 implies that one should expect to see more mergers and acquisitions in more concentrated (i.e., less competitive) industries. The economic intuition behind this result is as follows. A higher level of pre-merger industry concentration is associated with higher anticompetitive profits by Lemma 1, which raises the incentive to merge, *ceteris paribus*, and hence the potential payoff from exercising the merger option. Importantly, higher anticompetitive profits at the time of a merger lead to a higher opportunity cost of waiting to merge, and so firms in less competitive industries will optimally exercise their option to merge earlier.

This implication for the optimal timing of mergers in our Cournot–Nash framework is in sharp contrast to most of the earlier findings in the literature on irreversible investment under uncertainty. Notably, Grenadier (2002) demonstrates that firms in more competitive industries will optimally exercise their investment options earlier in a symmetric industry equilibrium model. Intuitively, more industry competition increases the opportunity cost of waiting to invest and thus accelerates investment option exercise. We attribute the difference in our results to differences in the economic modeling of industry competition and structure. In our asymmetric industry equilibrium model, anticompetitive profits result from merging two firms to form a new firm, which alters product market competition endogenously. In Grenadier’s (2002) model, anticompetitive profits result from exogenously reducing the number of identical firms that compete for an investment opportunity in the industry. Moreover, Grenadier (2002) studies an incremental investment problem, while we analyze a single discrete option exercise decision. We will consider scarcity of targets that can lead to competition among multiple bidders in Section 5. Bidder competition will hurt the acquirer, as is also shown by Morellec and Zhdanov (2005), and hence may attenuate the delayed option exercise due to the countervailing effect of product market competition that is central to our model.

Parts (iii) and (iv) of Proposition 6 characterize the ownership share. Part (iii) shows that the large firm bidder demands a greater ownership share than the small firm target if the two firms incur identical merger costs $X_I = X_S$. Intuitively, because the large firm has a higher pre-merger firm value, it demands a greater ownership share in our industry equilibrium. When the merger gains are endogenized the pre-merger industry concentration level also influences the ownership share. Part (iv) of Proposition 6 shows that a large merging firm demands a smaller ownership share in more concentrated industries, *ceteris paribus*. The intuition is that the pre-merger profit differential between the large and the small merging partners relative to the value of the merged firm declines with industry concentration. Thus, the large firm does not need to demand a greater share in more concentrated industries.

In our model, total merger returns come from the merger surplus. The merger surplus is generated by two incentives to merge: (i) gaining market power, and (ii) reducing production costs. As Perry and Porter (1985) and Salant et al. (1983) note, the gain in market power alone may not be sufficient to motivate a merger. To illustrate this point, we decompose the total merger surplus (net of merger costs) evaluated at the equilibrium merger threshold y^* into two components

$$V_M^a(y^*; n-1) - V_I(y^*; n) - V_S(y^*; n) - X_I - X_S = \underbrace{[\tilde{V}_M(y^*; n) - V_I(y^*; n) - V_S(y^*; n) - X_I - X_S]}_{S_1} + \underbrace{[V_M^a(y^*; n) - \tilde{V}_M(y^*; n)]}_{S_2}, \tag{42}$$

where $\tilde{V}_M(y^*; n)$ represents the value of the merged firm when there is no cost saving. We compute this value using Proposition 1 by assuming that the merged firm uses the large firm’s capital stock k to produce output and does not

¹³ The probability that a merger will take place in a time interval $[0, T]$ is given by

$$\Pr\left(\sup_{0 \leq \tau \leq T} Y(\tau) \geq y^*\right) = \mathcal{N}\left[\frac{\ln(y_0/y^*) + (\mu - \sigma^2/2)T}{\sigma\sqrt{T}}\right] + \left(\frac{y_0}{y^*}\right)^{-2(\mu - \sigma^2)/\sigma^2} \mathcal{N}\left[\frac{\ln(y_0/y^*) - (\mu - \sigma^2/2)T}{\sigma\sqrt{T}}\right],$$

where \mathcal{N} is the standard normal distribution function. This probability declines with the merger threshold y^* .

Table 1

Decomposition of total merger surplus. The expressions $V_M^a(y^*; n-1)$, $V_l(y^*; n)$, and $V_s(y^*; n)$ represent values of the merged firm, the pre-merger large firm, and the pre-merger small firm evaluated at the merger threshold. The expression $\tilde{V}_M(y^*; n)$ represents the value of the merged firm evaluated at the merger threshold if it uses the large firm's capital stock to produce output. We decompose the merger surplus (net of merger costs) $S \equiv V_M^a(y^*; n-1) - V_s(y^*; n) - V_l(y^*; n) - X_s - X_l$ into two components. One component, defined as $S_1 = \tilde{V}_M(y^*; n) - V_s(y^*; n) - V_l(y^*; n) - X_s - X_l$, is attributed to market power only. The other component, defined as $S_2 = V_M^a(y^*; n-1) - \tilde{V}_M(y^*; n)$, is attributed to cost savings. Parameter values are $K=1$, $k=0.2$, $b=0.5$, $r=0.08$, $\mu=0.01$, $\sigma=0.20$, and $X_l = X_s = 2$.

Large firms	$V_M^a(y^*; n-1)$	$\tilde{V}_M(y^*; n)$	$V_s(y^*; n)$	$V_l(y^*; n)$	S_1	S_2	S
$n=2$	183,482	130,376	61,395	122,053	-53,076	53,106	30
$n=3$	155,969	110,845	52,187	103,747	-45,094	45,124	30
$n=4$	135,527	96,333	45,345	90,147	-39,163	39,193	30

combine the two merging firms' capital assets to reduce production costs. Thus, the expressions S_1 and S_2 on the right-hand side of Eq. (42) represent the merger surplus attributed to market power and cost savings, respectively.

Table 1 illustrates these two components numerically for various values of n . One can see the effects of market power on the benefits (and hence returns) from mergers within our framework. First, note that the total merger surplus at the merger threshold is independent of n because it is equal to $2(X_s + X_l)/\beta$ by Eq. (41). Second, the large firm and the small firm do not have incentives to merge if the motivation is market power alone. That is, the associated merger surplus (S_1) is negative, even though the value of the merged firm is higher than the value of each merging firm. Third, it is the additional cost savings incentive represented by S_2 that makes the merger profitable. Finally, the table also reveals that the market power incentive is still present, although it does not motivate the merger by itself. This is because market power effect has a stronger effect as the industry becomes more concentrated (i.e., n is higher). Consequently, the component S_2 of the merger surplus generated by cost savings declines as the industry becomes more concentrated. Thus, the merger benefits in our numerical example are attributable to both cost savings and market power gains.

Example 2. Fig. 3 illustrates Proposition 6. The input parameter values are $a=100$, $b=0.5$, $K=1$, $k=0.2$, $n=2$, $r=8\%$, $X_l=2$, $X_s=2$, $\mu=1\%$, and $\sigma=20\%$. Recall that Eq. (1) implies that the number of small firms satisfies $m=2Kk^{-1}-2n=6$. Building on the insights from Example 1, the industry's parameter values a, b, K, k , and n , are selected such that a merger between two small firms is not profitable, i.e., $\Pi_l(n+1)-2\Pi_s(n) < 0$, but a merger between a small and a large firm is profitable, i.e., $\Pi_M^a(n-1)-\Pi_s(n)-\Pi_l(n) > 0$. Thus, Assumption 2 holds. To study the role of product market competition for mergers in isolation from frictions shared by other real options models of mergers, we use identical restructuring costs for the small firm target and the large firm acquirer.¹⁴ That is, we set $X_f=2$ for $f=s,l$. The risk-free rate is taken from the yield curve on Treasury bonds. Similarly, the growth rate of industry shocks has been selected to generate a reasonable probability for a merger to arise in this industry. Finally, the value of the diffusion parameter has been chosen to match the time-series volatility of an average S&P 500 firm's asset return.

As discussed earlier, the non-monotonicity of the profit differential with respect to the price sensitivity results from a tradeoff between a price effect and a quantity effect. Fig. 3a displays the impact of this economic phenomenon on the behavior of the merger threshold, i.e., $\partial y^*/\partial b$. Moreover, because the merger gains increase with industry concentration when Assumption 2 is satisfied, the merger threshold decreases with the number of large firms in the industry, n , as shown in Fig. 3b. In particular, when n rises from 2 to 5, the merger threshold y^* declines from 2.30 to 1.84. If we set the initial industry shock to $y_0=1$, these threshold values imply that the likelihood of a merger over a five-year horizon rises from 5.0% to 14.7%.

Another interesting feature of the optimal merger threshold is that it is a decreasing function of k , as shown in Fig. 3c. Notice that the variance of firm size, $k_i \in \{k/2, k\}$, in an industry with N firms is given by

$$\text{Var}(k_i) = \frac{(K-kn)k^3n}{2(2K-kn)[2K-k(n+1)]}, \tag{43}$$

which also increases with k . Thus, Fig. 3c and Eq. (43) reveal that, all else equal, takeovers are more likely in industries in which firm size is more dispersed. Although we are unable to prove this result theoretically, we have verified that it holds true for a wide range of parameter values. Intuitively, the endogenous synergy gains from merging increase when the size difference of the two merging partners' tangible assets rises. Finally, and as supported by Fig. 3, other standard comparative statics results apply within our model, so we do not discuss them.

4.2. Cumulative returns

We now turn to cumulative returns resulting from an asymmetric merger. The equity value of a large merging firm before a merger, denoted $E_l(Y(t); n)$, is equal to the value of its assets in place plus the value of the merger option

$$E_l(Y(t); n) = V_l(Y(t); n) + OM_l(Y(t), y^*, \zeta_l^*; n), \tag{44}$$

¹⁴ Intuitively, larger restructuring costs delay the timing of mergers. We will relax this assumption in Section 5.

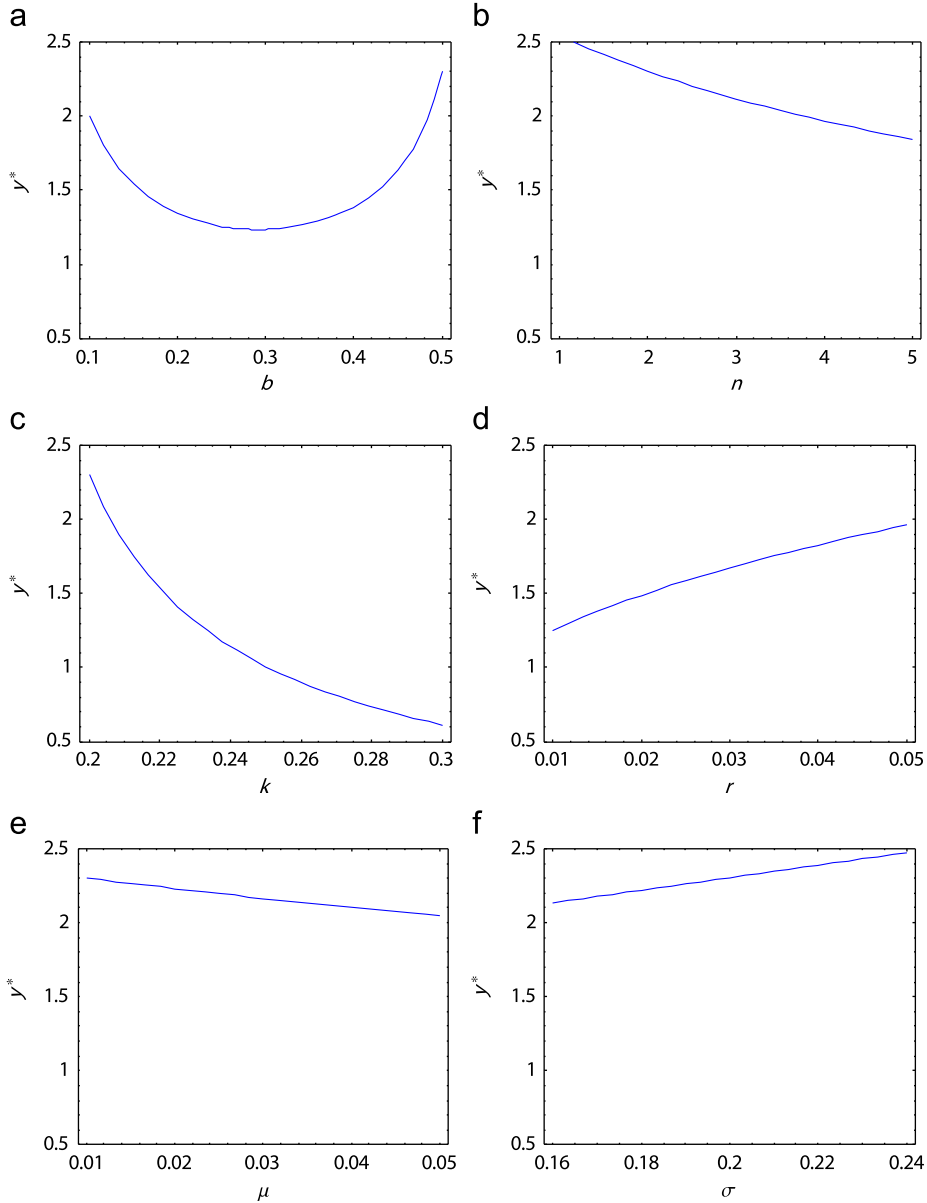


Fig. 3. Merger timing. This figure plots the merger threshold y^* as a function of the price sensitivity of demand, b , the number of large firms, n , the size of the large firm's tangible asset, k , the risk-free rate, r , the growth rate of industry shocks, μ , and the volatility of industry shocks, σ . Parameter values are $a=100$, $b=0.5$, $K=1$, $k=0.2$, $n=2$, $r=8\%$, $X_l=2$, $X_s=2$, $\mu=1\%$, and $\sigma=20\%$.

where $OM_l(Y(t), y^*, \zeta_l^*; n)$ is given in Eq. (32), and y^* and ζ_l^* are given in Proposition 6. Similarly, the equity value of a small merging firm before the merger, denoted $E_s(Y(t); n)$, is given by

$$E_s(Y(t); n) = V_s(Y(t); n) + OM_s(Y(t), y^*, \zeta_l^*; n), \tag{45}$$

where $OM_s(\cdot; n)$ is defined in Eq. (36).

We may express the cumulative stock returns as a fraction of the stand-alone equity value $V_f(Y(t); n)$ of the small firm $f=s$ and the large firm $f=l$. That is, the cumulative returns to the small and large merging firms at time $t \leq \tau_{y^*}$ are given by

$$R_{f,M}(Y(t), n) = \frac{E_f(Y(t); n) - V_f(Y(t); n)}{V_f(Y(t); n)} = \frac{OM_f(Y(t), y^*, \zeta_l^*; n)}{V_f(Y(t); n)}, \tag{46}$$

for $f=s, l$. The cumulative return to the merging firm f at the time of the merger announcement is equal to the expression in Eq. (46) evaluated at $t = \tau_{y^*}$ or $Y(t) = y^*$.

Similarly, we can compute the cumulative stock return to a rival firm at the time of the merger announcement. To do so, we first compute the equity value of a rival firm prior to the announcement of a merger at date $t \leq \tau_{y^*}$. It is equal to the value of assets in place before the merger plus an option value from the merger

$$V_f(Y(t); n) + \mathbb{E}^Y[e^{-r(\tau_{y^*}-t)}(V_f^a(Y(\tau_{y^*}); n-1) - V_f(Y(\tau_{y^*}); n)) | Y(t)], \tag{47}$$

for $f = s, l$. This option value results from the fact that the value of the rival firm becomes $V_f^a(Y(\tau_{y^*}); n-1)$ after the asymmetric merger since there are $n-1$ large firms, $m-1$ small firms, and a huge merged firm in the industry. We then define the cumulative return to a small or large rival firm before the merger as

$$R_{f,R}(Y(t); n) = \frac{\mathbb{E}^Y[e^{-r(\tau_{y^*}-t)}(V_f^a(Y(\tau_{y^*}); n-1) - V_f(Y(\tau_{y^*}); n)) | Y(t)]}{V_f(Y(t); n)}, \tag{48}$$

for $f = s, l$. We focus on the cumulative return at the time of the merger announcement, when $t = \tau_{y^*}$ or $Y(\tau_{y^*}) = y^*$.

Proposition 7. Consider an asymmetric merger between a large firm and a small firm. Suppose that Assumptions 1 and 2 hold. (i) The cumulative stock returns to the small and large merging firms at the time of restructuring are given by

$$R_{f,M}(y^*; n) = \frac{\Pi_M^a(n-1) - \Pi_l(n) - \Pi_s(n)}{\Pi_f(n)} \frac{X_f}{X_s + X_l} \frac{2}{\beta}, \tag{49}$$

for $f = s, l$. (ii) The cumulative stock returns to a small or a large rival firm at the time of restructuring are given by

$$R_{f,R}(y^*; n) = \left[1 - \frac{b^2}{\Delta(n)} \left(1 + \frac{2}{2 + 3bk} \right) \right]^{-2} - 1, \tag{50}$$

for $f = s, l$. (iii) All the above returns are positive and increase with n .

Proposition 7 highlights several interesting aspects of cumulative stock returns within our Cournot–Nash industry equilibrium framework. First, Eq. (49) reveals that the cumulative returns to the two merging firms have three types of determinants, including an anticompetitive effect, two size effects, and a hysteresis effect. The hysteresis effect is represented by $2/\beta$, which is a function of the risk-free rate, r , the growth rate of industry shocks, μ , and the volatility of industry shocks, σ . It implies that a higher volatility of industry shocks, σ , leads to higher cumulative returns to both the acquirer and the target. With more uncertainty surrounding the industry, the merger option is exercised when it is deeper in the money, resulting in higher cumulative stock returns. The two size effects are captured by $1/\Pi_f(n)$ and $X_f/(X_s + X_l)$, which respectively reflect the results that the smaller merging firm or the firm with a higher merger cost earns a higher return than the larger merging firm or the firm with a lower merger cost.

Second, and unlike existing real options models of mergers, our model with constant returns to scale does not produce an exogenous synergy effect for merger returns. Instead, we have an anticompetitive and cost reduction effect represented by the term $\Pi_M^a(n-1) - \Pi_l(n) - \Pi_s(n)$, which characterizes the merger benefits as a function of industry equilibria before and after the control transaction. This effect reflects the fact that after a merger, there are fewer small and large firms in the industry, and a huge merged entity emerges. Hence, both the market structure and the competitive landscape of the industry change at the time of the merger. In addition, the huge merged firm combines the tangible assets of the two merging firms, thereby reducing production costs. The fraction $[\Pi_M^a(n-1) - \Pi_l(n) - \Pi_s(n)]/\Pi_f(n)$ in Eq. (49) commingles the first size effect with the anticompetitive effect, indicating the percentage gain in profits resulting from a control transaction. Notably, this percentage gain differs for a small and a large merging firm as it depends on the firm's size and hence its relative contribution to the merger benefits.

Third, merging firms earn higher returns in more concentrated industries. The intuition is that there is a stronger anticompetitive effect on the equilibrium price after a merger in those industries. As a consequence, merging firms derive higher returns from entering into a takeover.

Fourth, returns to merger and rival firms are invariant to the sensitivity to industry-wide demand shocks. Recall that only the anticompetitive effect of returns depends on a . By inspection of Eqs. (25)–(27), notice that all firms' profit multipliers are a quadratic function of a . Thus, even though exposure to industry-wide shocks crucially affects the optimal timing of mergers, it does not affect the returns from restructuring decisions in our industry equilibrium framework.

Fifth, Eq. (50) shows that the cumulative return to a small or a large rival firm at the time of a merger announcement is positive too. Like the cumulative returns to merging firms, the cumulative returns to rival firms increase with industry concentration. The intuition is that the industry's equilibrium price rises after the merger, and rival firms also benefit from this price increase. This benefit increases with industry concentration. Like merger returns, rival returns are therefore higher in more concentrated industries. Notice, however, that the magnitude of rival returns does not depend on firm size, as rival firms do not change their capital stock at the time of a control transaction but only adjust their equilibrium output choices.

Example 3. We illustrate Proposition 7 by a numerical example with baseline parameter values used in Examples 1 and 2. Recall that we let $X_l = X_s$ to focus on the portion of merger returns that is due not simply to different restructuring costs, which is also a feature of other real options models of mergers and hence not unique to our framework. Fig. 4 graphs the

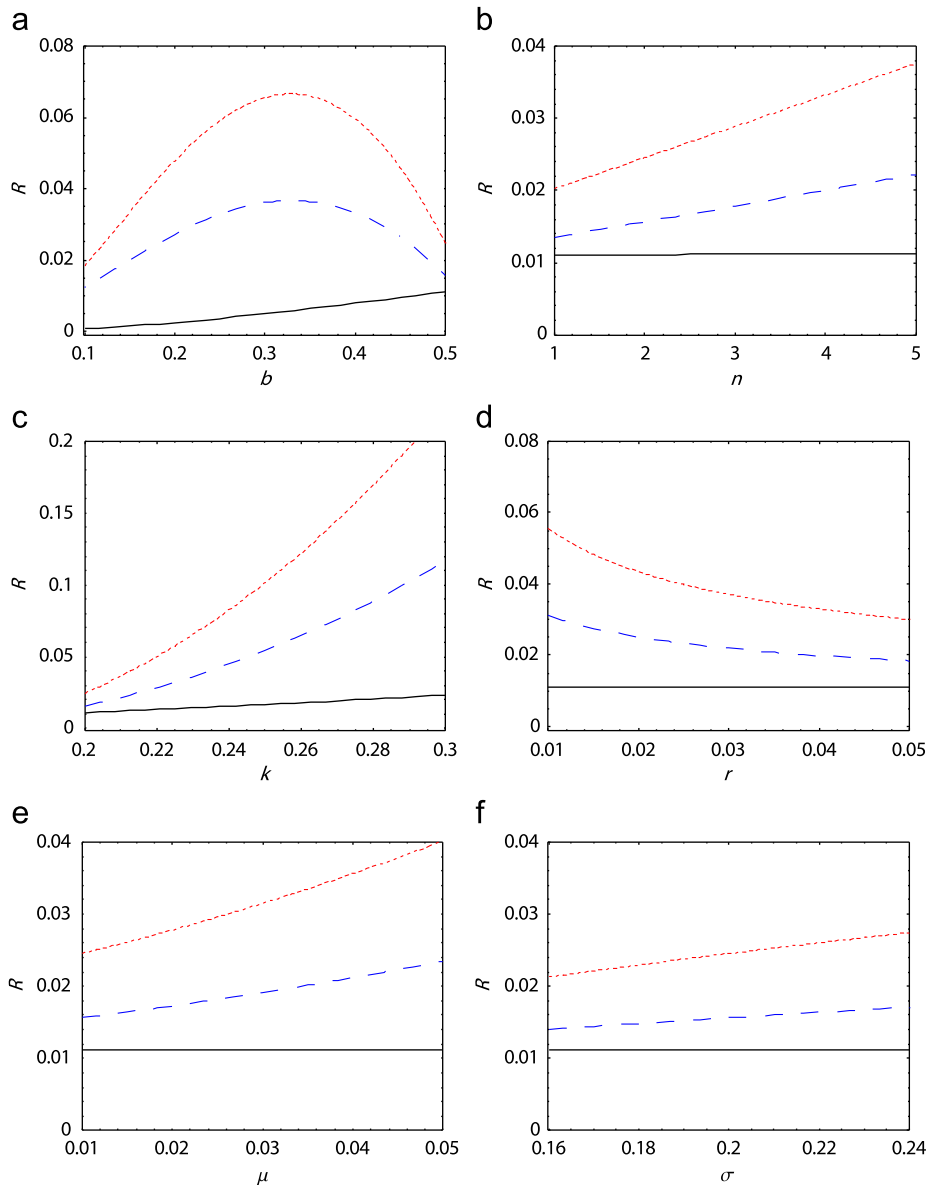


Fig. 4. Cumulative stock returns. This figure depicts the cumulative stock returns of a small merging firm (dotted line), a large merging firm (dashed line), and a rival firm (solid line) as a function of the price sensitivity of demand, b ; the number of large firms, n ; the size of the large firm's tangible asset, k ; the risk-free rate, r ; the growth rate of industry shocks, μ ; and the volatility of industry shocks, σ . Parameter values are $a=100$, $b=0.5$, $K=1$, $k=0.2$, $n=2$, $r=8\%$, $X_1=2$, $X_2=2$, $\mu=1\%$, and $\sigma=20\%$.

cumulative returns of a small merging firm (dashed line) and a large merging firm (dotted line) given in Eq. (49) as well as the return to a rival firm (solid line) given in Eq. (50) as functions of various industry characteristics, such as the price sensitivity of demand, b , and the number of large firms, n . The figure reveals several interesting aspects of the determinants of merger and rival returns, such as firm size, profitability, and the firm's (relative) contribution to the creation of the merger benefit.

First, and somewhat surprisingly, the return of a small merging firm (i.e. target) exceeds the return of a large merging firm (i.e. acquirer). Mathematically, the anticompetitive effect, $\Pi_M^q(n-1) - \Pi_I(n) - \Pi_S(n)$, is scaled by the firm's pre-merger profitability or status quo, $\Pi_f(n)$, in Eq. (49), so that merger returns reflect a percentage gain in profits that differs for small and large merging firms. Crucially, this percentage gain depends on the pre-merger firm size and hence the firm's relative contribution to the post-merger synergy gains. The intuition for this interesting finding is therefore that the smaller firm benefits relatively more from a merger in our industry equilibrium framework and hence enjoys a larger relative synergy gain (i.e. merger return). As mentioned earlier, mergers typically involve acquisitions of a small firm by a large firm. This implication of the model therefore supports the available evidence without reliance on additional, behavioral assumptions, such as misvaluation (see, e.g., Shleifer and Vishny, 2003).

Second, merger returns vary non-monotonically with the price sensitivity of demand, b , which is consistent with our findings in the earlier examples. Yet, rival returns increase with the price sensitivity of demand, b . The reason for this interesting difference between merger and rival returns is that industry rivals benefit from the (positive) price effect but are not hurt by the (negative) quantity effect of the Cournot–Nash equilibrium. In fact, because rivals do not change their firm size, k_i , they optimally produce a slightly higher quantity subsequent to a successful takeover deal.

Third, the graph in Fig. 4b studies the effect of the number of large firms, n , on merger and rival returns. The figure reveals that a higher level of concentration in the industry (i.e. a larger value of n) leads to higher returns for all firms in the industry.

Fourth, merger and rival returns are increasing in k , as shown in Fig. 4c. Recall that the variance of firm size in Eq. (43) also increases with k . Thus, returns to merging and rival firms arising from restructuring are higher in industries with more dispersion in firm size. Finally, and as confirmed by Fig. 4, other comparative statics results are largely due to the hysteresis effect that is shared by other real options models of mergers, so we do not discuss them.

5. Merger equilibrium with multiple bidders

So far, we have focused on an asymmetric merger with a single bidder. We now consider two bidders competing for a small firm target and suppose Assumption 3 holds throughout this section, so that both bidders have incentives to merge with the target. We consider two cases. In the first case, the two bidders are different in that one is a large firm and the other is a small firm. In the second case, the two bidders are identical in that either both are large firms or both are small firms. Intuitively, bidder competition puts the target into an advantageous position and allows target shareholders to extract a higher premium from the bidding firms. Notably, we do not rely on, e.g., asymmetric information for bid premium to obtain. Merger benefits are derived endogenously from an industry equilibrium (under perfect information) rather than exogenously specified. Thus, product market competition may interact with bidder competition.

5.1. Two different bidders

Suppose that one bidder is a large firm and the other is a small firm. The bidding game is as follows. Once the contest is initiated, the two bidding firms submit bids to the target in the form of a fraction of the merged firm's equity to be owned by target shareholders upon the takeover. The bidder who offers, in present value terms, the highest monetary payoff to the target shareholders wins the contest. Given the winner's ownership share, the winning bidder and the target select their merger timing independently. In equilibrium, they must agree on the merger timing.¹⁵

In our model, a large firm has a production cost advantage over a small firm. Thus, the value of the merged firm is higher when the small firm target merges with a large firm bidder than when it merges with a small firm bidder. Formally, we can use Eqs. (17) and (27) to show that

$$V_M^a(y; n-1) > V_I(y; n+1). \quad (51)$$

Eq. (51) implies that the large firm will win the takeover contest as long as the takeover is profitable to it because it can always slightly bid more than the small firm bidder and deliver more value to target shareholders. This strategy is costly to the large firm when its merger costs are sufficiently higher than the small firm bidder's merger costs. In this case, the takeover may not be profitable to the large firm bidder, and it would rather drop out.

From the preceding discussions, we conclude that either the large firm or the small firm may win the takeover contest, depending on the relative effects of production and merger costs. In the analysis below, we will focus only on the empirically more plausible case where the large firm wins the contest. The analysis of the case where the small firm wins the contest is contained in a previous version of this paper and is available upon request.

Before presenting our analysis, we define the breakeven share ζ_s^{BE} for the small firm bidder as

$$\zeta_s^{BE} V_I(y; n+1) - V_s(y; n) - X_s = 0. \quad (52)$$

If the small firm bidder places a bid higher than $1 - \zeta_s^{BE}$, then it will realize a negative value by entering the deal. In this case, the small firm bidder would be better off losing the takeover contest. Similarly, we define the breakeven share ζ_l^{BE} for the large firm bidder as

$$\zeta_l^{BE} V_M^a(y; n-1) - V_I(y; n) - X_l = 0. \quad (53)$$

We now discuss the bidders' strategies when the large firm bidder wins the contest. In this case, the losing small firm bidder may still influence equilibrium, depending on whether it is strong or weak. As a result, there are two possibilities to consider:

1. The losing small firm bidder is weak in the sense that merging with the large firm bidder and accepting the ownership share $1 - \zeta_l^{BE}$ is more profitable to the target than merging with the small firm bidder and accepting the ownership

¹⁵ Our analysis follows similar steps as in Morellec and Zhdanov (2005), in which the target cares about the monetary value of a bid. We also use ownership share as a device to solve the model. Any monetary value of a bid can be transformed into a corresponding ownership share by dividing the monetary value of the bid by the combined firm value.

share $1 - \zeta_s^{BE}$:

$$(1 - \zeta_l^*)V_M^a(y; n-1) > (1 - \zeta_s^{BE})V_l(y; n+1), \tag{54}$$

where ζ_l^* is the equilibrium share without bidder competition given in Proposition 6. For the small firm bidder to win, it must offer an ownership share of at least $1 - \zeta_s^{BE}$ to target shareholders, which implies a negative net payoff to the small firm bidder. Thus, it would rather drop out of the takeover contest and the equilibrium is the same as in Proposition 6.

2. The small firm bidder is strong in the sense that merging with the large firm bidder and accepting the ownership share $1 - \zeta_l^*$ is less profitable to the target than merging with the small firm bidder and accepting the ownership share $1 - \zeta_s^{BE}$:

$$(1 - \zeta_l^*)V_M^a(y; n-1) < (1 - \zeta_s^{BE})V_l(y; n+1). \tag{55}$$

Under this condition, the small firm bidder has an incentive to bid an amount slightly less than $1 - \zeta_s^{BE}$ in order to win the contest. Anticipating this bidder's competition, the large firm bidder will place a bid higher than $1 - \zeta_l^*$ until equality holds in Eq. (55). We define ζ_l^{\max} as the ownership share satisfying this equality:

$$(1 - \zeta_l^{\max})V_M^a(y; n-1) = (1 - \zeta_s^{BE})V_l(y; n+1). \tag{56}$$

The value ζ_l^{\max} is the maximum share that the large firm bidder can extract from the merged firm such that it still wins the takeover contest. If the large firm demands a share higher than ζ_l^{\max} , or places a bid lower than $1 - \zeta_l^{\max}$, the small firm will outbid the large firm and win the contest. Thus, the best response of the large firm is to place a bid $1 - \zeta_l^{\max}$. Note that the ownership share ζ_l^{\max} of the large firm bidder must be greater than the breakeven share ζ_l^{BE} for the large firm to participate. If this condition is violated, the large firm would rather drop out.

Given the above bidding strategies, the winning bidder will select a merger time to maximize the option value to merge. The target shareholders also select a merger time to maximize their option value to merge, given the ownership share proposed by the winning bidder. The equilibrium ownership share is determined so that the merger timing selected by the winning bidder and the target shareholders agrees. Fig. 5 illustrates equilibria when the large firm bidder wins the takeover contest. The increasing function represents the target's strategy. The solid (dashed) decreasing line represents the large firm bidder's strategy when the losing bidder is weak (strong). In particular, the dashed line represents y as a function of ζ_l^{\max} derived from Eq. (56) after we substitute ζ_s^{BE} from Eq. (52). The crossing points of the reaction functions in Fig. 5 represent option exercise equilibria, which are formally presented in Proposition 8.

Proposition 8. Consider a small firm bidder and a large firm bidder competing for a small firm target. Suppose Assumptions 1 and 3 hold. Let

$$A(n) \equiv \frac{\Pi_M^a(n-1) - \Pi_l(n) - \Pi_s(n)}{\Pi_l(n+1) - 2\Pi_s(n)} \tag{57}$$

denote the relative merger benefits. (i) If

$$\frac{X_l}{X_s} < \frac{2(\beta-1)A(n)}{\beta} - 1, \tag{58}$$

then the equilibrium is the same as that with a single large firm bidder described in Proposition 6. The cumulative stock returns to the winning large firm bidder and the small firm target are given in Propositions 7. (ii) If

$$\frac{2(\beta-1)A(n)}{\beta} - 1 < \frac{X_l}{X_s} < \frac{2(\beta-1)A(n)}{\beta-2} - \frac{\beta}{\beta-2}, \tag{59}$$

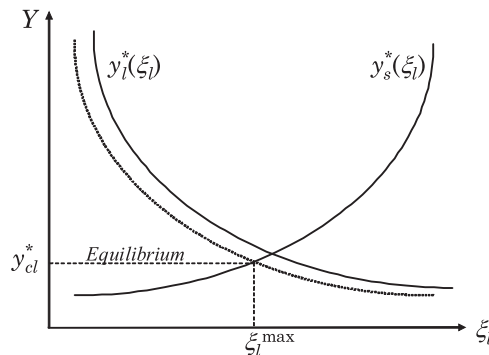


Fig. 5. Reaction functions with multiple bidders. The figure plots the reaction functions of the acquirer and the target, as a function of the bidder's ownership share. The increasing line represents the target's strategy. The solid (dashed) decreasing line represents the large firm bidder's strategy when the losing bidder is weak (strong). The crossing points of the reaction functions represent the option exercise equilibria of Proposition 8.

then the large firm bidder wins the contest and the small firm bidder is strong. The takeover takes place the first time the industry shock (Y_t) reaches the threshold value

$$y_{cl}^*(n) = \sqrt{\frac{2(\beta-1)X_s(r-2(\mu+\sigma^2/2))}{(\beta-2)\Pi_l(n+1)-2\Pi_s(n)}} \tag{60}$$

The share of the merged firm accruing to the winning large firm bidder is given by

$$\xi_l^{\max}(n) = 1 - \frac{\beta\Pi_l(n+1)-2\Pi_s(n)}{2(\beta-1)\Pi_M^a(n-1)} \tag{61}$$

The cumulative returns to the winning large firm bidder and the small firm target are given by

$$R_{l,M}(y_{cl}^*; n) = \frac{\Pi_M^a(n-1)-\Pi_l(n)-\Pi_s(n)}{\Pi_l(n)} - \frac{(\beta-2)X_l + \beta X_s}{2(\beta-1)X_s} \frac{\Pi_l(n+1)-2\Pi_s(n)}{\Pi_l(n)} \tag{62}$$

and

$$R_{s,M}(y_{cl}^*; n) = \frac{\Pi_l(n+1)-2\Pi_s(n)}{(\beta-1)\Pi_s(n)} \tag{63}$$

Proposition 8 provides a characterization of two possible merger equilibria with different bidders. While, in principle, the small firm bidder could win the takeover contest if it had a large enough merger cost advantage, we only consider equilibria in which the large firm bidder always wins the takeover contest. We believe that this case is more relevant in corporate practice because merger costs include fees to investment banks and lawyers, and should not vary substantially for firms of different sizes (see Gorton et al., 2009).¹⁶

The two conditions (58) and (59) measure the relative cost advantage and the relative merger benefit. They are the key to determining the equilibrium outcomes in the presence of bidder competition. Part (i) of **Proposition 8** shows that if the small firm bidder's merger cost advantage is not large enough in that condition (58) holds, then its competition with the large firm bidder does not matter for the large firm bidder's behavior. As a result, the equilibrium is the same as that with a single large firm bidder described in **Proposition 6**.

Part (ii) of **Proposition 8** establishes that if the small firm bidder's merger cost advantage is sufficiently large, then its competition with the large firm bidder allows the target to extract additional returns from the winning bidder. That is, if condition (59) holds, the merger return to the winning large firm bidder given in Eq. (62) is lower than without the bidder competition given in Eq. (49), while the merger return to the small target firm in Eq. (63) is greater than without bidder competition given in Eq. (49).

To interpret **Proposition 8** further, we define the bid premium resulting from bidder competition as the percentage increase in the target equity value over the case without bidder competition. These equity values are evaluated at the equilibrium time of the merger. Clearly, there is a positive bid premium only if the losing bidder is strong as in part (ii). In this case, we can formally define the bid premium resulting from bidder competition as

$$\frac{[(1-\xi_l^{\max})V_M^a(y_{cl}^*; n-1) - (1-\xi_l^*)V_M^a(y_{cl}^*; n-1)]}{(1-\xi_l^*)V_M^a(y_{cl}^*; n-1)} = \frac{\xi_l^* - \xi_l^{\max}}{1-\xi_l^*} \tag{64}$$

We can use condition (59) and the expressions for ξ_l^* and ξ_l^{\max} to demonstrate that $\xi_l^* > \xi_l^{\max}$. Consequently, the bid premium is positive. Note that both condition (59) and the bid premium depend on industry characteristics, such as the industry concentration level, n , and the price sensitivity of demand parameter, b . As a result, there is an interaction between product market competition and bidder competition in our model.

Example 4. A numerical example illustrates the results reported in **Proposition 8**, focusing on the interesting interactions between bidder competition and industry competition. Baseline parameter values are: $a=100$, $k=0.2$, $K=1$, $b=0.4$, $n=4$, $r=8\%$, $\mu=1\%$, and $\sigma=20\%$. To ensure that the large firm bidder wins the takeover contest and the small firm bidder can be potentially strong as in part (ii) of **Proposition 8**, we set $X_l=20$ and $X_s=1$ so that condition (59) holds.

Fig. 6 displays the bid premium in Eq. (64) as a function of the number of large firms, n , and the price sensitivity of demand, b . The figure highlights several key elements of the different equilibria resulting from jointly analyzing bidder competition and product market competition. First, the left panel shows that the bid premium is first an increasing and then a declining function of the price sensitivity parameter b . When b is low enough, the bid premium is close to zero. When b is high enough, condition (58) holds. As a result, the small firm bidder does not influence the equilibrium bidding process, and there is no bid premium.¹⁷

¹⁶ We have also noted earlier that large firm bidders and small firm targets are consistent with the empirical evidence. For example, Moeller et al. (2004) report that contested deals are more likely for large acquirers than for small acquirers, which implies that large firm bidders tend to win more often in takeover contests.

¹⁷ While the bid premium resulting from the interaction of bidder and industry competition in our setting can be higher or lower for different parameter values, it is only one component of the combined run-up and markup observed in corporate practice.

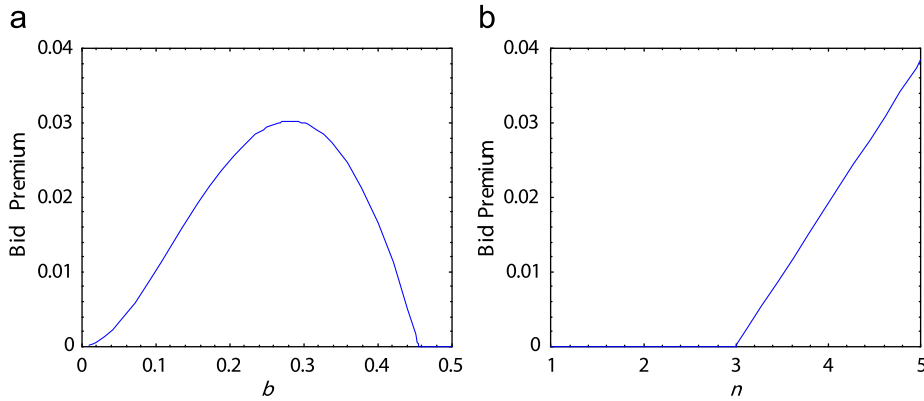


Fig. 6. Bidder competition and industry competition. The figure presents the bid premium $(\xi_l^* - \bar{\xi}_l^{\max}) / (1 - \xi_l^*)$ as a function of the number of large firms n and the price sensitivity of demand b . Parameter values are $a=100$, $b=0.4$, $K=1$, $k=0.2$, $n=4$, $r=8\%$, $X_l=20$, $X_s=1$, $\mu=1\%$, and $\sigma=20\%$.

Second, the right panel of the figure illustrates our earlier discussion that only when the industry is sufficiently concentrated, bidder competition matters. In the numerical example, bidder competition induces an additional bid premium if there are at least three large firms in the industry. Moreover, in the region where the bid premium is non-trivial it increases with industry concentration. Absent of a formal model, one might have been tempted to conclude that bidder competition generally raises the price of a contested deal irrespective of industry competition. However, this example underlines that the degree of industry competition may play a central role for if and how a bid premium emerges in equilibrium.

5.2. Two identical bidders

We now consider the alternative case in which there are two identical firms competing for a small target. We suppose that both bidders are large firms. Hence the production cost advantage from the previous subsection with different bidders is not present in this subsection. The analysis of the case with two identical small firms is similar and hence is omitted here.

Without loss of generality, we label the two large firms as firm 1 and firm 2. From the preceding subsection, we conclude that either firm 1 or firm 2 may win the takeover contest. We conjecture that the equilibrium bid must be equal to ξ_l^{BE} in Eq. (53), so that each bidder wins with equal probability and earns zero surplus. The intuition is as follows. Suppose firm 1 wins the contest and obtains ownership share ξ_1 . To earn nonnegative surplus, it must be the case that $\xi_1 \geq \xi_l^{BE}$. But if $\xi_1 > \xi_l^{BE}$, then firm 2 can offer ownership share $1 - \xi_l^{BE} - \varepsilon$ to the target for a sufficiently small $\varepsilon > 0$. The target's payoff if merging with firm 2 will be

$$(1 - \xi_l^{BE} - \varepsilon) V_M^a(y; n-1), \tag{65}$$

which is greater than the payoff if merging with firm 1 for a sufficiently small $\varepsilon > 0$

$$(1 - \xi_1) V_M^a(y; n-1). \tag{66}$$

Thus, firm 2 will win the contest and still earns a positive surplus. This contradicts with our previous supposition. We deduce that, in equilibrium, the winning large firm will bid its breakeven share ξ_l^{BE} . The next proposition provides the solution of the takeover contest with two identical, large bidders.

Proposition 9. Consider two large firm bidders competing for a small firm target. Suppose Assumptions 1 and 2 hold. (i) The takeover takes place the first time the industry shock (Y_t) reaches the threshold value

$$y^*(n) = \sqrt{\frac{\beta X_s [\beta X_s + (\beta - 2) X_l] [r - 2(\mu + \sigma^2 / 2)]}{\beta - 2 \beta X_s [\Pi_M^a(n-1) - \Pi_l(n) - \Pi_s(n)]}}. \tag{67}$$

(ii) The share of the merged firm accruing to the winning large firm bidder is given by

$$\xi^*(n) = \frac{X_l(\beta - 2)[\Pi_M^a(n-1) - \Pi_s(n)] + \beta X_s \Pi_l(n)}{[\beta X_s + (\beta - 2) X_l] \Pi_M^a(n-1)}. \tag{68}$$

(iii) The cumulative returns to the winning large firm bidder and the small firm target are given by

$$R_{l,M}(y^*; n) = 0 \tag{69}$$

and

$$R_{s,M}(y^*; n) = \frac{2X_s[\Pi_M^a(n-1) - \Pi_l(n) - \Pi_s(n)]}{[\beta X_s + (\beta - 2) X_l] \Pi_s(x)}. \tag{70}$$

Proposition 9 establishes the results for merger timing, merger terms, and merger returns for the case of competition among two identical (large) bidders. In this case, competition is fiercer than in the case of different bidders in the previous

section, which puts the target into an even more advantageous position and hence produces even higher returns to target shareholders. Importantly, the two identical bidders' competition squeezes away all surplus to the two bidders and allows the target to extract all merger surplus.

6. Conclusion

This paper develops a real options model of mergers and acquisitions that jointly determines the industry's product market equilibrium and the timing and terms of takeovers. The analysis explicitly recognizes the role of product market competition and derives equilibrium restructuring strategies by solving an option exercise game between bidding and target shareholders. Our results are generally consistent with the extant empirical evidence. In addition, our model also generates a number of novel empirical implications:

1. Product market competition among heterogeneous firms does not speed up the acquisition process. Instead, we find that increased industry competition delays the timing of takeovers. Thus, mergers are more likely in more concentrated industries.
2. The relative scarcity of acquirers and targets in the industry is reflected in the equilibrium equity share (i.e., merger terms). In particular, a large acquirer (small target) demands a lower (higher) ownership share in the merged firm if the industry is less competitive.
3. Merger returns are higher for the smaller merging firm (e.g. target) than the larger merging firm (e.g. acquirer) when the firms have identical merger costs. Returns to merging and rival firms arising from restructuring are higher in more concentrated industries.
4. Takeovers are more likely in industries in which dispersion in firm size is higher. In addition, returns to merging and rival firms arising from restructuring are higher in industries with more dispersion in firm size.
5. With multiple bidders for a scarce target and similar merger costs, the large firm bidder wins the takeover contest when competing with a small firm bidder. Only in a sufficiently concentrated industry, bidder competition speeds up the takeover process and leads the large firm acquirer to pay a bid premium to discourage the competing small firm competitor. If it exists, this bid premium decreases with industry competition. By contrast, in a sufficiently competitive industry, the small firm bidder does not matter, and the equilibrium outcome corresponds to the one without bidder competition.
6. If the two bidding firms are identical, then bidder competition will squeeze away all surplus to the bidders and allow target shareholders to extract all merger surplus.

The rich set of predictions generated by integrating a real options model of mergers into an industry equilibrium framework suggests that further extensions of our model will be valuable. Some possible extensions would add complexity to our analysis by refining the link between industry structure and takeover activity. For example, our analysis follows most research in the literature by assuming exogenously the initiation of mergers in the sense that the merger structure (i.e., what firm merges with what firm, and which remains independent) is exogenously imposed in the absence of antitrust law. As a result, our model indicates that industry rivals benefit from mergers without having to bear the restructuring costs, leading to a so-called "free-rider problem", i.e., merging firms provide a positive externality to rival firms. This free rider problem may give firms incentives to further delay takeovers in more competitive industries.

More generally, there may be a race to merge in that multiple firms compete to merge with one firm or multiple other firms. In this case, merging firms may face a preemption threat, which would be technically challenging within our setup. As is well known in the literature on the continuous-time models of preemption games, one has to define strategies appropriately and address the issue of coordination failures (see, e.g., [Simon and Stinchcombe, 1989](#)). Some researchers study real options models of preemption games extending an early contribution by [Fudenberg and Tirole \(1985\)](#). [Weeds \(2002\)](#) presents a model in which two firms invest in competing research projects with uncertain returns. She assumes that successful innovation by one firm eliminates all possible profits for the other, giving rise to a first-mover advantage. [Mason and Weeds \(2010\)](#) study an irreversible investment problem in which two firms compete for a project. There is an advantage to being the first to invest and externalities to investing when the other firm also does so. For preemption games with two firms, there are typically two types of equilibria. In the first type, the two firms invest simultaneously. In the second type, the two firms invest sequentially. [Argenziano and Schmidt-Dengler \(2007\)](#) extend [Fudenberg and Tirole's \(1985\)](#) duopoly model to an arbitrary, finite number of firms. Their analysis is quite complex. In their case, each firm's investment time may be different, but it is also possible that some firms' investment times are clustered. One could adapt the analysis of [Argenziano and Schmidt-Dengler \(2007\)](#) to our setup with many firms. However, the solution would be more complex because we would have to consider each pair's merger decisions taking into account the endogenous change of industry structure after a merger.

Furthermore, we have to consider mergers when each firm makes individual merger decisions and responds to mergers by other firms, which is a difficult problem even in a static model (see, e.g., [Qiu and Zhou, 2007](#)). In this case, multiple mergers may arise sequentially, and hence the order of mergers may be an important element of the dynamic evolution of the industry's competitive landscape through time. Similarly, modeling new entry into and/or exit from the industry

would also have important implications that are beyond the scope of our article. Prima facie, these extensions could probably generate more or less preemptive behavior, depending on the intensities of entry, exit, etc., and hence generate less or more delays of takeovers in more competitive industries. Research that combines some of these elements with our analysis is likely to be a fruitful avenue for future research in mergers and acquisitions, and more generally, in corporate finance. Our paper can potentially serve as a stepping stone for other researchers to investigate mergers motivated by altering industry structure in the presence of competition.

Finally, another interesting extension would be to introduce a random outcome to the merger-approval process. That is, after two firms announce their intention to merge, the probability that the merger is approved by regulatory authorities would be less than one. Yet, some of the merger costs would still have to be incurred even if the merger was disallowed. While our model predicts that mergers are more likely in more concentrated industries, the probability of approval would presumably be correspondingly lower in more concentrated industries. As a result of a higher disapproval probability, the merger timing would be delayed and the merger returns would be lower.

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Appendix A

Proof of Proposition 1. In the Cournot–Nash industry equilibrium, each firm $i = 1, \dots, N$ has the objective to

$$\max_{q_i(t)} \pi_i(t), \tag{A.1}$$

while taking other firms' output strategies as given, where $\pi_i(t)$ is defined in Eq. (4). This maximization problem has the first-order condition for each firm i

$$aY(t) - bQ(t) = (b + 2k_i^{-1})q_i(t). \tag{A.2}$$

Using Eqs. (1) and (5), we can solve the system of first-order conditions in Eq. (A.2) to obtain the equilibrium expression for $q_i^*(t)$ in Eq. (7). Aggregating individual firms' output choices yields the industry's optimal output level $Q^*(t)$ in Eq. (9). The industry's equilibrium price process $P^*(t)$ in Eq. (10) immediately follows from substituting $Q^*(t)$ into Eq. (2) and simplifying. □

Derivation of HHI indexes: The Herfindahl–Hirschman (HHI) index is defined as the sum of the squares of the market shares of each firm in the industry. That is, the HHI index for an industry with n large firms (i.e., $k_i = k$) and m small firms (i.e., $k_i = k/2$) at time t is given by

$$10,000 \left[n \left(\frac{q_l(t)}{Q(t)} \right)^2 + m \left(\frac{q_s(t)}{Q(t)} \right)^2 \right]. \tag{A.3}$$

Using Proposition 1 and letting $nk + m k/2 = 1$, we can derive that the HHI index is given by

$$\frac{n(2k^{-2} - b^2) + 2(b + k^{-1})^2 k^{-1}}{[b(2k^{-1} - n) + 2k^{-2}]^2}. \tag{A.4}$$

This HHI index is monotonically increasing in the number of large firms, n . To see this, notice that the partial derivative of this HHI index w.r.t. n is

$$10,000 \frac{(b^3 k^4 n - 2b k^2 n - 6b^2 k^2 - 2b^3 k^3 - 8b k - 4)(b k^2 n - 2b k - 2)}{k^6 [b(2k^{-1} - n) + 2k^{-2}]^4}, \tag{A.5}$$

which is positive because $nk < 1$ and hence

$$(b^3 k^4 n - 2b k^2 n - 6b^2 k^2 - 2b^3 k^3 - 8b k - 4) < -2b k^2 n - 6b^2 k^2 - b^3 k^3 - 8b k - 4 < 0 \tag{A.6}$$

and

$$(bk^2n - 2bk - 2) < -bk - 2 < 0. \quad (\text{A.7})$$

Similar arguments imply that the HHI index for an industry with $n-1$ large firms (i.e., $k_i = k$), $m-1$ small firms (i.e., $k_i = k/2$), and one huge firm (i.e., $k_i = 3k/2$) at time t is given by:

$$10,000 b^2(b+k^{-1})^2 \frac{(6bk+5)(bk+2)^2 + \frac{(2-b^2k^2)n}{(bk+1)^2} \frac{k-2}{k}}{[\Delta(n) - b^2(1+2/(3bk+2)) - (b+k^{-1})(b+2k^{-1})]^2}. \quad (\text{A.8})$$

Finally, the HHI index for an industry with $n-2$ large firms (i.e., $k_i = k$), m small firms (i.e., $k_i = k/2$), and one humongous firm (i.e., $k_i = 2k$) at time t is given by

$$10,000 \frac{k(2K(bk+1)^2(2bk+1)^2 - 2k(2b^2k^2 - 1)(bk+2)^2 - k(2bk+1)^2(b^2k^2 - 2)n)}{[bk^2(n+2bk(n+1)+4) - 2K(bk+1)(2bk+1)]^2}, \quad (\text{A.9})$$

which completes the computation of the three concentration measures used in footnote 8. \square

Proof of Proposition 2. Using Proposition 1, we can derive firm i 's profits

$$\pi_i(t) = Pq_i - q_i^2/(2k_i) = \left(\frac{aY(t)}{1+B}\right)^2 \left(1 - \frac{\theta_i}{2bk_i}\right) \frac{\theta_i}{b}. \quad (\text{A.10})$$

We need to substitute the values of θ_i and k_i for a small, a large, or a merged firm to compute their profits and firm values. Substituting $k_s = k/2$ and $k_l = k$, $\theta_s = b/(b+2k^{-1})$, and $\theta_l = b/(b+k^{-1})$ into the expression of instantaneous operating profits in Eq. (A.10) produces the closed-form solutions in Eqs. (16) and (17). Using these equations to evaluate Eq. (6), we can derive the expression for equilibrium firm value $V_i(y; n)$ for $i = s, l$ that is reported in Eq. (15). \square

Proof of Proposition 3. We first find the critical value Δ^* in Eq. (19) by solving the equation

$$\Pi_l(n+1) - 2\Pi_s(n) = 0. \quad (\text{A.11})$$

Economically, Eq. (A.11) represents a breakeven condition for the incentive to merge. Note that the functional form of $\Delta(n)$ in Eq. (14) has the useful property

$$\Delta(n) = \Delta(n+1) + b^2. \quad (\text{A.12})$$

Thus, using Eqs. (16) and (17), we can write

$$\Pi_l(n+1) - 2\Pi_s(n) = \frac{a^2(b+2k^{-1})^2(b+k^{-1}/2)}{\Delta(n+1)^2} - \frac{2a^2(b+k^{-1})^3}{\Delta(n)^2}. \quad (\text{A.13})$$

When we insert Eqs. (A.12) and (A.13) into the breakeven condition in Eq. (A.11), rearrange, and simplify, we obtain

$$\frac{2a^2(b+k^{-1})^3}{\Delta(n)^2} \left[\frac{A^2}{(1-b^2/\Delta(n))^2} - 1 \right] = 0, \quad (\text{A.14})$$

where the positive constant A is given in Eq. (20). By solving Eq. (A.14) for $\Delta(n)$, we can determine the critical value Δ^* in Eq. (19). Since one can easily verify that the term $\Pi_l(n+1) - 2\Pi_s(n)$ increases with the number of large firms, we can therefore derive the three conditions for the incentive to merge in Eqs. (21)–(23). \square

Proof of Proposition 4. Using Proposition 1, we can show that the post-merger output levels of a small firm and a large firm are given by

$$q_s^a(t; n-1) = \frac{a(b+k^{-1})}{\Delta(n) - b^2 \left(1 + \frac{2}{2+3bk}\right)} Y(t), \quad (\text{A.15})$$

$$q_l^a(t; n-1) = \frac{a(b+2k^{-1})}{\Delta(n) - b^2 \left(1 + \frac{2}{2+3bk}\right)} Y(t), \quad (\text{A.16})$$

and the merged firm produces output at the level

$$q_M^a(t; n-1) = \frac{3a}{3b+2k^{-1}(b+k^{-1})(b+2k^{-1})} \frac{Y(t)}{\Delta(n)-b^2\left(1+\frac{2}{2+3bk}\right)} \tag{A.17}$$

In addition, the post-merger industry output and price are given by

$$Q^a(t; n-1) = \frac{a}{b} \left[1 - \frac{(b+k^{-1})(b+2k^{-1})}{\Delta(n)-b^2\left(1+\frac{2}{2+3bk}\right)} \right] Y(t), \tag{A.18}$$

$$P^a(t; n-1) = \frac{a(b+k^{-1})(b+2k^{-1})}{\Delta(n)-b^2\left(1+\frac{2}{2+3bk}\right)} Y(t). \tag{A.19}$$

Substituting the equilibrium output choice $q_s^a(t)$, $q_l^a(t)$, and $q_M^a(t)$ in Eqs. (A.15)–(A.17) with $k_M = 3k/2$ and $\theta_M = b/(b+\frac{2}{3}k^{-1})$ into the expression of instantaneous operating profits in Eq. (A.10) produces the closed-form solutions in Eqs. (25)–(27). Using these equations to evaluate Eq. (6), we can derive the expression for equilibrium firm value $V_f^a(y; n-1)$ for $f = s, l, M$ that is reported in Eq. (24). □

Proof of Proposition 5. The arguments are similar to the proof of Proposition 3. We first find the critical value Δ^a in Eq. (29) by solving the equation

$$\Pi_M^a(n-1) - \Pi_s(n) - \Pi_l(n) = 0. \tag{A.20}$$

Using Eqs. (16), (17) and (27), we express the breakeven condition (A.20) as follows:

$$\frac{3(b+2k^{-1})^2(b+k^{-1})^2(3b+k^{-1})/(3b+2k^{-1})^2}{\left[\Delta(n)-b^2\left(1+\frac{2}{2+3bk}\right)\right]^2} - \frac{(b+2k^{-1})^2(b+k^{-1}/2)+(b+k^{-1})^3}{\Delta(n)^2} = 0. \tag{A.21}$$

After rearranging and simplifying, we can rewrite Eq. (A.21) as

$$\frac{(b+k^{-1}/2)(b+2k^{-1})^2(b+k^{-1})^3}{\Delta(n)^2} \left[\frac{D^2}{\left(1-b^2\left(1+\frac{2}{2+3bk}\right)/\Delta(n)\right)^2} - 1 \right] = 0, \tag{A.22}$$

where the positive constant D is given in Eq. (30). By solving Eq. (A.22) for $\Delta(n)$, we can determine the critical value Δ^a in Eq. (29). Since one can easily verify that the term $\Pi_M^a(n-1) - \Pi_s(n) - \Pi_l(n)$ increases with the number of large firms, we can therefore derive the three conditions for the incentive to merge that correspond to Eqs. (21)–(23). □

Proof of Lemma 1. We use Propositions 2 and 4 to show that

$$\begin{aligned} & \Pi_M^a(n-1) - \Pi_s(n) - \Pi_l(n) \\ &= \frac{3a^2(b+2k^{-1})^2(b+k^{-1})^2(3b+k^{-1})/(3b+2k^{-1})^2}{\left[\Delta(n)-b^2\left(1+\frac{2}{2+3bk}\right)\right]^2} - \frac{a^2(b+k^{-1})^3}{\Delta(n)^2} - \frac{a^2(b+2k^{-1})^2(b+k^{-1}/2)}{\Delta(n)^2} \\ &= \frac{a^2(b+k^{-1})^3 + a^2(b+2k^{-1})^2(b+k^{-1}/2)}{\left[\Delta(n)-b^2\left(1+\frac{2}{2+3bk}\right)\right]^2} \times \left\{ D^2 - \left[1 - \frac{b^2\left(1+\frac{2}{2+3bk}\right)}{\Delta(n)} \right]^2 \right\}, \end{aligned} \tag{A.23}$$

where D is defined in Eq. (30). Under Assumptions 2 or 3, the expression in curly brackets in Eq. (A.23) is positive. In addition, since $\Delta(n)$ declines with n , we know that $\Pi_M^a(n-1) - \Pi_s(n) - \Pi_l(n)$ increases with n .

We can show the profit ratio $[\Pi_M^a(n-1) - \Pi_s(n) - \Pi_l(n)]/\Pi_f(n)$ is equal to a positive constant times the expression

$$\left[1 - \frac{b^2\left(1+\frac{2}{2+3bk}\right)}{\Delta(n)} \right]^{-2} \left\{ D^2 - \left[1 - \frac{b^2\left(1+\frac{2}{2+3bk}\right)}{\Delta(n)} \right]^2 \right\}, \tag{A.24}$$

which, under Assumptions 2 or 3, increases with n . □

Proof of Proposition 6. Using Eq. (31), we can derive

$$OM_l(y, y_l^*, \zeta_l; n) = [\zeta_l V_M^a(y_l^*; n-1) - V_l(y_l^*; n) - X_l] \mathbb{E}^y [e^{-r\tau_{y_l^*}}], \tag{A.25}$$

for some undetermined threshold $y_l^* \geq y$. By Karatzas and Shreve (1999), we know that

$$\mathbb{E}^y [e^{-r\tau_{y_l^*}}] = \left(\frac{y}{y_l^*}\right)^\beta, \tag{A.26}$$

where β is the positive root of Eq. (33). Thus, we obtain Eq. (32). Solving the first-order condition (34), we obtain Eq. (35). Similarly, we can derive Eq. (38). We can then use Eqs. (35) and (38) to determine the equilibrium sharing rule ζ_l^* in Eq. (40) by solving the equation $y_l^* = y_s^*$ for ζ_l . Substituting ζ_l^* into either Eq. (35) or Eq. (38) yields the option value-maximizing merger threshold reported in Eq. (39). Finally, the comparative statics results in parts (ii) and (iv) of the proposition follow from Lemma 1 and Propositions 2 and 4. □

Proof of Proposition 7. Evaluating Eqs. (46) and (48) at the merger threshold $Y(t) = y^*$ and using the results from Propositions 4 and 6, we find the cumulative stock returns in Eqs. (49) and (50). Finally, consider part (iii). By Lemma 1 and Assumptions 2 and 3, the cumulative stock returns to the small and large merging firms are positive and increase with n . Since $\Delta(n) > 0$ and drops with n by Eq. (14), it follows from Eq. (50) that the cumulative returns to rival firms are also positive and increase with n . □

Proof of Proposition 8. We first consider part (i), where the large firm bidder wins the contest, and the small firm bidder is too weak to matter. In this case, the equilibrium is the same as that with a single large firm bidder. The equilibrium merger threshold y^* and ownership share ζ_l^* of the large firm bidder are given in Proposition 6. Using Eq. (52), we can derive the breakeven share of the small firm bidder at the merger threshold y^*

$$\zeta_s^{BE} = \frac{X_s + V_s(y^*; n)}{V_l(y^*; n+1)} = \frac{X_s + \Pi_s(n)(y^*)^2 / (r - 2(\mu + \sigma^2/2))}{\Pi_l(n+1)(y^*)^2 / (r - 2(\mu + \sigma^2/2))} = \frac{(\beta - 2)X_s[\Pi_M^a - \Pi_l(n) - \Pi_s(n)] + \beta(X_s + X_l)\Pi_s(n)}{\beta(X_s + X_l)\Pi_l(n+1)}. \tag{A.27}$$

For condition (54) to hold, we must have

$$(1 - \zeta_l^*)\Pi_M^a > (1 - \zeta_s^{BE})\Pi_l(n+1). \tag{A.28}$$

Substituting Eq. (A.27) for ζ_s^{BE} and Eq. (40) for ζ_l^* into Eq. (A.28) yields Eq. (58). We thus obtain part (i) of the proposition.

We next consider part (ii), where the large firm bidder wins the contest, and the small firm bidder matters. Using Eqs. (52) and (56) as well as Propositions 2 and 4, we can derive the merger threshold selected by the winning large firm bidder as a function of ζ_l^{\max} :

$$y_B(\zeta_l^{\max}) = \sqrt{\frac{X_s(r - 2(\mu + \sigma^2/2))}{\Pi_l(n+1) - \Pi_s(n) - (1 - \zeta_l^{\max})\Pi_M^a(n-1)}}. \tag{A.29}$$

Using Eq. (38), we can derive the merger threshold selected by the small firm target's given the winning bidder's demanded ownership share ζ_l^{\max}

$$y_T(\zeta_l^{\max}) = \sqrt{\frac{\beta X_s}{\beta - 2} \frac{r - 2(\mu + \sigma^2/2)}{(1 - \zeta_l^{\max})\Pi_M^a(n-1) - \Pi_s(n)}}. \tag{A.30}$$

In equilibrium, the winning bidder and the target must agree on the merger timing in that $y_B(\zeta_l^{\max}) = y_T(\zeta_l^{\max}) \equiv y_{cl}^*$. We can then solve the system of Eqs. (A.29) and (A.30) for the winning bidder's ownership share ζ_l^{\max} and the merger threshold y_{cl}^* when the losing small firm bidder is strong. They are given in Eqs. (60) and (61).

For part (ii) to happen, condition (58) must be violated. We thus obtain the first inequality in (59). In addition, the large firm bidder must be willing to participate. That is, we require $\zeta_l^{\max} > \zeta_l^{BE}$, and use Eq. (53) to compute the breakeven share at the equilibrium trigger y_{cl}^*

$$\zeta_l^{BE} = \frac{X_l + V_l(y_{cl}^*; n)}{V_M^a(y_{cl}^*; n-1)} = \frac{(\beta - 2)X_l[\Pi_l(n+1) - 2\Pi_s(n)] + 2(\beta - 1)X_s\Pi_l(n)}{2(\beta - 1)X_s\Pi_M^a(n-1)}, \tag{A.31}$$

where y_{cl}^* is given in Eq. (60). Substituting Eq. (A.31) in Eq. (61) for ζ_l^{\max} into the condition $\zeta_l^{\max} > \zeta_l^{BE}$, we obtain the second inequality in (59). Evaluating Eq. (46) for $f=s$ and for $f=l$ at the merger threshold $Y(t) = y_{cl}^*$ in Eq. (60), when the large firm bidder receives the share ζ_l^{\max} in Eq. (61), and using the results from Propositions 2 and 4, we find the cumulative stock returns in Eqs. (62) and (63). We thus obtain part (ii). The computation of cumulative stock returns follows the steps outlined in the proof of Proposition 7. □

Proof of Proposition 9. We have shown in Section 5.2 that the equilibrium bid ξ^* is given by $\xi^* = \xi_l^{BE}$. Using Eq. (53), we can derive the bidder's reaction function as

$$y_B(\xi^*) = \sqrt{\frac{X_l(r-2(\mu+\sigma^2/2))}{\xi^* \Pi_M^a(n-1) - \Pi_l(n)}} \quad (\text{A.32})$$

Using Eq. (38), we can derive the merger threshold selected by the small firm target's given the winning bidder's demanded ownership share ξ^*

$$y_T(\xi^*) \equiv \sqrt{\frac{\beta X_s}{\beta - 2} \frac{r-2(\mu+\sigma^2/2)}{(1-\xi^*) \Pi_M^a(n-1) - \Pi_s(n)}} \quad (\text{A.33})$$

In equilibrium, the winning bidder and the target must agree on the merger timing in that $y_B(\xi^*) = y_T(\xi^*) \equiv y^*$. We can then solve the system of Eqs. (A.32) and (A.33) for the winning bidder's ownership share ξ^* and the merger threshold y^* when the losing bidder is a large firm. They are given in Eqs. (67) and (68). The computation of cumulative stock returns follows the steps outlined in the proof of Proposition 7. \square

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