Chapter 26 - Pricing Options

26-1:

The leveraged stock and option decision trees are given as follows:

Three options are required to balance the leveraged stock so

A. The fair market value of the option is $8.577

\[ C = \frac{50 - \frac{25.00}{1.03}}{3} = \frac{50 - 24.27}{3} = \frac{8.57605178}{3} = \frac{8.58}{3} \]

B. The value of the leveraged stock is $75 or $0; the value of the option is $25 or $0. Thus it takes three options to balance one share. The hedge ratio is 3.

C. The fair market value of the option is $8.577; the option is trading at $7 so we buy the option low and sell the option equivalent high.

Buy 3 Discovery Café (DVC) $75 call option contracts
(3 contracts * 100 shares/contract * $7/share) ($2,100.00)

Sell 100 shares Discovery Café (DVC) @ $50 ($7500 margin deposited in interest bearing securities) $5,000.00

Loan at 3% ($2,427.18)

$472.82
D.
If Discovery Café (DVC) goes to $100
   Exercise calls buying 300 shares at $75  ($22,500.00)
   Cover the short position and retrieve margin
   Sell 200 shares @ $100  $20,000.00
   Collect repayment of $2,427.18 loan with 3% interest  $2,500.00
                                     $0.00

If Discovery Café (DVC) goes to $25
   Calls expire worthless
   Buy 100 Discovery Café at $25  ($2,500.00)
   Cover the short position and retrieve margin
   Collect repayment of $2,427.18 loan with 3% interest  $2,500.00
                                     $0.00

This works out the way we expected because we were able to deposit the 150% margin required on the short position in interest bearing securities. Abstracting from the margin requirement allows us to confirm the theoretical value of this strategy at $472.82

\[
\pi = 3 \text{ contracts} \times \frac{100 \text{ shares}}{\text{contracts}} \times \frac{8.57605178 - 7}{\text{share}} = 472.82
\]

More realistically we would have
   Buy 3 Discovery Café (DVC) calls @ $7  ($2,100.00)
   Sell 100 Discovery Café (DVC) @ $50  $5,000.00
   150% margin  ($7,500.00)
   Borrow $4,854.37 at 3%  $4,854.37
                                     $254.37

If Discovery Café (DVC) goes to $100
   Exercise calls buying 300 shares at $75  ($22,500.00)
   Cover the short position and retrieve margin  $7,500.00
   Sell 200 shares @ $100  $20,000.00
   Repay Loan  ($5,000.00)
                                     $0.00

If Discovery Café (DVC) goes to $25
   Calls expire worthless
   Buy 100 Discovery Café at $25  ($2,500.00)
   Cover the short position and retrieve margin  $7,500.00
   Repay $2,427.18 loan with 3% interest  ($5,000.00)
                                     $0.00

The loan of $4,854.37 comes from the model itself. The portfolios are constructed to break even regardless of outcome. We calculate the amount of the loan we must repay ($5,000.00) and then borrow the Net Present Value of that amount.
26-2:

The leveraged stock and option decision trees are given as follows:

Two options are required to balance the leveraged stock so

A. The fair market value of the option is $26.19

\[
C = \frac{\frac{100 - 50.00}{1.05}}{2} = \frac{100 - \frac{47.62}{1.05}}{2} = \frac{100 - 47.62}{2} = 26.19
\]

B. The value of the leveraged stock is $150 or $0; the value of the option is $75 or $0. Thus it takes two options to balance one share. The hedge ratio is 2

\[
\text{Hedge ratio} = \frac{200 - 50.00}{(200 - 125) - 0} = 2
\]

C. The call option is trading at $28 so we buy the option equivalent low ($26.19) and sell the option high ($28). The theoretical value of the strategy is $361.90 (2 * 100 * (28 - 26.190476)). Our initial outlay is thus -$361.90. The investment is negative since we begin by collecting more money than we pay out.

Sell 2 Tardis Intertemporal (TI) calls @$28 $5,600.00
Buy 100 Tardis Intertemporal (TI) @$100 ($10,000.00)
Borrow at 5% $4,761.90

\[
\text{Total Initial Outlay} = 5,600 + (-10,000) + 4,761.90 = -3,658.10
\]

\[
\text{Theoretical Value of Strategy} = 2 \times 100 \times (28 - 26.190476) = 361.90
\]

\[
\text{Initial Outlay} = -3,658.10
\]

\[
\text{Strategy Value} = 361.90
\]

\[
\text{Hedge Ratio} = 2
\]

\[
\text{Fair Market Value of Option} = 26.19
\]
D.

If Tardis Intertemporal (TI) goes to $200

- Calls exercised generating a loss of $200-$125 per share ($15,000.00)
- Sell 100 TI @ $200 ($20,000.00)
- Repay $4,761.90 loan with 5% interest ($5,000.00)

Total: $0.00

If Tardis Intertemporal (TI) goes to $50

- Calls expire worthless
- Sell 100 TI at $50 ($5,000.00)
- Repay $4,761.90 loan with 5% interest ($5,000.00)

Total: $0.00

Note that this form of arbitrage, because it focuses on calculating the price of the option, forces the future value to zero. If you set the present value to zero, which you can do because you are given the price of the option, you should be able to calculate the profit of $380.00 on a net investment of $0.00. You should also be able to confirm that at 5% $361.90 is the present value of $380.00 in one year.

26-3:

\[
d_1 = \left[ \ln \left( \frac{90}{100} \right) + \left( 0.05 + \frac{0.50^2}{2} \right) \right] \frac{0.25}{0.50 \sqrt{0.25}}
\]

\[
d_1 = -0.1054 + 0.04375
\]

\[
d_1 = -0.2466
\]

\[
M(d_1) = M[-0.25] = 0.40129
\]

\[
d_2 = -0.2466 - 0.50/0.25
\]

\[
d_2 = -0.4966
\]

\[
M(d_2) = M[-0.50] = 0.30855
\]

\[
P_{\text{Call}} = P_{\text{Stock}} N(d_1) - P_{\text{Exercise}} M(d_1) e^{-rt}
\]

\[
P_{\text{Call}} = 90 \times 0.40129 - 100 \times 0.30855 \times e^{-0.08 \times 0.25}
\]

\[
P_{\text{Call}} = 36.1161 - 30.471713
\]

\[
P_{\text{Call}} = 5.644387
\]
A. With the increased volatility of the underlying stock we should be willing to pay $5.64 rather than the $3.93 calculated for this call option. This is the same as saying that you would be willing to more in auto theft insurance when the probability of having your car stolen increases.

B. There is a 30.9% probability that the call will expire in-the-money.

C. 

26-4:

When volatility increases from 40% to 50% the fair market value of the put increases from $12.69 to $14.40

$$ P_{Put} = P_{Call} - P_{Stock} + [ P_{Exercise} + D ] e^{-rt} $$

$$ = \$5.64 - \$90 + \$100 \ e^{-0.05 \times 0.15} $$

$$ = \$14.40 $$
26-5:

\[ d_1 = \frac{\ln \left( \frac{90}{80} \right) + \left( 0.05 + \frac{0.40^2}{2} \right) \times 0.25}{0.40 \sqrt{0.25}} \]

\[ = \frac{0.11778 + 0.0325}{0.20} \]

\[ = 0.7514 \]

\[ M(d_1) = M(0.75) = 0.77337 \]

\[ d_2 = d_1 - 0.40 \sqrt{0.25} \]

\[ = 0.5514 \]

\[ M(d_2) = M(0.55) = 0.70884 \]

\[ P_{Call} = P_{Stock} N(d_1) - P_{Exercise} N(d_2) e^{-rt} \]

\[ = $90 \left[ 0.77337 \right] - $80 \left[ 0.70884 \right] e^{-0.05 \cdot 0.25} \]

\[ = $90 \left[ 0.77337 \right] - $80 \left[ 0.70884 \right] 0.9875778 \]

\[ = $69.6093 - $56.00277 \]

\[ = $13.6065 \]

A. We should be willing to pay $13.60

B. \( \Delta = 0.77 \) This means that the value of the call increases by $0.77 every time the price of Hypothetical resources increases by $1. So if HR moves from $90 to $91 then the fair market value of the $80 call increases from $13.60 to $14.37.

C. There is a 70.9% probability that the call will expire in the money.

\[ P_{Put} = P_{Call} - P_{Stock} + \left[ P_{Exercise} + D \right] e^{-rt} \]

\[ = $13.60 - $90 + $80 e^{-0.05 \cdot 0.25} \]

\[ = $2.61 \]

D. We should be willing to pay $2.61

E. If the $80 call has a 70.9% probability of expiring in-the-money then the $80 put has a 29.1% probability of expiring in-the-money.
26-6:

\[ d_1 = \frac{\ln \left( \frac{90}{90} \right) + \left( 0.05 + \frac{0.40^2}{2} \right) 0.25}{0.40 \sqrt{0.25}} \]

\[ = \frac{0.0000 + 0.0325}{0.20} \]

\[ = 0.1625 \]

\[ N(d_1) = N(0.16) = 0.56357 \]

\[ d_2 = 0.1625 - 0.40\sqrt{0.25} \]

\[ = -0.0375 \]

\[ N(d_2) = N(-0.04) = 0.48404 \]

\[ P_{Call} = P_{Stock} N(d_1) - P_{Strike} N(d_2) e^{-rt} \]

\[ = \$90 \left[ 0.56357 \right] - \$90 \left[ 0.48404 \right] e^{-0.05\times0.25} \]

\[ = \$90 \left[ 0.56357 \right] - \$90 \left[ 0.48404 \right] 0.9875778 \]

\[ = \$50.7213 - \$43.0224 \]

\[ = \$7.6988 \]

A. We should be willing to pay $7.70

B. \( \Delta = 0.56 \) This means that the value of the call increases by $0.56 every time the price of Hypothetical resources increases by $1. So if HR moves from $90 to $91 then the fair market value of the $80 call increases from $7.70 to $8.26.

C. There is a 48.4% probability that the call will expire in the money. The option is right-on-the-money, so we expect this probability to be 50%. We can try to blame the rounding we do when looking up the normal distribution, but even calculated to 16 decimal places we calculate an \( N(d_1) \) of 48.5%. There is actually a 3.2% probability that the shares end up at $90. So there is a 48.4% probability that the call expires in-the-money, a 48.4% probability that the put expires in-the-money, and a 3.2% probability that both the call and the put expire right on the money and worthless.

D. We should be willing to pay $6.58

\[ P_{Put} = P_{Call} - P_{Stock} + \left[ P_{Strike} + D \right] e^{-rt} \]

\[ = \$7.70 - \$90 - \$90 e^{-0.05\times0.25} \]

\[ = \$6.582 \]

E. 48.4%