Chapter 3 - Measuring Portfolio Risk

3-1:

Just expand the standard formula and regroup. This question should bring back fond memories of high school algebra.

\[ \sigma^2 = E[(X - E[X])^2] \]

\[ \sigma^2 = E[X^2] - 2 \cdot E[X] \cdot E[X] + E[X]^2 \]

\[ \sigma^2 = E[X^2] - 2 \cdot E[X] \cdot E[X] + E[X]^2 \]

\[ \sigma^2 = E[X^2] - 2 \cdot E[X]^2 + E[X]^2 \]

\[ \sigma^2 = E[X^2] - E[X]^2 \quad \text{Q.E.D.} \]
3-2:

To visualize the risk associated with these stocks we use the properties of the normal distribution: that an observation lies

within one standard deviation of the mean 68% of the time,
within two standard deviations of the mean 95% of the time, and
within three standard deviations of the mean 99.7% of the time.

<table>
<thead>
<tr>
<th></th>
<th>mean</th>
<th>Std Dev</th>
<th>Var</th>
<th>-3σ</th>
<th>-2σ</th>
<th>-1σ</th>
<th>Mean</th>
<th>+1σ</th>
<th>+2σ</th>
<th>+3σ</th>
<th>CV</th>
</tr>
</thead>
<tbody>
<tr>
<td>PG</td>
<td>1.07%</td>
<td>3.43%</td>
<td>0.0012</td>
<td>-9.2%</td>
<td>-5.79%</td>
<td>-2.36%</td>
<td>1.07%</td>
<td>4.50%</td>
<td>7.93%</td>
<td>11.36%</td>
<td>3.206</td>
</tr>
<tr>
<td>BA</td>
<td>2.42%</td>
<td>3.58%</td>
<td>0.0013</td>
<td>-8.3%</td>
<td>-4.74%</td>
<td>-1.06%</td>
<td>2.42%</td>
<td>6.00%</td>
<td>9.58%</td>
<td>13.16%</td>
<td>1.479</td>
</tr>
<tr>
<td>GOOG</td>
<td>6.24%</td>
<td>11.31%</td>
<td>0.0128</td>
<td>-27.7%</td>
<td>-16.38%</td>
<td>-5.07%</td>
<td>6.24%</td>
<td>17.55%</td>
<td>28.86%</td>
<td>40.17%</td>
<td>1.813</td>
</tr>
</tbody>
</table>

Just by looking at the standard deviation, we can see that Google is much riskier than either PG or Boeing. The graph makes this even more obvious. If we define a really bad day as a three sigma event then we would lose 27.7% on Google but we would lose only 9.2% and 8.3% on P&G and Boeing respectively.

*Figure (Answers) 3-2:*

However, the coefficient of variation shows that PG is the riskier stock relative to the expected return. We can see that the standard deviation for Boeing is almost the same as that for PG, but the expected return for Boeing is more than twice the expected return for PG.